

Observation of atoms consisting of π^+ and π^- mesons

L.G. Afanasyev, A.S. Chvyrov, O.E. Gorchakov, M.A. Ivanov, V.V. Karpukhin,
A.V. Kolomyichenko, V.I. Komarov, V.V. Kruglov, A.V. Kuptsov, L.L. Nemenov,
M.V. Nikitin, Zh.P. Pustyl'nik

*Laboratory of Nuclear Problems, Joint Institute for Nuclear Research,
Head Post Office, P O Box 79, 101000 Moscow, Russian Federation*

A.V. Kulikov, S.V. Trusov, V.V. Yazkov

Institute for Nuclear Physics, Dubna Branch, Moscow State University, 141980 Dubna, Russian Federation

G.G. Mkrtchyan

Yerevan Physical Institute, Erevan, Armenia

and

A.P. Kurov

Institute for High Energy Physics, Serpukhov, 142284 Protvino, Moscow region, Russian Federation

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In the experiment 272 ± 49 atoms consisting of π^+ and π^- mesons were observed. The atoms were produced in a Ta target by 70 GeV protons and π^+ and π^- mesons were detected from the atom break-up in the same target.

1. Introduction

In all processes with emission of opposite charge particles production of Coulomb bound states (atoms) is possible. Production of $\pi^+\pi^-$ atoms (and of other hadronic atoms) in inclusive processes was considered and a method of their observation and lifetime measurement was proposed [1]. The atoms are produced in S-states with the cross section

$$\frac{d\sigma_n^A}{d\mathbf{p}_A} = (2\pi)^3 \frac{E_A}{M_A} |\Psi_n(0)|^2 \frac{d\sigma_0}{d\mathbf{p}_1 d\mathbf{p}_2}, \quad (1)$$

where \mathbf{p}_A , E_A and M_A are the momentum, energy and mass of the $\pi^+\pi^-$ atom ($A_{2\pi}$) in the lab system, respectively, $|\Psi_n(0)|^2 = P_B^3/\pi n^3$ is the atomic wave function squared at the origin with the principal quantum number n and the orbital momentum $l=0$, P_B is the Bohr momentum in $A_{2\pi}$, $d\sigma_0/d\mathbf{p}_1 d\mathbf{p}_2$ is the

double inclusive production cross section for $\pi^+\pi^-$ pairs from short lived sources without taking into account $\pi^+\pi^-$ Coulomb interaction in the final state, \mathbf{p}_1 and \mathbf{p}_2 are the π^+ and π^- momenta in the lab system. The momenta of π^+ and π^- mesons obey the relation $\mathbf{p}_1 = \mathbf{p}_2 = \frac{1}{2}\mathbf{p}_A$. The $A_{2\pi}$ are produced in atomic states with different principal quantum numbers n and are distributed according to n^{-3} : $W_1 = 83\%$, $W_2 = 10.4\%$, $W_3 = 3.1\%$, $W_{n \geq 4} = 3.5\%$.

The lifetime τ_n of $A_{2\pi}$ with the principal quantum number n and $l=0$ is determined by the charge-exchange process $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and may be written through the S-wave $\pi\pi$ scattering lengths a_0 and a_2 with isospin values 0 and 2 [2]:

$$\frac{1}{\tau_n} = \frac{8\pi}{9} \left(\frac{2\Delta m}{\mu} \right)^{1/2} (a_0 - a_2)^2 |\Psi_n(0)|^2, \quad (2)$$

where $\Delta m = M_A - 2m_{\pi^0}$, m_{π^0} is the π^0 -meson mass

and μ is the $A_{2\pi}$ reduced mass. The lifetime dependence on n is determined by the value of $|\Psi_n(0)|^2$ and from (2) one obtains $\tau_n = \tau_1 \cdot n^3$.

It follows from (2) that the τ_1 measurement with 10% precision would allow to determine $|a_0 - a_2|$ in a model independent way with 5% precision.

In chiral perturbation theory [3–5] one finds [4] $a_0 = (0.20 \pm 0.01)m_\pi^{-1}$, $a_2 = (-0.042 \pm 0.002)m_\pi^{-1}$. Inserting these values in (2) τ_1 can be calculated: $\tau_1 = (3.7 \pm 0.3) \times 10^{-15}$ s.

