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Statistical self-similarity of spatial variations of snow cover: verification of the hypothesis and application in the snowmelt runoff generation models

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Abstract:

An analysis of snow cover measurement data in a number of physiographic regions and landscapes has shown that fields of snow cover characteristics can be considered as random fields with homogeneous increments and that these fields exhibit statistical self-similarity. A physically based distributed model of snowmelt runoff generation developed for the Upper Kolyma River basin (the catchment area is about 100 000 km^2) has been used to estimate the sensitivity of snowmelt dynamics over the basin and flood hydrographs to the parameterization of subgrid effects based on the hypothesis of statistical self-similarity of the maximum snow water equivalent fields. Such parameterization of subgrid effects enables us to improve the description of snowmelt dynamics both within subgrid areas and over the entire river basin. The snowmelt flood hydrographs appear less sensitive to the self-similarity of snow cover over subgrid areas than to the dynamics of snowmelt because of a too large catchment area of the basin under consideration. However, for certain hydrometeorological conditions and for small river basins this effect may lead to significant changes of the calculated hydrographs. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS snow; self-similarity; distributed modelling; runoff

INTRODUCTION

A distinctive feature of snow cover is its high spatial variability. The small-scale variations of snow depth and density caused by spatial variation in topography, vegetation, and local meteorological conditions are superimposed on the large-scale variability of snowfall associated with global atmospheric circulation and physiographic zones. Distributed numerical modelling of snowmelt runoff generation is based on dividing a river basin into grid cells. Within each cell, the model inputs and coefficients are considered to be spatially uniform. The absence of data representing the small-scale subgrid variations of snow cover characteristics may lead to errors in modelling snowmelt runoff generation even for small grid sizes. For example, it is very difficult to take into account the spatial variations of snow cover induced by ravine networks, small-scale variation of mountainous relief, and differences in types and density of vegetation. The measurements of snow depths or snow water equivalents are commonly carried out irregularly in space, and do not often provide direct representation of the spatial snow variability that is necessary for correct estimation of the river basin water balance or for describing the influence of snow on the runoff generation processes.

It is well established that the spatial variations of snow depths and snow water equivalents for a given region can be considered as random fields variables for which areal statistical distributions follow lognormal or two-parametric gamma probability laws (e.g. Kuzmin, 1963; Gottschalk and Jutman, 1979; Kuusisto, 1984; Killingtveit and Sand, 1991; Shook and Gray, 1997). However, these random fields may be strongly heterogeneous, and the statistical parameters needed for construction of probability distributions may vary

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in space and as a function of the area size. The number of measurement points needed for reasonable estimation of spatial variance or higher statistical moments is commonly sufficient only for areas that are significantly larger than the grid cells of numerical runoff models or, *vice versa*, only for a small part of these grid cells. Thus, it is necessary to assign the statistical parameters for domains without measurements or transfer these parameters from the larger or smaller domains. An opportunity to solve this problem is presented by investigating regularities in the stochastic spatial structure of snow characteristics and searching for relationships between variations of these characteristics for different spatial scales.

Early work in this field includes the theory of random fields with homogeneous increments in meteorology (Gandin, 1963; Monin and Jaglom, 1967), and in geology the development of geostatistics (Matheron, 1967). The theory of fractals (see Mandelbrot (1982) and references cited therein) has also been used successfully to understand the relationships between statistical parameters of different spatial scales. The linkages between geostatistical characterization of random fields and their fractal models have been demonstrated (Bruno and Raspa, 1989). Both these approaches, under the certain property of random fields commonly called statistical self-similarity, enabled us to transfer statistical parameters for one scale to statistical parameters for other scales using a simple scaling transformation of the random variable. Thus, the problem of accounting for subgrid variations of a spatial variable was simplified by applying the hypothesis on the statistical self-similarity. The fractal theory and the hypothesis on the statistical self-similarity has been applied for modelling of random fields of elevations (Mark and Aronson, 1984), rainfall characteristics (Lovejoy and Mandelbrot, 1985), soil constants (Burrough, 1983), and a number of other geophysical variables (Kundzewicz, 1995).

The fractal and statistical self-similarity concepts were also used successfully for some research on the spatial structure of snow cover; however, most of these investigations were associated with analysis of snow covered areas. Fractal geometry was used for describing snow patches and change in snow covered areas during the snowmelt processes (Shook and Gray, 1996, 1997). Based on the analysis of the four sets of snow covered areas variograms, derived from remotely sensed snow cover images and images obtained in the laboratory, Blöschl (1999) demonstrated that snow covered areas have fractal properties that are preserved for a large range of spatial scales. Shook and Gray (1996), using the snow measurement data along 1000 m transects on flat stubble fields near Saskatoon, Canada, showed that the spatial distribution of snow depth and snow water equivalent were fractals for small scales. They also demonstrated that the scale for which the snow cover was fractal depended on the topography and vegetation. The objectives of our study are the following: (1) to verify the hypothesis of statistical self-similarity for snow cover for wide ranges of spatial scales and physiographic conditions; (2) to use relationships following from this property for parameterization of subgrid effects in a distributed model of snowmelt runoff generation; and (3) to determine the effectiveness of this parameterization for improving the modelling of snowmelt runoff.

