Experimental Evidence for a Growing Surface Wave and Acoustic Beam Narrowing upon Reflection from Fluid-Solid Interfaces

Oleg A. Sapozhnikov,1, 2 Alexander A. Karabutov, Jr.,1 and Vladimir G. Mozhaev1

1Department of Acoustics, Physics Faculty, Moscow State University
Leninskie Gory, Moscow 119992, Russia
2Center for Industrial and Medical Ultrasound, Applied Physics Laboratory,
University of Washington, Seattle, WA, USA
oleg@acs366.phys.msu.ru

Abstract—The secular equation for acoustic waves at fluid-solid interfaces yields the common leaky wave and its complement. This complementary wave grows instead of decays with propagation and is time-reversed compared to the leaky wave. Moreover, this growing wave has not yet been observed experimentally, perhaps due to difficulty of its excitation. Experimental observation of this wave was one goal of our work. The second goal was to study mirror reflection of an acoustic beam of special shape when the incident angle is equal to the Rayleigh critical angle. An obliquely incident beam is known to split after reflection into two components: a specular beam and a broad beam generated by the leaky waves. The interference of these two components results in "Scholte displacement" of the reflected beam along the interface and overall beam broadening. Our hypothesis was that by time reversing the reflection at the critical angle, the reflection beam can be made narrower rather than broader.

Keywords: leaky Rayleigh wave, reflection, Scholte displacement, time reversal, shadowgraphy

I. INTRODUCTION

Surface acoustic waves at liquid-solid and gas-solid interfaces are widely studied for over last 50 years [1, 2]. It is known that at the interface between an isotropic solid and a vacuum there is always a surface mode: the Rayleigh wave, which has a velocity slightly lower than the bulk shear wave and it is not attenuated as it propagates along the solid surface [3]. If the vacuum is replaced by a fluid with sound speed less than the Rayleigh wave velocity, the surface wave is attenuated by radiating into the fluid and becomes a leaky Rayleigh wave (also known as pseudo-Rayleigh wave), that is faster than the pressure wave in fluid but slower than the bulk waves in the solid [4]. The leaky Rayleigh mode has exponentially growing amplitude when moving off the interface into the fluid. Although this seems physically difficult to achieve, it is experimentally observable in the form of a separate signal when recording waves in the fluid at a distance from a near-interface point source in fluid: this follows from the fact that the pole of the corresponding secular equation is close to the real axis [5]. In addition to the leaky wave (which is not a "true" surface wave), there also is a true surface mode, similar to the Stoneley wave on the solid-solid interface, which has a velocity slightly less than sound speed in the fluid and it is not attenuated as it propagates along the interface. This wave is referred to as the Scholte wave [6]. The leaky Rayleigh wave can be considered as surface wave that decays due to radiation into the fluid, relative to which it is a supersonic source. In most of the practical cases this wave propagates many wavelengths $\lambda$ before it attenuates: for instance, in case of the aluminum-water interface the amplitude decays $e^{-2.7}$ times at approximately $5.4 \times \lambda$ [1]. Formally, the leaky Rayleigh wave structure can be found by considering a perturbation that travels along the interface, with exponential decay into the solid. The dependence in the fluid is also exponential, but not necessarily decaying. As mentioned, a true interface wave decays in both media, whereas the leaky wave can have exponential growth in the fluid. The equation of motion and the boundary conditions then result in the characteristic secular equation [4]. Among the Scholte wave and the mentioned leaky wave, it has a solution in form of a wave complement to the leaky wave. This wave is usually neglected from the consideration, because it is a growing wave, which, at first glance, seems physically unrealistic.

The growing interface wave is in fact a time-reversed compared to the leaky Rayleigh wave. Its existence follows directly from the fact that the governing equations in a non-attenuating media are invariant to the change of sign of time. Note that in the leaky wave the energy propagates only in the direction off the interface, i.e. the wave in the fluid is purely outgoing. In the growing surface wave, therefore, the wave in fluid should be purely incoming, which means it is completely absorbed by the interface (no reflection); all its energy transforms to the surface wave. This property of the growing wave may be of interest in increasing the efficiency of surface wave sources used in nondestructive testing. The growing wave has not yet been observed experimentally due to difficulty of its excitation. This wave can be created in a limited space region if an acoustic beam with a specially chosen structure and angle of incidence is directed toward the interface from the fluid side. Due to damping of reflection by
the growing surface wave, the beam has to narrow upon reflection.

In this paper we report on observation and numerical modeling of both leaky and growing surface waves on the solid-fluid interface, and demonstrate the effect of beam narrowing.