2. $A_{2\pi}$ detection method and setup description

In the present paper we describe the experiment of the observation of $\pi^+\pi^-$ atoms carried out at the 70 GeV proton synchrotron (U-70) at Serpukhov. Pionic atoms and $\pi^+\pi^-$ pairs (“free” pairs) were produced in a 8 μm thick tantalum target (“thick” target) inserted into the internal proton beam. The atoms can either annihilate into $\pi^0\pi^0$ pairs or break up (ionize) into $\pi^+\pi^-$ pairs (“atomic” pairs) inside the same target. The “free” and “atomic” pairs get into the 40 m long vacuum channel (the acceptance is 3.8×10^{-5} sr) at 8.4° to the proton beam and are detected by the setup in the 0.8–2.4 GeV/c pion momentum interval.

The number of “atomic” pairs depends on the atom lifetime τ , the break-up cross section and the target thickness. Assuming $\tau_1 = 3.7 \times 10^{-15}$ s the annihilation length of $A_{2\pi}$ in the 1S-state at $\gamma=10$ is 11 μm and the $A_{2\pi}$ mean free path in Ta is 6 μm independent of the γ -factor for $\gamma > 6$. In this experiment 8 atoms on average were generated per 10^{11} p -Ta interactions into the setup acceptance and $\sim 40\%$ of the atoms broke up in the target into “atomic” pairs detected by the setup. The checking measurements were carried out with a 1.4 μm thick tantalum target (“thin” target), where only $\sim 10\%$ of the atoms broke up.

Pions in “atomic” pairs have small relative momenta $Q < 3$ MeV/c, and therefore approximately equal energies $E_+ \approx E_-$ in the lab system and a small opening angle $\theta_{1,2} \approx 6/\gamma$ mrad.

The experimental setup shown in fig. 1 has a relative momentum resolution of about 1 MeV/c. The channel is connected to the accelerator vacuum pipe without any partition and is shielded against the ac-

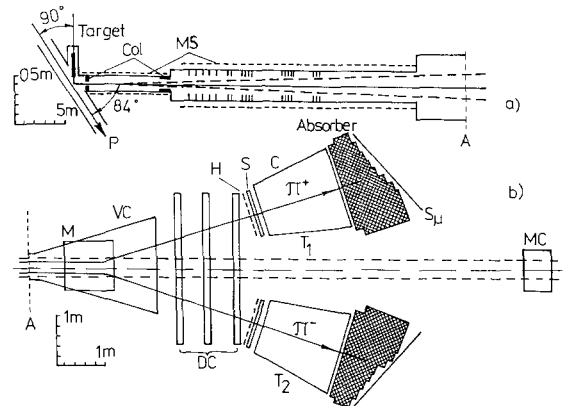


Fig. 1 Experimental setup (a) channel scheme: p – internal proton beam, Target – target mechanism, Col – collimator, MS – magnetic shield, (b) magnet and detectors M – poles of spectrometer magnet, VC – vacuum chamber, DC – drift chambers, H – scintillation hodoscopes, S, S_μ – scintillation counters, C – gas Cherenkov counters, Absorber – cast-iron absorber, MC – monitor counters

celerator and Earth magnetic fields. The channel ends with a vacuum chamber placed between the spectrometer magnet poles ($B = 0.85$ T).

Charged particles were detected by the telescopes T_1 and T_2 . The track coordinates were measured by drift chambers. The time interval between detector hits in T_1 and T_2 was measured by scintillation hodoscopes. Electrons and positrons were rejected by gas Cherenkov counters, and muons by scintillation counters placed behind absorbers. Besides π mesons other charged hadrons were also detected. Monitoring of the proton–target interactions was carried out by γ -flux measurements. The number of proton–target interactions was 7×10^8 per 0.7 s spill, and the detector counting rate $\sim 10^5$ per spill.

The first level trigger was formed by the coincidence signal $(H_1 S_1 \bar{C}_1 \bar{S}_{\mu 1}) \cdot (H_2 S_2 \bar{C}_2 \bar{S}_{\mu 2})$ (see fig. 1). A special processor selected tracks having small angles to the channel axis in a vertical plane and a vertical coordinate difference $|Y_1 - Y_2| \leq 80$ mm. The number of events per spill written to magnetic tape was about 90. The total statistics contains 1.3×10^7 events using “thick” and “thin” targets.