THEORETICAL BACKGROUND

According to Gupta and Waymire (1990), statistical self-similarity is a property of a given random variable $S(x)$, where the probability distribution of $S(x)$ within any cell F_k of an area F is the same as the distribution over the whole area F, if a scaling transformation of this variable within F_k is made. An example of a scaling transformation is when the variable $S(x)$ is multiplied by a factor r^H, where r is a constant depending on the ratio of F_k to F and H is a constant depending on a measure of spatial correlation of $S(x)$. In this case, the conditions of the equality of probability distributions of $S(x)$ within the areas F_k and F can be presented as the following relation between corresponding statistical moments $E[{S_k}^n]$ and $E[{S_F}^n]$ of order n:

$$E[{S_k}^n] = r^{nH} E[{S_F}^n] \tag{1}$$

As seen from Equation (1), if we confine ourselves to the first two moments, then one of the conditions of the statistical self-similarity is that the coefficient of spatial variation is a constant and does not vary with size of area under consideration. Kuchment *et al.* (1986) used this assumption to take into account the subgrid variations of saturated hydraulic conductivity, however, without proper theoretical interpretation and validation.

If the random field $S(x)$ is heterogeneous, but its increment $I(h) = S(x+h) - S(x)$ is homogeneous and isotropic, then it is possible to construct the variogram

$$\gamma(h) = E[S(x+h) - S(x)]^2 \tag{2}$$

where $S(x)$ and $S(x+h)$ are values of the random variable at points x and $x+h$ respectively.

If the variogram of the value of $S(x)$ has the power structure

$$\gamma(h) = \alpha h^{2H} \tag{3}$$

where α and H are constants, then for the increments with steps of h and rh the following equality can be written:

$$I(rh) = r^H I(h) \tag{4}$$

Determining the statistical moments for both sides of Equation (4), we derive Equation (1). Therefore, the random variables whose variograms demonstrate power structure are statistically self-similar.

An example of the self-similar random variable is the Brownian random process with the increments $I(h)$ being the Gaussian white noise and the variogram expressed by the function in Equation (3) at $H = 0{\cdot}5$. In a more general model of a self-similar random process suggested by Mandelbrot (see references in Mandelbrot (1982)) and called the fractional Brownian process, the variogram was represented by the function in Equation (3) at $0 < H < 1$. If $H > 0{\cdot}5$, the increments of this process are positively correlated and large-scale variations prevail; if $H < 0{\cdot}5$, the increments are negatively correlated and small-scale variations prevail. As a measure of irregularity of a random surface and correlation of the large-scale and small-scale variations, Mandelbrot also introduced the fractal dimension D such that

$$D = E + 1 - H \tag{5}$$

where E is the topological dimension. Therefore, it is possible to estimate the fractal dimension of a fractal surface by studying the fractal dimension of its section or points along a straight line. According to Matheron (1967) and Feder (1988) by averaging the variogram in Equation (3) over two embedded circles or squares with areas F and F_k having a centre at point x_0, we obtain:

$$m_k - S_0 = r^H (m_F - S_0) \tag{6}$$

$$\sigma_k^2 = r^{2H} \sigma_F^2 \tag{7}$$

where $S_0 = S(x_0)$; m_k and σ_k^2 are respectively the mean and the variance of $S(x)$ over the area F_k; m_F and σ_F^2 are respectively the mean and the variance of $S(x)$ over the area F; $r = \sqrt{F_k/F}$. These relations allow one to carry out scaling of statistical parameters of $S(x)$ at relevant choice of embedded areas and $S(x_0)$. If one assigns $S(x_0) = 0$, then the coefficient of spatial variation of $S(x)$ for F_k and F is equal. If there are measured values of m_k and m_F, it is possible to carry out only scaling of the variance, using Equation (7).

VERIFICATION OF THE HYPOTHESIS ON STATISTICAL SELF-SIMILARITY OF SNOW COVER

The available information about snow cover distribution includes two forms of data: (1) point measurements along straight-line snow courses, and (2) snow cover data calculated by averaging the snow course measurements. The snow courses are commonly chosen to represent the micro- and meso-scale variability of snow cover for different types of landscape and relief. The required length of snow courses varies from 100 m to several kilometres, and the distance between sampling points can be 10, 25, 50 or 100 m depending on local conditions (e.g. Kuzmin, 1963; Garstka, 1964; Kuusisto, 1984). To analyse the stochastic structure of spatial snow characteristics, it was necessary to have sufficient number of snow courses inside the area under consideration. In this case, the linear scales of the area covered by snow are usually more than 15 to 20 km. Taking into account these peculiarities of snow cover measurements, we investigated the statistical structure of snow cover for the snow courses and for the areas separately.

Figure 1a represents a typical example of the snow depth variation along the 7 km snow course on an open plot near Moscow, based on the snow measurements carried by Moscow University Hydrological Department (the distance between data points was 25 m). The variabilities of the snow depth over the first 700 m of this snow course using a sample spacing of 25 m and over the entire snow course using a sample spacing of 250 m are shown in Figure 1b and c respectively. As seen in Figure 1, the snow depth variability over this snow course at different spatial scales had a similar structure.

