

Referee report on the paper

The images of non-commutative polynomials evaluated on 2×2 matrices

by A. Kanel-Belov, S. Malev and L. Rowen

The authors consider the problem, attributed to Kaplansky, asking whether the set of values of a multilinear polynomial on $M_n(K)$ is a vector space. They solve it for $n = 2$ (and K quadratically closed). It is quite remarkable that a problem that sounds like an exercise for students, “Prove that a set is a vector space”, can be solved only in a quite special case using rather sophisticated tools. Anyway, there is no doubt that this is a difficult problem, and that any progress is worthwhile.

The paper is well-written and I do not have any specific suggestions for improvements. Perhaps the authors could also mention in the introduction that the question has a positive answer in the case where $p = x_1x_2 - x_2x_1$ (Albert and Muckenhoupt, Michigan Math. J. 4 (1957), 1-3). But even for this simplest polynomial the problem is already nontrivial.

I like the paper and I recommend its publication in Proc. AMS.