STABILIZATION OF HYPERBOLIC PLYKIN ATTRACTOR BY THE PYRAGAS METHOD

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In the present contribution consideration is being given to an autonomous physical system which is characterized by the presence of the attractor of a hyperbolic type. We study the possibility of controlling and stabilizing the Plykin attractor of this type by the Pyragas method. The choice of the method of control. As such it is possible to use an external signal or the introduction of additional delayed feedback. (Both methods can be realized primarily during the schematic simulation then in a real experiment). It might also be interesting to think about the realization of a more complicated scheme of control of the type suggested in the work for the stabilization of unstable periodic orbits belonging to the attractor.

Introduction

Hyperbolicity is a fundamental feature of chaotic systems. It is as follows: a tangent space \( \Sigma \) of such systems is a combination of three subspaces; stable \( E^s \), unstable \( E^u \) and \( E^0 \) neutral. Close trajectories which correspond to converge exponentially to \( E^s \) each other when \( t \to +\infty \), and those which correspond to \( E^u \) - when \( t \to -\infty \). In the subspace \( E^0 \) the vectors contract and expand more slowly than the exponential velocity. When the degree of contraction and expansion in the subspaces \( E^s \) and \( E^u \) changes from point to point along the trajectory, such systems are called non-uniformly hyperbolic. Dynamic systems with uniform hyperbolicity of all the trajectories are called Anosov systems.

The set \( \Lambda \) is called a hyperbolic attractor of a dynamic system if \( \Lambda \) - is a closed topologic transitive hyperbolic set and there exists such a vicinity \( U \supset \Lambda \) that \( \Lambda = \bigcup_{k=0}^{\infty} f^k U \). Smale - Williams’ solenoid and Plykin’s attractor [1] are well-known hyperbolic attractors. Plykin’s sphere is obtained by the transformation of the disc domain into itself where \( S^2 \) - a unitary disc in \( R^2 \). Then \( f : T \to T, f(x, y, z) = (\cos \varphi \sin \phi, \sin \varphi \sin \phi, \cos \phi) \), where \( k>2 \) determines the compression “by thickness”, sets the disc as a subset \( T \subset R^3 \). Let there be a smooth family of non-linear controlled systems of ordinary differential equations \( \dot{x} = F(x, \mu, u), x \in M \subset R^n, \mu \in L \subset R^k, u \in U \subset R^m, F \in C^\infty \) depending on the vector of controlling parameters \( u \). Suppose that it is necessary to stabilize unstable limiting cycle of the period \( T \), which is the solution of the family when \( u = 0 \) and \( \mu = \mu^* \). Let the system have a regular attractor when the parameters are of the same value \( u = 0 \) a \( \mu = \mu^* \). Then the stabilization of the cycle \( x^*(t, \mu^*) \) is carried out by means of the feedback with the delay being in the form of \( u(t) = K(x(t-T) - x(t)) \), where \( K \) - is the matrix of coefficients. There-with the initial conditional \( x(0) \) is chosen in a sufficiently small vicinity of the cycle. Then the solution \( x(t) \) of the system \( \dot{x} = F(x(t), \mu^*, K(x(t-T) - x(t))) \) with the feedback with \( \mu = \mu^* \) can converge to the sought - for unstable cycle \( x^*(t, \mu^*) \) [2].
The use of the Pyragas method for the formation of regular dynamics in autonomous hyperbolic attractors

Let us take into consideration the system of the type [1]:

\[
\begin{align*}
\dot{X} &= -2\epsilon Y^2 \Omega_1 \left( \cos(\omega_2 \cos \omega_1 t) - X \sin(\omega_2 \cos \omega_1 t) \right) + \\
&\quad kY \Omega_2 \left( \cos(\omega_2 \sin \omega_1 t) - X \sin(\omega_2 \sin \omega_1 t) \right) \sin \omega_1 t, \\
\dot{Y} &= 2Y \Omega_1 \left( X \cos(\omega_2 \cos \omega_1 t) + 2^{-1}(1 - X^2 + Y^2) \sin(\omega_2 \cos \omega_1 t) \right) - \\
&\quad k \Omega_2 \left( X \cos(\omega_2 \sin \omega_1 t) + 2^{-1}(1 - X^2 + Y^2) \sin(\omega_2 \sin \omega_1 t) \right) \sin \omega_1 t + F(K, \tau), \\
\Omega_1 &= (2X \cos(\omega_2 \cos \omega_1 t) + (1 - X^2 - Y^2) \sin(\omega_2 \cos \omega_1 t)) (1 + X^2 + Y^2)^{-2}, \\
\Omega_2 &= (-2X \sin(\omega_2 \sin \omega_1 t) + (1 - X^2 - Y^2) \cos(\omega_2 \sin \omega_1 t)) (1 + X^2 + Y^2)^{-1} + 2^{-1/2}.
\end{align*}
\]

Here \(X, Y\) - dynamic variables, \(\epsilon\) and \(k\) - coefficient of connection, \(\omega_{1,2} = (\pi/2, \pi/4)\) - inherent frequency oscillations, \(F_{Y,\tau} = K[Y(t - \tau) - Y(t)]\).

Result

The phase portraits and Fourier spectrums presented demonstrate the behavior of the system (1). \(K = 0\) corresponds to the chaos Fig. 1, Fig. 2 corresponds to \(K = 1.8\) and \(\tau = 1.8\) the stable state, corresponds to the hyperbolic chaotic state. The blank part with the given values of the parameters depicts chaotic features. In dark areas the dynamics of the system undergoes qualitative changes, and the hyperbolic attractor degenerates into the limiting cycle, and the spectrum from the continuous corresponding to chaotic oscillations changes into an equidistant one with the frequencies corresponding to the basic frequency and its harmonics.

References


Figure 1: On the left a phase portrait of the system (1) and corresponding spectral density, and on the right the dynamic variable is represented on the scale of time are shown when $\epsilon = 0.72, k = 1.9$.

Figure 2: The system (1) corresponding to the spectral density is shown when $\tau = 1.8, K = 1.8, \epsilon = 0.72, k = 1.9$