Figure 2 shows the variogram of the snow depth variations for the snow course under consideration together with the variograms of snow depth variations in other landscapes constructed on the basis of the snow measurements in the following regions: (a) Tien-Shan region (mountainous terrain); (b) North of Alaska (tundra); (c) Valday region (forest zone); (d) the Oka River basin (forest zone); (e) the Don River basin

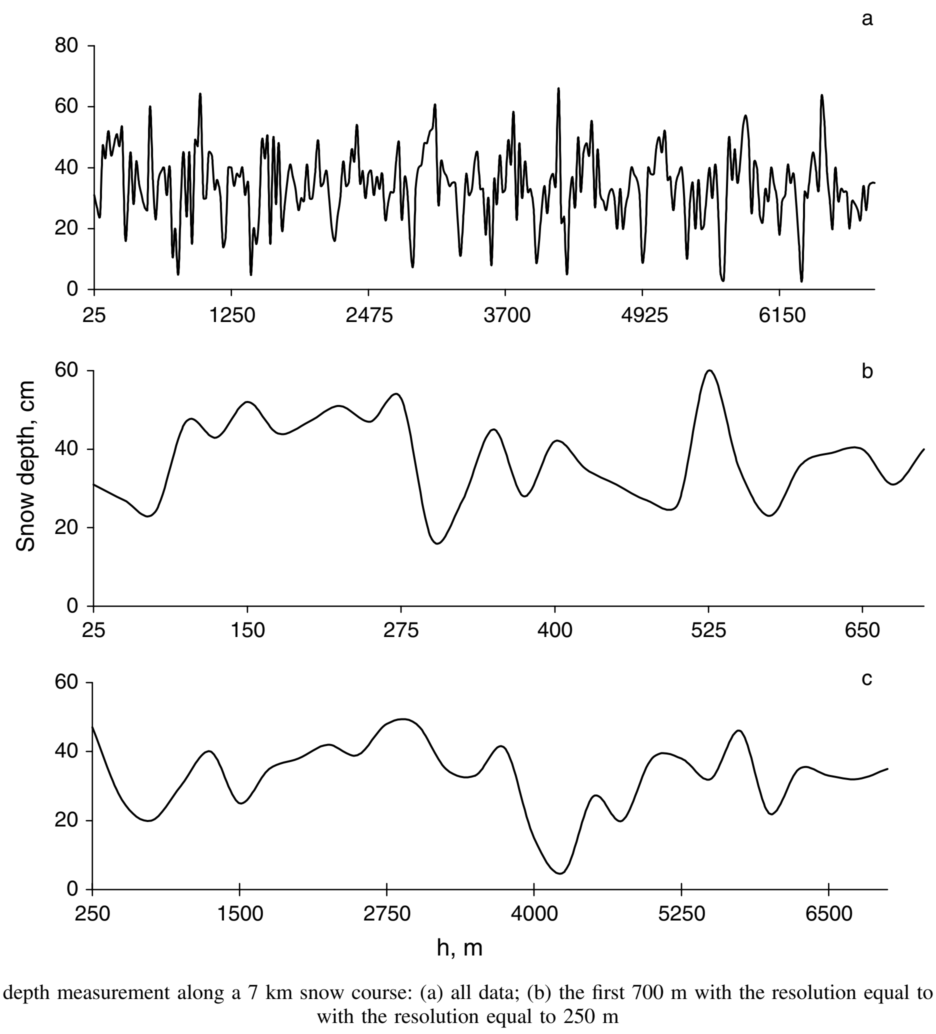

Figure 1. Snow depth measurement along a 7 km snow course: (a) all data; (b) the first 700 m with the resolution equal to 25 m; (c) 7 km with the resolution equal to 250 m