II. MATERIALS AND METHODS

A. Experimental Arrangement

A photo of the core of the experimental set-up is shown in Fig. 1. The ultrasound source (A) is a broadband, 0.5-3 MHz, 2.5 cm diameter, single element, plane transducer (Panametrics V194) driven by a tone burst from a function generator (Agilent 33250A). Reflectors (B and C) are aluminum blocks with fine angular adjustments.

The system is immersed in water. The water tank has optically transparent windows. Ultrasonic field in water is visualized using shadowgraphy imaging and stroboscopic flashing of a semiconductor laser [7]. The light flash is of 100 ns duration, which is much shorter that the wave cycle and thus allows to “freeze” the wave shadow. By recording such instantaneous pictures at various delays between the pulse emission and laser flashing, a movie of the wave shadow dynamics is prepared.

During the experiment, the ultrasound pulsed beam from the source A is directed on the reflector B at incident angle close to the critical Rayleigh angle, arcsin (c0/cg), where c0 is speed of sound in the fluid, cg is Rayleigh wave speed. For the aluminum-water interface this angle is ≈30°. At such angle of incidence the surface wave is effectively excited; it then propagates along the reflector surface B and reradiates acoustic energy to water, thus forming a wide reflecting beam. The second reflector (C) is positioned perpendicularly to this reflecting beam. Upon reflection from the mirror C, the wave propagates back to the reflector B. At this time the growing acoustic wave is formed at the interface B. See more details in the Results.

B. Numerical Modeling Approach

The wave propagation in the region consisted of fluid and solid sub-regions is studied numerically using the method of finite differences. The elastic wave propagation in an isotropic medium is governed by the equation of motion

\[ \rho \frac{\partial v_i}{\partial t} = \partial \sigma_i / \partial x_i \]

and, as the signals are small, Hooke’s law

\[ \sigma_i = \lambda (\nabla u_i) + \mu (\nabla u_i + \nabla u_i^T) \]

where \( \rho \) is density, \( v_i = \partial u_i / \partial t \) and \( u_i \) are velocity and displacement components, \( \sigma_i \) are components of elastic stress tensor, \( \lambda \) and \( \mu \) are Lamé constants. Two-dimensional geometry is considered for simplicity (contrary to experiment, which is three dimensional, see Fig. 1). To solve the equations in finite differences, they are discretized using a central differencing scheme with staggered grids both in space and in time. To avoid the parasitic numerical reflections from the calculation region border, a perfectly matched layer is placed around the region. Velocity and stress are initially set to zero. A flat ultrasound source is modeled by the corresponding boundary condition on its surface. The boundary conditions on the solid-fluid interfaces are not written explicitly: instead, the entire region was modeled as an inhomogeneous region with a piecewise uniform distribution of parameters. The details of the finite difference numerical scheme are presented in [8].

III. RESULTS

A. Modeling

Typical modeling results are illustrated in Fig. 2. A transducer of 2.5 cm size is shown in the upper left corner. The reflectors’ contours are shown by thin lines. At \( t=0 \) the source starts to radiate a 2-cycle pulse at 0.5 MHz. The incidence angle is equal to the critical Rayleigh angle 30.25°. The pulse propagates in water and at \( t=25 \mu s \) reaches the surface of the first reflector. Then it starts to reflect specularly \( (t=35 \mu s) \). At the same time a strong elastic perturbation is formed in the aluminum block beneath the interface. This perturbation is the leaky Rayleigh wave. It propagates along the surface and slowly decays due to radiation of the outgoing wave to water \( (t=50 \mu s) \).

Eventually a wide reflected beam is formed with a quasi-plane wavefront and non-uniform lateral structure \( (t=70 \mu s) \). This result is not new. An obliquely incident beam is known to split after reflection into two components: a specular beam and a broad beam generated by the leaky wave. The interference of these two components results in overall displacement of the reflected beam along the interface and its broadening [9].