The measurements and simulation allowed to obtain the setup resolution on the momentum $\sigma_p/p = 0.008$, on the vertical plane angle of deviation from the target direction $\sigma_{\varphi 1} = \sigma_{\varphi 2} = 1.2$ mrad and on

the angle between particles at the magnet entrance $\sigma_{\theta_{1,2}} = 0.1$ mrad. Also were obtained the distributions of "atomic" pairs on the Q projections to the $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$ direction (Q_L) and to the plane perpendicular to \mathbf{p} (Q_T). The distributions of the Q_L and of the Q_T components Q_X and Q_Y are Gaussian-like and have the standard deviations $\sigma_{Q_L} = 1.3$ MeV/c, $\sigma_{Q_X} = \sigma_{Q_Y} = 0.60$ MeV/c for the "thick" target and $\sigma_{Q_L} = 1.3$ MeV/c, $\sigma_{Q_X} = \sigma_{Q_Y} = 0.44$ MeV/c for the "thin" target. The momentum resolution and all standard deviations are averaged in 0.8–2.4 GeV/c pion momentum range.

3. Data processing

At data processing the space reconstruction of events was fulfilled. The corrections on the residual magnetic field in the channel and on the horizontal component of the spectrometer magnet field were taken into account. The particle momenta and the track coordinates at the magnet entrance were calculated under the assumption that the particles came from the target. The angles φ_{y1} and φ_{y2} in T_1 and T_2 between the track projections and the direction towards the target were also determined. The FWHM of the φ_{y1} and φ_{y2} distributions for particles coming from the target is 2.5×10^{-3} rad (fig. 2) in accordance with simulation. Pairs originating in the target

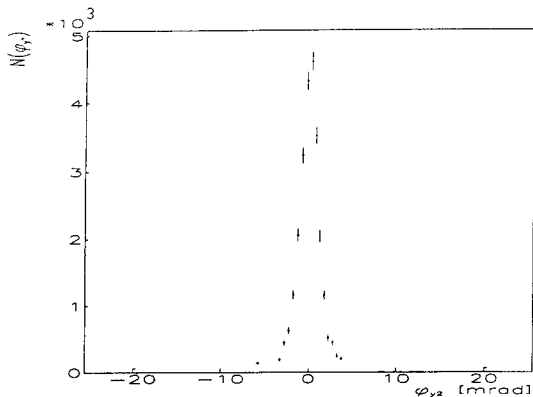


Fig. 2. Distribution of π^- mesons from time-correlated $\pi^+\pi^-$ pairs over the vertical plane deviation from the target direction. The peak is formed by π^- mesons produced in the target. The smooth background is caused by π^- mesons from K decays near the spectrometer magnet

were selected by applying a cut $\varphi_y \leq 3.5 \times 10^{-3}$ rad, where $\varphi_y = (\varphi_{y1}^2 + \varphi_{y2}^2)^{1/2}$, and by some other requirements.

The selected events are distributed in the time difference t_H between hits of the hodoscopes as shown in fig. 3. The distribution contains a true coincidence peak ($\sigma = 0.8$ ns) and a uniform background from accidental coincidences. The intervals $\Delta t_1 = \Delta t_3 = 8.0$ ns were used to determine the number of accidental events N_a in the signal region, and the interval $\Delta t_2 = 2.56$ ns to obtain the sum N_{ta} of true and accidental events. In the interval Δt_2 the ratio of true to accidental events is 0.36.