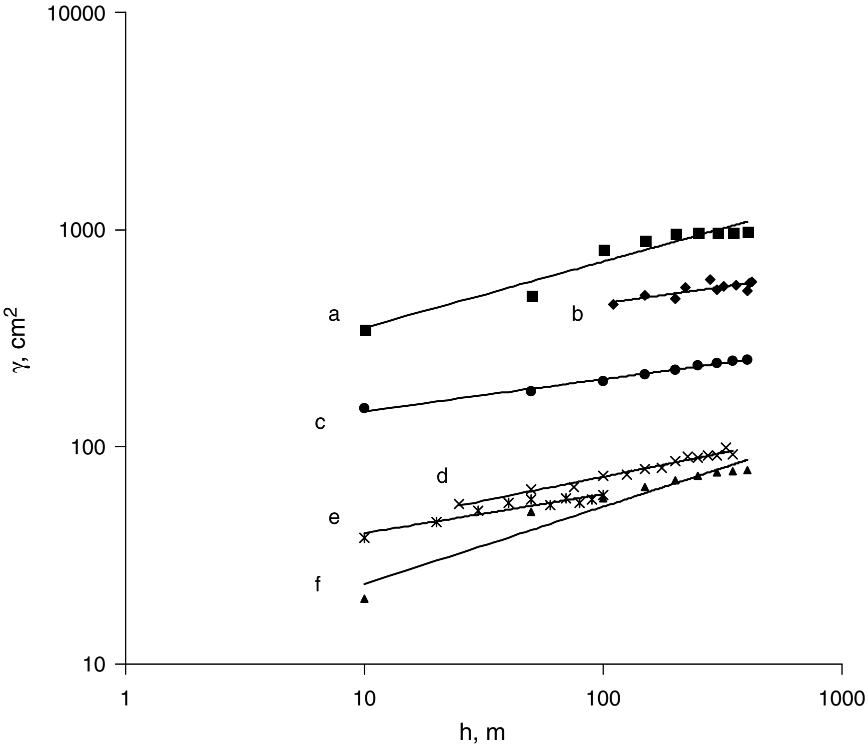

Figure 2. Variograms of snow depth for scales from tens to hundreds meters: (a) Tien-Shan region; (b) Alaska; (c) Valday region; (d) oka river basin; (e) don river basin; (f) lower Volga region

(forest steppe zone); and (f) Lower Volga region (steppe zone). The data used to construct these variograms were taken from Dunaeva *et al*. (1960), Trifonova (1962), Brown and Johnson (1965), and Kopanev (1971). As seen in Figure 2, the spatial variograms of snow depth in logarithmic coordinates were approximated quite well by linear functions, and we can consider the variations of the snow depth at chosen snow courses as the fractional Brownian processes.

The fractal dimensions of snow depth determined using Equation (5) (E was taken to be equal to unity because the data of linear snow courses were used) are shown in Table I, and vary within a narrow range. The values of H showed in Table I were close to the estimates of H obtained by Shook and Gray (1996) for scales 20–100 m.

Table I. Fractal dimension of the snow depth

Region	Number of data points	Lower limit of distances (m)	Upper limit of distances (m)	Exponent of variogram ±standard deviation	Fractal dimension of surface
Tien-Shan region (mountains)	100	20	2000	$0{\cdot}30 \pm 0{\cdot}03$	1·85
Valday region (forest plain)	50	20	1000	$0{\cdot}15 \pm 0{\cdot}01$	1·93
Dubovskaya water balance station (steppe)	100	20	2000	$0{\cdot}18 \pm 0{\cdot}02$	1·91
North of Alaska (tundra)	100	100	1420	$0{\cdot}15 \pm 0{\cdot}04$	1·92
Oka River basin (open plain)	280	25	7000	$0{\cdot}22 \pm 0{\cdot}01$	1·89
Lower Volga region (steppe)	100	20	2000	$0{\cdot}36 \pm 0{\cdot}03$	1·82

Because the spatial variation of snow density is small by comparison with the snow depth, and snow depth and snow water equivalent are strongly correlated values, the fields of snow depth and snow water equivalent have similar structure. In order to investigate the areal structure of the snow cover, we used measurements of maximum (before melting) snow water equivalent in six physiographic regions of East Europe and the Kolyma River basin (North-east Russia). We obtained these data from the Agrometeorological Annuals, which were published by the Hydrometeorological Service of the USSR. Below is a short description of the regions.

(1) This region with an area of 48 000 km^2 is situated in the northwest of Russia and partially includes the drainage area of the North Dvina, Mesen, Pechora and Upper Volga Rivers. The region has low relief dominated by forest vegetation, except for the northern part where the vegetation consists largely of mixed forest and tundra. The zonal distribution of snow cover is expressed weakly, but, in general, the forest has higher snow cover than the tundra.

(2) The upper and middle part of the Volga River basin (31 000 km^2 in area) is mainly a plain rugged relief with small differences in elevation and with forest vegetation.

(3) The region is a sub-basin of the Kama River basin with a size of 42 000 km^2. The upper part of the region is mountainous and is bounded to the east by the Ural Mountains; the middle and lower part of the region is a hilly plain.

(4) This region is located in the Don River basin (mainly the Sosna River basin) and covers an area of 21 000 km^2. It is a rugged plain in the forest–steppe zone.

(5) This region covers an area of 34 000 km^2 in the basins of the Dniester, South Bug and Pripyat (the west part of the Ukraine). About 30% of the region is in the foothills of the Carpathian Mountains with forest vegetation; the rest of the region is a forest–steppe zone.

(6) The upper part of the Kolyma River basin (drainage area is 99 400 km^2) is a mountainous area with elevations ranging from 1000 to 1200 m to 1700 to 2000 m. Most of this area is covered by tundra and taiga vegetation. A significant portion of the basin is barren ground. This basin is located in the zone of continuous permafrost.

The spatial variograms of the maximum snow water equivalents for all six physiographic regions are depicted in Figure 3, which shows that all these variograms are well approximated by the power functions and, consequently, the conditions of homogeneity of increments and the self-similarity are satisfied. The number of sample points, the long-term mean of the areally averaged maximum snow water equivalents and their standard deviations, the exponent of power variogram, and the fractal dimension for each region are given in Table II (E was taken to be equal to two).