The goal of the second reflector is to reverse in time the wave process. Note that numerically the time reversal procedure is fairly simple: it is enough to change the sign of velocity components without changing the stress components. We did this test and confirmed that after such a numerical time reversal the modeled wave indeed propagated backwards repeating all its history back in time. The test has thus demonstrated the good accuracy of the numerical algorithm. However, experimentally such an ideal time reversal is not possible. One of the solutions here is to use multi-element arrays to record the wave structure, reverse each element signal in time numerically, and re-radiate them back [10]. This approach is difficult to put into practice. One the other hand, in case of a purely plane wave with a symmetric temporal waveform the time reversal is achieved upon a simple reflection from a perpendicularly oriented mirror. In case of quasi-plane wave this kind of reflection does not provide exact
time reversal, but if the wave beam is sufficiently wide it is reasonable to expect good result. The reflected wave image at \( t=100 \, \mu s \) confirms this assumption. It is seen that the wave propagating back to the first reflector does not produce any reflected wave, as should be upon true time reversal. It is also seen that the elastic perturbations beneath the interface grow, transforming the incident beam energy to the surface mode. The last image in Fig. 2 (\( t=140 \, \mu s \)) illustrates the fact that the pulsed beam, that is finally formed after the time-reversed beam reflection, is again the same narrow as the initial beam (compare with \( t=25 \, \mu s \)). The time-reversal is not perfect: there are some weak ripples both in water and aluminum. However, there amplitude is relatively small. Therefore, the modeling shows that using a properly oriented flat reflector one can achieve the growing surface wave formation and corresponding narrowing of the reflected beam.

**Experiment**

The experiment is based on the pulsed shadowgraphy that is capable of identifying sufficiently fine details of the wave structure. Figure 3 presents main results. The wave shadow is shown for three times: 8 \( \mu s \) (initial wave), 62 \( \mu s \) (after first reflection from the bottom plate), and 99 \( \mu s \) (when the wave incidents on the bottom plate the second time).

The reflected beam at 62 \( \mu s \) illustrates the main features of the leaky wave generation and the corresponding non-specular reflection. The acoustic beam broadens and its effective axis is shifted to the right-hand side. This shift is sometimes referred to as “Schoch displacement” [11]. Note that the wave amplitude has a minimum approximately in the middle of the specular reflection region (near the arrow tail), which is sometimes called the “null strip” effect [12, 13]. As it was mentioned before, this can be interpreted as a result of an interference of a specular beam and a broad beam generated by the surface wave. The pulsed shadowgraphy allows seeing more than that: in particular, in the region of specular reflection the wavefronts are distorted and 180°-phase shift occurs across the null-strip region. Optical visualization used in the previous papers [12, 13, 14] was based on the c.w. schlieren technique and thus was not able to reveal this 180°-shift effect.

Shadowgraphy at subsequent time 99 \( \mu s \) shows that the wide incident beam does not create noticeable reflected signals from the bottom reflector. This means that around that time the growing surface acoustic wave did exist, i.e. instead of being radiated by the leaky surface wave, energy of the incident beam was completely absorbed by the surface wave, and the surface wave grew as it propagated. Note that this is almost identical
to the theoretical image shown in Fig.2 (t=100 µs). The shadowgraphy shows only the fluid part of this new interface wave. The perturbations in solid can be also visualized in transparent solids, but this was not the case in our experiment with aluminum reflectors.

When the front of the growing surface wave reached the site of incidence of the beam from the transducer, the wave structure is strongly distorted and the specularly reflected pulsed beam is created (not shown in Fig. 3). This specularly reflected beam was narrower than the incident wave as hypothesized and calculated numerically.

IV. DISCUSSION

The growing surface wave has several features that can be useful in practice. One of them is perfect matching between the incident part of the wave and its interface component. Albeit the entire growing wave is physically impossible to achieve (because the amplitude in the fluid can not grow infinitely), one can imagine configurations when such a wave would exist within a limited spatial region. Consider a rectangular transducer in fluid that radiates an acoustic beam onto the fluid-solid interface at the Rayleigh angle. Instead of usual quasi-uniform distribution of the wave amplitude along the source, let the distribution be non-uniform, with an exponential growth in the vertical plane from the lower edge of the transducer to its upper edge. If the exponent is properly chosen, such a transducer will radiate a non-reflecting incident beam everywhere except the beam edges. Such a configuration thus provides the Rayleigh wave generator with efficiency close to 100%. Note that the maximum efficiency achievable with uniformly excited transducers can not exceed 80% due to reflection from the surface [9].

Another important feature of the growing surface wave is the fact that it is a pure phase conjugate of the corresponding leaky wave. Based on that, we have demonstrated that the time reversal nature of the wave process makes it possible to create a wide acoustic beam with a specially chosen lateral distribution, which produces significantly narrower reflected beam. Therefore, a beam compression and thus intensity increase can be achieved without lenses or curved mirrors

ACKNOWLEDGMENTS

The work was supported by RFBR, INTAS, ISTC, and NSBRI SMS00402. We appreciate the help of Valery Rozhkov in making the set-up shown in Fig. 1.

REFERENCES