The true coincidences N_t are caused mainly by $\pi^+\pi^-$ pairs produced in the target. The fraction of $\pi^+\pi^-$ pairs generated in the accelerator vacuum pipe and in the beryllium target holder was measured to be less than 3% of the total number of N_t . The measurements and simulation have shown that $\pi^+\pi^-$ pairs from K^+ , K^- and K_L^0 decays are strongly re-

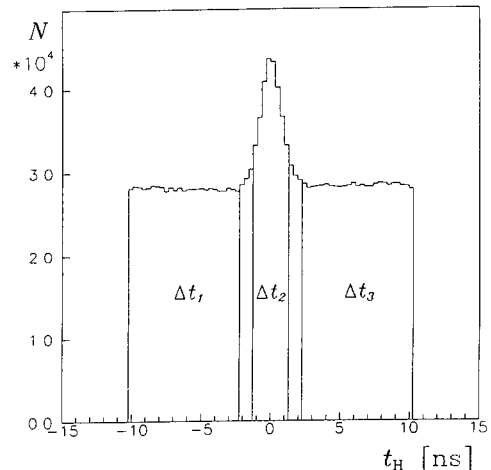


Fig. 3. Distribution of the time difference between hits in the hodoscopes. The peak is formed by time-correlated $\pi^+\pi^-$ pairs, and the uniform background is due to accidental coincidences.

duced by cutting on ϕ_y , and that number is about $10^{-2} \times N_t$. The admixtures of K^+K^- and $p\bar{p}$ pairs are equal to $10^{-4} N_t$ and $5 \times 10^{-3} N_t$, respectively. The e^+e^- pair admixture due to some inefficiency of the Cherenkov counters is $6 \times 10^{-3} N_t$. The π^+K^- , π^-K^+ , $\pi^+\bar{p}$ and π^-p pairs are absent in the true coincidence peak because the π , K and p times of flight from the target to the hodoscopes are essentially different.

From the above it follows that the number of $\pi^+\pi^-$ pairs is more than 97% of the total number of true events detected.

In order to get a better separation of the "atomic" from the "free" pairs we analyzed the distribution of the events in the variable F instead of Q :

$$F = \left[\left(\frac{Q_L}{\sigma_{QL}} \right)^2 + \left(\frac{Q_X}{\sigma_{QX}} \right)^2 + \left(\frac{Q_Y}{\sigma_{QY}} \right)^2 \right]^{1/2}. \quad (3)$$

The true event distribution in F (and in other variables) was found from the obvious relation

$$\frac{dN_t}{dF} = \frac{dN_{ta}}{dF} - \left(\frac{\Delta t_2}{\Delta t_1 + \Delta t_3} \right) \frac{dN_a}{dF}. \quad (4)$$

The distribution (4) was fitted for $F > 3$ (where "atomic" pairs are absent) by an approximating distribution. The number of "atomic" pairs is then determined by the difference between the number of $\pi^+\pi^-$ pairs in the interval $F < 2$ and the corresponding number of "free" pairs, obtained for $F < 2$ by an extrapolation of the curve fitted to the data in the region $F > 3$.

4. Approximation procedure for the $\pi^+\pi^-$ pair distribution

To obtain the approximation of the "free" pair distribution we have taken as a base the accidental $\pi^+\pi^-$ pair distribution $dN_a^{\pi\pi}/dF \equiv \Phi(F)$ because the latter and the true $\pi^+\pi^-$ pair distribution dN_t^0/dF , without taking into account the final state interaction, should have the same shape. This follows from the fact that both distributions are proportional to the product of the single inclusive production cross sections of π^+ and π^- mesons [6]. So dN_t^0/dF equals to $\Phi(F)$. The distribution $\Phi(F)$ was obtained from the accidental event distribution dN_a/dF by subtracting the π^-p and πK accidental pairs, their con-

tribution was found with the Lund model. The fraction of protons in the momentum interval 0.8–2.4 GeV/c is calculated to be $N_p/N_\pi = 0.65$. This relative number of protons was measured in the interval $0.8 \leq p \leq 1.4$ GeV/c and coincides with Lund model calculations to within 5%. The fraction of K -mesons relative to pions is below 0.3%.

The distribution $\Phi(F)$ is a sum of the pair distribution $\Phi(F)W_S(F)$ from short lived sources (pairs from direct processes and from decays of ρ , ω , ϕ , ...) and of the pair distribution $\Phi(F)[1 - W_S(F)]$ from long lived sources (one or both π mesons arise from η or K_S^0 decays). Here the weight $W_S(F)$ is the probability that two pions in an accidental pair originate from short lived sources. It was calculated with the Lund model.