The calculated values of H, both for the snow courses in different landscapes and also for the physiographic regions, have relatively small differences. The fractal dimensions for the forested area tend to have slightly higher values. The relatively small value of the fractal dimensions for the Kolyma River basin is explained by its mountainous relief. The values of H for all variograms were less than 0·5, and thus the spatial increments of the snow depth or the snow water equivalent were negatively correlated, indicating that these characteristics appeared relatively noisy and the short–range effects in their variations dominated (the sign of the derivations from the mean values of these functions often alternates). Such antipersistent properties of spatial variation are also typical for spatial distribution of rainfall (Gupta and Waymire, 1990), topography (Mark and Aronson, 1984), and a number of soil parameters (Burrough, 1983). However, regardless of dominance of short-range effects, the correlation of increments extended over arbitrarily large spatial scales.

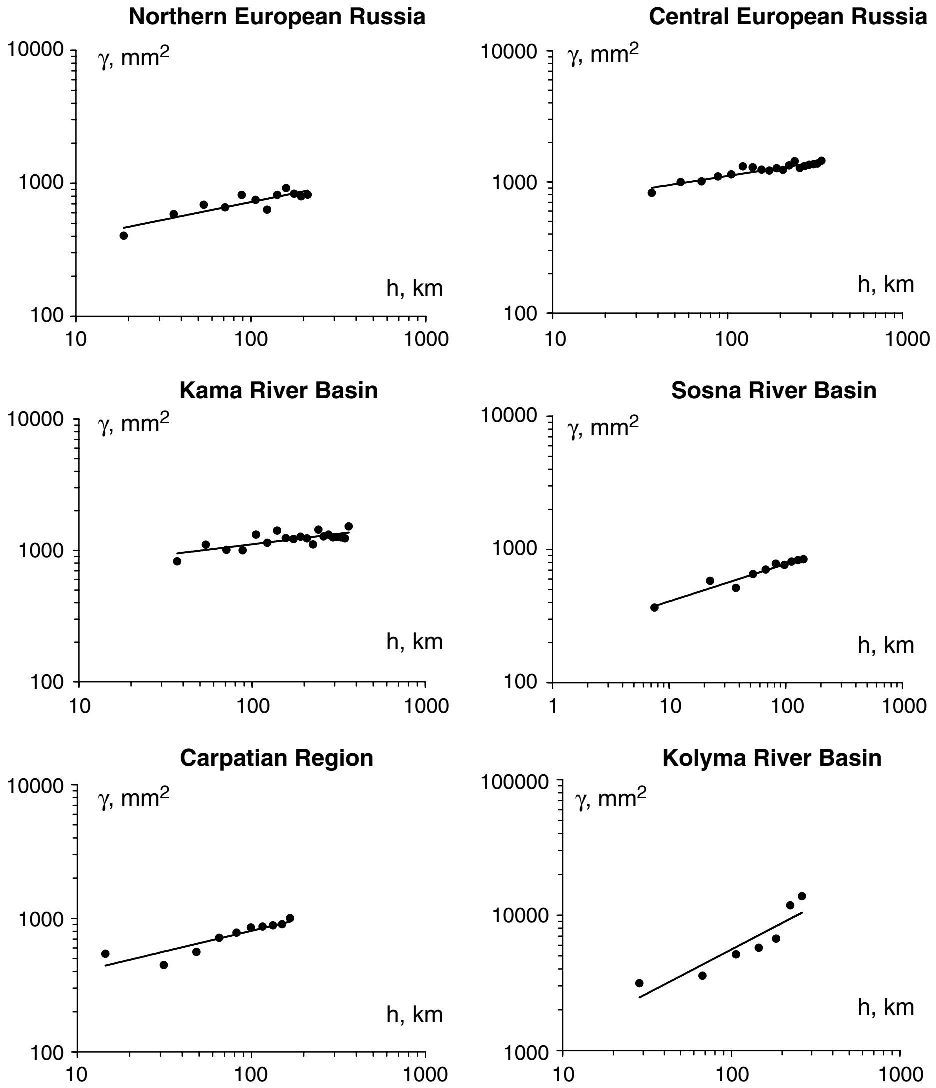

Figure 3. Variograms of the snow water equivalent for scales from tens to hundreds kilometres

Table II. Fractal dimension of the maximum snow water equivalent

Region	Number of data points	Mean areally averaged snow water equivalent (mm)	Standard deviation of snow water equivalent (mm)	Exponent of variogram ±standard deviation	Fractal dimension
Northern European Russia	65	142	34	0·30 ± 0·05	2·85
Central European Russia	41	116	37	0·22 ± 0·03	2·89
Kama River basin	50	114	33	0·16 ± 0·03	2·92
Sosna River basin	30	82	32	0·28 ± 0·02	2·86
Carpatian Region	40	50	32	0·31 ± 0·05	2·84
Kolyma River basin	20	94	49	0·67 ± 0·07	2·66

THE APPLICATION OF THE HYPOTHESIS ON STATISTICAL SELF-SIMILARITY OF SNOW COVER CHARACTERISTICS

In order to estimate the sensitivity of modelled snowmelt runoff to scaling of snow cover spatial variance, we applied the distributed runoff generation model developed for the Upper Kolyma River basin (Kuchment *et al.*, 2000).