The typical size of the pion production region in the case of short lived sources is 1–3 fm which is much smaller than the Bohr radius of the $\pi^+\pi^-$ atom ($r_B = 387$ fm). Thus the Coulomb interaction in the final state was taken into account by multiplying $\Phi(F)W_S(F)$ with the Coulomb factor $A_C(\beta)$ [7]. The corrections caused by the strong interaction were not introduced into this distribution because the corresponding correlation function in the analyzed interval of F is constant [8]. The pion pairs from long lived sources do not interact in the final state, and therefore no correction to the distribution $\Phi(F) \times [1 - W_S(F)]$ was applied.

The Coulomb interaction has a strong influence on the "free" pair production cross section from short lived sources at $Q < 10$ MeV/c. The Coulomb factor $A_C(\beta)$ [7] depends on the relative velocity β of the $\pi^+\pi^-$ pair in its CMS:

$$A_C(\beta) = 2\pi\eta / [\exp(2\pi\eta) - 1], \quad \eta = -\alpha/\beta, \quad \alpha = \frac{1}{137}. \quad (5)$$

The relativistic corrections to $A_C(\beta)$ under the present experimental conditions do not exceed 0.5% [9]. An observation of the Coulomb interaction effect in the $\pi^+\pi^-$ system was realized in the experiment [8] carried out with the present setup, and this effect was confirmed later [10]. Henceforth we call the pairs from short lived sources "Coulomb" pairs and those from long lived sources "decay" pairs.

From the above we can write the approximating distribution in the form

$$\frac{dN_t}{dF} = q\Phi(F)\{W_s(F)A_c(\beta) + f[1 - W_s(F)]\}, \quad (6)$$

where q is a normalization factor, f is a free parameter which accounts for the uncertainty of $W_s(F)$. With such a description of the data the particle production dynamics as well as the setup efficiency are taken into account. In the analysis interval $0 \leq F \leq 40$ the deviation of $W_s(F)$ from a constant is only a few percent, and hence it is well-founded to write the approximation function as

$$\frac{dN_t}{dF} = q\Phi(F)[A_c(\beta) + f_1], \quad (6a)$$

where f_1 accounts for the “decay” pair fraction.

5. Determination of the $A_{2\pi}$ number

The distributions dN_t/dF for “thick” and “thin” targets are shown as points with errors in figs. 4a and 5a and contain 5.9×10^4 and 4.4×10^4 events, respectively, in the interval $0 \leq F \leq 40$. To improve the statistical precision the distributions for “thick” and “thin” targets were fitted jointly by the distribution (6) because the parameter f does not depend on the target thickness. The parameter value was found to be $f = 1.8 \pm 0.3$. The fitting distributions (6) for events obtained for “thick” and “thin” targets are also presented in figs. 4a and 5a as histograms. The ratios of the experimental distribution to the fit for the “thick” and “thin” targets are shown in figs. 4b and 5b.

The following numbers of excessive pairs for the “thick”, n_A^{tk} , and the “thin” target, n_A^{tn} , in the interval $F \leq 2$ were obtained:

$$\begin{aligned} n_A^{tk} &= 272 \pm 49, \quad \bar{\chi}^2 = 1.28, \\ n_A^{tn} &= 35 \pm 41, \quad \bar{\chi}^2 = 0.75. \end{aligned} \quad (7)$$

The excessive pair number for the “thin” target, normalized to the proton interaction number for the “thick” target, is $n_A^{tn} = 47 \pm 55$.

We carried out a comparison of the excessive pair number (7) with the expected number of “atomic” pairs. In this analysis the “Coulomb” pair numbers, experimentally obtained for $Q < 2$ MeV/c,

$$N_C^{tk} = 983 \pm 54, \quad N_C^{tn} = 757 \pm 42, \quad (8)$$

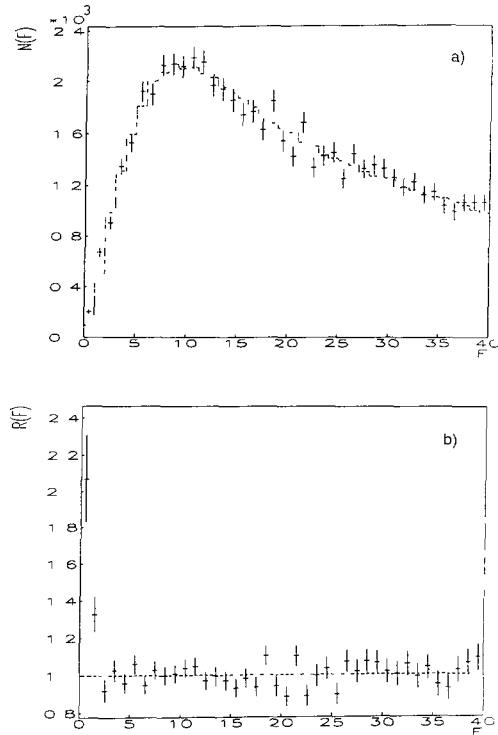


Fig 4. (a) Experimental distribution of $\pi^+\pi^-$ pairs produced in the “thick” target as a function of F (points with errors) and approximating distribution of “free” pairs on the same variable (dashed histogram) (b) The ratio of the experimental to the approximating distribution. The deviation of the ratio from unity in the two first bins is due to extra pairs originating from ionization of $A_{2\pi}$ in the target matter.

were used as follows.

The ratio of the produced atom number to the “Coulomb” pair number in the interval $Q < 2$ MeV/c was found by a numerical integration of (1) and of the Coulomb pair distribution in this interval, taking into account the multiple scattering in the target and the setup resolution

$$N_A^{tk} = 0.97 N_C^{tk}, \quad N_A^{tn} = 0.95 N_C^{tn}. \quad (9)$$

The break-up probability of $A_{2\pi}$ in the target was calculated using ionization and excitation cross sections [11], the $A_{2\pi}$ lifetime $\tau_1 = 3.7 \times 10^{-15}$ s and the quantum number distribution of the produced $A_{2\pi}$:

$$W^{tk} = 0.384, \quad W^{tn} = 0.105. \quad (10)$$

Note that in the present experimental conditions W^{tk} depends only weakly on τ_1 , $W^{tk}(\tau_1 = 2 \times 10^{-15})$

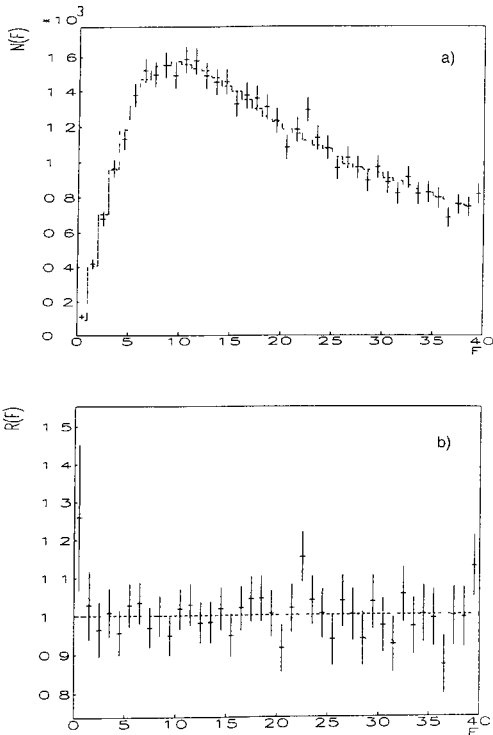


Fig. 5. The same distributions as in figs. 4a and 4b but for the "thin" target. The absence of extra pairs in the first two bins is caused by the low $A_{2\pi}$ ionization probability in the "thin" target.

$s)=0.35$, $W^{ik}(\tau_1=6 \times 10^{-15} \text{ s})=0.40$, and W^{in} does practically not depend on τ_1 .

It was calculated that 71% and 74% of the "atomic" pairs produced in the "thick" and "thin" target, respectively, lie in the interval $F \leq 2$. Then we obtain the following expected "atomic" pair numbers:

$$n_A^{ik} = 260 \pm 14, \quad n_A^{in} = 56 \pm 3. \quad (11)$$

The errors in (11) reflect only the precision in the "Coulomb" pair number.