The model is based on a finite-element schematization on the river basin and describes the following processes of runoff generation in the permafrost regions: snow cover formation and snowmelt, thawing of the ground, evaporation, basin storage dynamics, overland, subsurface, and channel flow. The maximum snow water equivalent S values were determined for each finite-element with aid of the Thiessen method using the records at 20 snow measurements stations. The schematization of the basin and the location of data points are shown in Figure 4.

In order to describe variations of S over each subgrid area, the following procedure was developed: we assumed that the areal distribution of the initial value of S within each subgrid area satisfied lognormal probability law. The mean value of S over the kth subgrid area (m_k) was assigned equal to the value of S measured at the closest snow station. The variance for subgrid areas σ_k^2 was assigned in two ways: (1) as

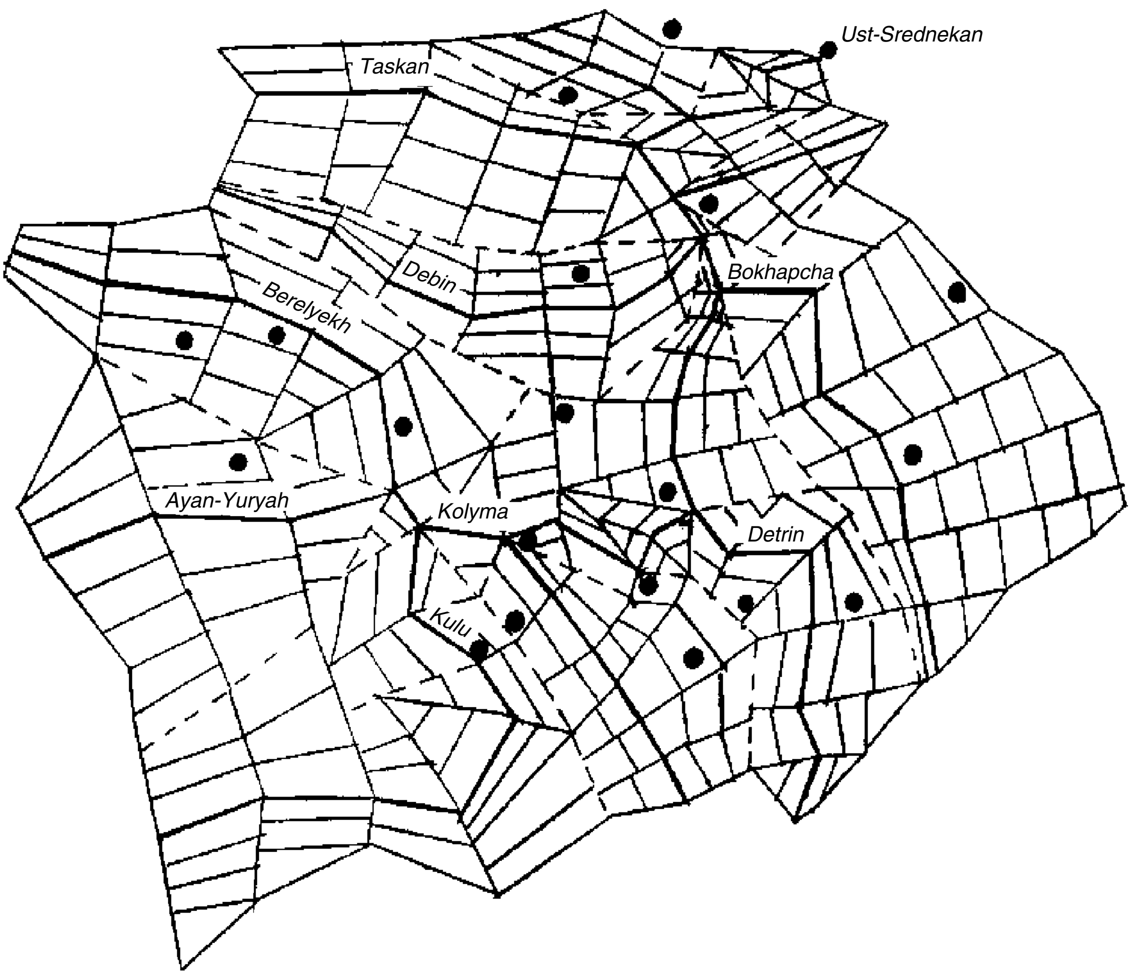

Figure 4. Schematization of the Upper Kolyma River basin and the location of data points

a constant over the whole basin area σ_F^2 (this approach was used during the calibration of the model); and (2) as calculated, according to Equation (6):

$$\sigma_k^2 = \left(\frac{F_k}{F}\right)^{0\cdot 67} \sigma_F^2 \tag{8}$$

The value σ_F^2 was estimated using 20 data points (the standard error of estimation was about 13% under the condition that the values of S were distributed by lognormal probability law).

The reduction of the subgrid variance reduced the portion of subgrid area with large values of the snow water equivalent and the snowmelt ends earlier. Figures 5 and 6 reveal these effects for two snowmelt events (Springs of 1967 and 1975). In both cases, the calculated snow-covered area declined more rapidly when the subgrid variance of the snow water equivalent was determined as a function of the subgrid area than if this variance was assigned to be equal to the constant value determined for the whole river basin. However, the difference in the distributions of snow-free area was significantly larger for the 1967 Spring event. We explain this by a more uniform distribution of snow cover over the basin.