The excessive pair number in the interval $0 \leq F \leq 1$ (n_{1A}) and in the interval $1 \leq F \leq 2$ (n_{2A}) are

$$\begin{aligned} n_{1A}^{ik} &= 106 \pm 19, & n_{2A}^{ik} &= 166 \pm 43, \\ n_{1A}^{in} &= 23 \pm 16, & n_{2A}^{in} &= 12 \pm 36. \end{aligned} \quad (12)$$

The ratio R_A^{exp} of n_{2A}^{ik} to n_{1A}^{ik} is in agreement with the corresponding calculated ratio R_A^{calc} for the 1S state, but differs from the corresponding one for the "free" pairs, R_f^{exp} :

$$R_A^{\text{exp}} = 1.56 \pm 0.47, \quad R_A^{\text{calc}} = 2.2,$$

$$R_f^{\text{exp}} = 5.09 \pm 0.30. \quad (13)$$

R_f^{exp} was obtained by averaging this ratio for both targets.

The presence of excessive $\pi^+\pi^-$ pairs for the "thick" target, their absence for the "thin" target (7) and the agreement of the observed excess of pairs with the expected value in the full interval $F \leq 2$ (11) as well as in the two separate intervals (13) allow us to conclude that the observed excessive $\pi^+\pi^-$ pairs in the "thick" target arose from the $\pi^+\pi^-$ atoms produced and broken up in the same target.

6. Test of the stability of the result

We tested the influence of uncertainties in the setup resolution and the e^+e^- pair admixture, of the F interval width in the analysis, of the π^-p admixture and of the approximating distribution type on the "atomic" pair number selected.

The setup resolution influences the shape of the "atomic" and "free" pair distributions. The uncertainty of the resolution in p and $\Theta_{1,2}$ cited above is 10%. It was shown that a 10% simultaneous resolution variation leads to a 4.3% change in n_A^{ik} .

The influence of the e^+e^- pair admixture in the distribution dN_i/dF (that is 0.6% in this experiment) on the number of "atomic" pairs was checked by adding to this distribution a 2% and 4% admixture of e^+e^- pairs relative to the total number of analyzed events. The corresponding changes in n_A^{ik} are 0.5% and 2.5%, respectively. Therefore one can conclude that a small admixture of e^+e^- pairs does not affect the final results.

The number of "atomic" pairs remains constant within 5% if the fit interval is enlarged from $F \leq 20$ to $F \leq 100$.

A change of n_A^{ik} was also obtained when replacing in (6) the distribution $\Phi(F)$ by dN_a/dF , i.e. the π^-p and πK admixtures were not subtracted from the accidental events:

$$\begin{aligned} n_A^{ik} &= 267 \pm 49, & \bar{\chi}^2 &= 1.21, \\ n_A^{in} &= 29 \pm 41, & \bar{\chi}^2 &= 0.81. \end{aligned} \quad (14)$$

To evaluate the influence of the model dependent

function $W_S(F)$ on the “atomic” pair number the analysis was performed with the formula (6a) where one neglects the difference in the production dynamics of “Coulomb” and “decay” pairs:

$$\begin{aligned} n_A^{\text{tk}} &= 268 \pm 49, \quad \overline{\chi^2} = 1.28, \\ n_A^{\text{tn}} &= 31 \pm 41, \quad \overline{\chi^2} = 0.76. \end{aligned} \quad (15)$$

By comparing (7) with (14) and (15) we conclude that the number of “atomic” pairs does not depend on corrections calculated with the Lund model.

7. Conclusions

In the present experiment we observed 272 ± 49 $\pi^+\pi^-$ atoms produced in an inclusive process. The method to observe new hadronic atoms proposed in ref. [1] was successfully applied.

It is worthwhile to note that fitting the distribution dN_i/dF by the function (6a) and determining the term f_1 allows to obtain a relation between the number of $\pi^+\pi^-$ pairs generated from short lived and long lived sources. The relative number of $\pi^+\pi^-$ pairs from long lived sources, determined in the present experiment, is $(26 \pm 5)\%$ of the total number of $\pi^+\pi^-$ pairs produced in the target. The same value determined by means of the Lund model amounts to 17%. The applicability of this method to separate $\pi^+\pi^-$ pairs generated from short lived and long lived sources was indicated in our previous publication [8].

Later [12] the necessity was pointed out to take into account the different sources of $\pi^+\pi^-$ pairs (short lived and long lived) when Coulomb corrections are introduced into experimental distributions of identical pions.

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