The results from comparing the observed hydrographs with the hydrographs calculated when σ_k was constant for the whole basin, and when σ_k was determined for each finite-element area taking into account their sizes, are given in Figure 7. Table III contains a comparison of the peak discharges calculated using

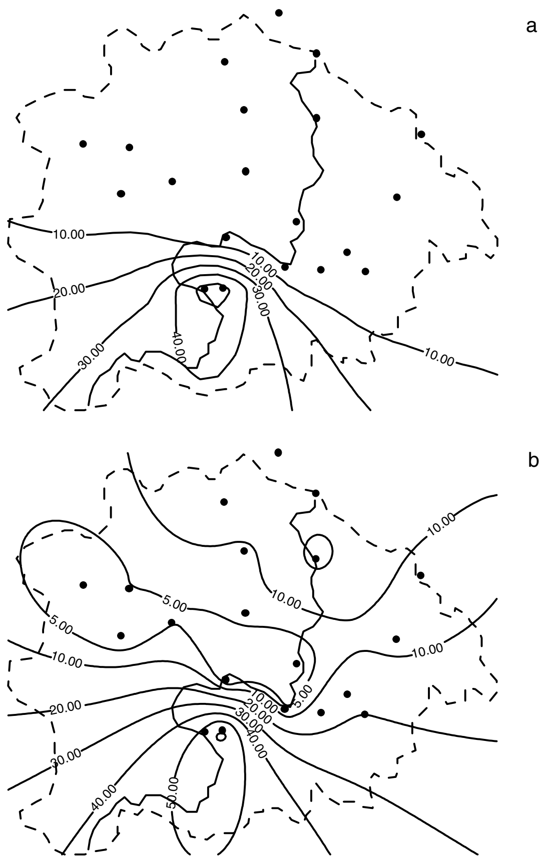

Figure 5. Spatial distribution of calculated snow water equivalent S (mm) for 20 May 1967: (a) the variance of S depended on the size of subgrid areas; (b) the variance of S was assigned as a constant for the whole basin

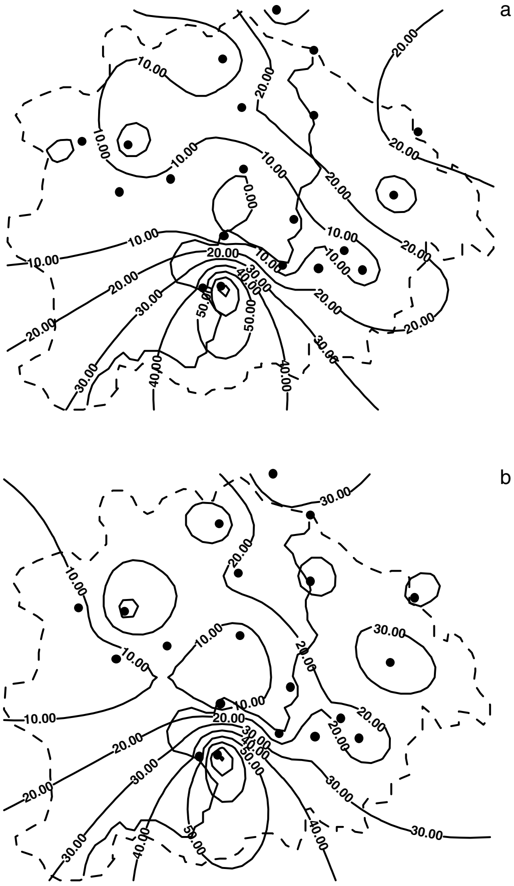

Figure 6. Spatial distribution of calculated snow water equivalent S (mm) for 20 May 1975: (a) the variance of S depended on the size of subgrid areas; (b) the variance of S was assigned as a constant for the whole basin

both ways of assigning variance σ_k. Because the lag time of the Upper Kolyma River basin is large enough there is some smoothing of differences in timing of snowmelt water inputs from different basin areas, and the sensitivity of the modelled snowmelt runoff to scaling of snow cover spatial variance was significantly less than the sensitivity of snowmelt water inputs. However, as seen from Figure 7 and Table III, in most cases there was a perceptible increase of snowmelt peak discharges when the size of area was taken into account when assigning σ_k. Such increases reached 20% for 1974 and 30% for 1967 when the snowmelt periods were long. Scaling of snow cover spatial variance did not improve calculation of the hydrographs (Figure 7). This was a result of calibrating the model parameters without this scaling. At the same time, it is possible to assume that, with some hydrometeorological conditions and for small river basins, this scaling may allow one to improve the calibration of model parameters and the accuracy of calculating runoff generation processes.

CONCLUSIONS

(1) The analysis of snow measurements carried out in different physiographic regions showed that fields of snow cover characteristics may be considered as random fields with homogeneous increments.

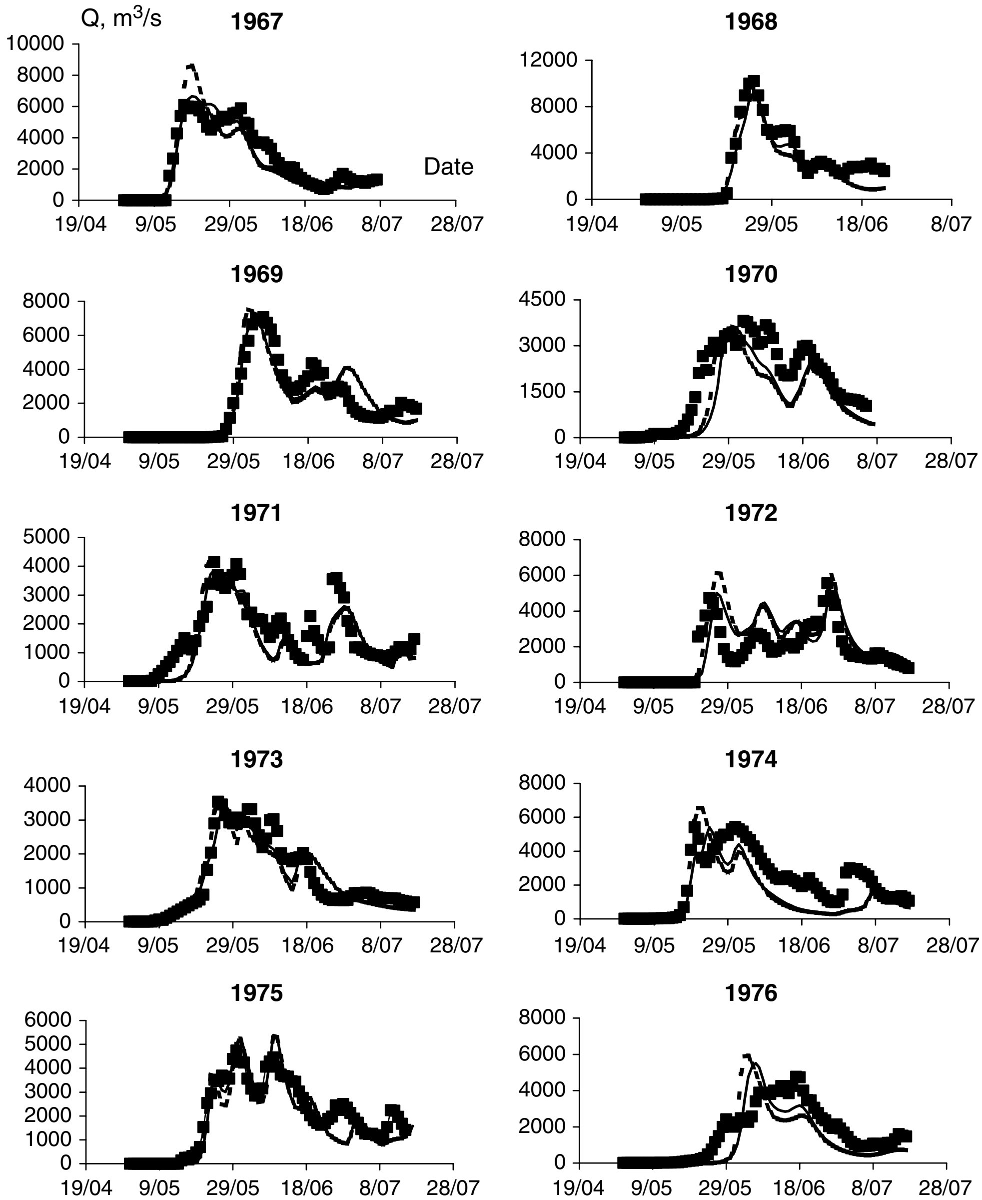

Figure 7. Observed (squares) and calculated hydrographs of the Kolyma River (dashed line: the variance of the initial snow water equivalent depended on the size of subgrid areas; solid line: the variance of the initial snow water equivalent was assigned as a constant for the whole basin)

(2) The variograms of these fields were approximated by a power function, and, consequently, these fields were considered statistically self-similar.
(3) To transfer from statistical parameters of snow distributions for large areas to statistical parameters for small areas, it was possible to use Equations (6) and (7).
(4) Use of self-similarity of snow cover for parameterization of subgrid effects enabled us to improve the description of snowmelt dynamics both within subgrid areas and for the whole river basin.

Table III. Peak discharges of snowmelt floods calculated with different approaches to describing subgrid changes of snow water equivalent

Year	Peak discharge (m^3 s^{-1})		Deviation (%)
	$\sigma_{k.} \neq \sigma_F$	$\sigma_{k.} = \sigma_F$	
1976	5921	5506	8
1975	5304	5388	−2
1974	6530	5421	20
1973	3654	3433	6
1972	6131	5577	10
1971	4226	3872	9
1970	3639	3556	2
1969	7527	6951	8
1968	10 578	10 653	−1
1967	8601	6658	29
Average	6211	5702	9

(5) The sensitivity of snowmelt flood hydrographs to the variance of the snow water equivalent on the size of subgrid areas was less than for snowmelt dynamics. However, for some hydrometeorological conditions and for small river basins this effect may lead to significant changes in the calculated hydrographs.

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