RESEARCH OF THE MULTICOMPONENT PUPIL'S MODEL ON THE COMPUTER

Mayer, R. V.[1*]

1 The Glazov Korolenko State Pedagogical Institute, Glazov, Russia.
* Corresponding author: robert_maier@mail.ru.

CrossMark
click for updates

OPEN ACCESS

UDC: 37.02
Citation: Mayer, R. V. (2015). Research of the multicomponent pupil's model on the computer. In *Advanced Studies in Science: Theory and Practice* (Vol. 4) (81-95). London, UK. doi: 10.17809/25(2015)-09.
Editing: Edited by the Editorial Board of the Global Partnership on Development of Scientific Cooperation LLC.
Received: November 19, 2015.
Accepted: December 10, 2015.
Published: December 28, 2015.

Data Availability Statement: All relevant data are within the paper and its Supporting Information files.
DOI: 10.17809/25(2015)-09

Abstract

In article several three-component models of pupils are analysed. The model 1 considers the following: (i) during trainings, the amount of weak (poor) pupil's knowledge increases, and part of weak knowledge transforms into stronger (solid) knowledge; (ii) after the end of the training, strong knowledge gradually turns into less strong one, and the amount of weak knowledge decreases on the exponential law. Model 2 allows considering that at the increase in the lag from teacher's requirements, the pupil's motivation to learning and the efforts spent by them at first increases, reaches a maximum, and then decreases. Model 3 is a development of the previous models and considers that an increase in speed of teacher's statements about the new material, the transmission coefficient of the "teacher–pupil" channel is equal 1 at first, and then it smoothly decreases to 0. By methods of imitating modeling, it hase been found: (i) there is an increase in pupil's knowledge who is trained at 11–year school; (ii) there is a change in pupil's knowledge in that case, when a teacher divides the theoretical material into several parts and alternates them with a number of practical tasks; (iii) there is also the dependence of the maximum quantity of the acquired knowledge and the corresponding speed of the statement from a number of the training material's parts.

Key words:

Computer modeling; didactics; education; mathematical methods; pedagogy; pupil; simulations; teacher.

ИССЛЕДОВАНИЕ МНОГОКОМПОНЕНТНОЙ МОДЕЛИ УЧЕНИКА НА КОМПЬЮТЕРЕ

Майер Р. В.[1*]

1 Глазовский государственный педагогический институт им. В.Г.Короленко, г. Глазов, Россия.
* Автор, ведущий переписку: robert_maier@mail.ru.

Аннотация

В статье проанализированы несколько трехкомпонентных моделей ученика. Модель 1 учитывает следующее: 1) во время обучения у ученика увеличивается количество непрочных знаний, причем часть непрочных знаний превращаются в более прочные; 2) при отсутствии обучения прочные знания постепенно превращается в менее прочные, а количество непрочных знаний уменьшается по экспоненциальному закону. Модель 2 позволяет учесть, что мотивация ученика к обучению и затрачиваемые им усилия при увеличении отставания от требований учителя сначала возрастают, достигают максимума, а затем уменьшаются. Модель 3 является развитием предыдущих моделей и учитывает, что при увеличении скорости изложения нового материала учителем коэффициент передачи канала связи "учитель–ученик" сначала равен 1, а затем плавно уменьшается до 0. Методами имитационного моделирования исследовано: 1) увеличение знаний ученика при обучении в 11–летней школе; 2) изменение знаний ученика в случае, когда учитель делит теоретический материал на несколько частей и чередует их с практическими заданиями; 3) зависимость объема усвоенных знаний и коэффициента обученности ученика от скорости сообщения информации при различном количестве порций.

Ключевые слова:

Компьютерное моделирование; дидактика; образование; математические методы; педагогика; ученик; имитации; учитель.

Научная статья включена в Российский индекс научного цитирования.
www.elibrary.ru

Introduction

The development of the theory of training assumes the application of methods of mathematical and computer (simulation) modeling. A real pupil is replaced with an abstract model, which behavior is described by one or several equations (Atkinson, Baujer & Kroters, 1969; Leontev & Gohman, 1984; Roberts, 1986). The study of various mathematical and computer models of a pupil for the purpose of their specification and evolution has the particular interest. The condition of a didactic system is characterized by quantities of this or that type of knowledge acquired by a pupil, which is measured in conventional units. The known models of the training process (Atkinson, Baujer, & Kroters, 1969; Dobrynina, 2009; Dorrer & Ivanilova, 2007; Itelson, 1964; Ivashkin & Nazojkin, 2011; Kudrjavcev, Vashik, Strogalov, Alisejchik & Peretruhin, 1996; Leontev, & Gohman, 1984; Mayer, 2014; Mayer, 2013; Roberts, 1986; Solovov & Menshikov, 2001; Firstov, 2011) are based on the assumption that all elements of learning material (ELM) are acquired and forgotten equally easily. It is possible to assume that the model of a didactic system describes the training process better if the model takes into account: 1) transition of weak (poor) knowledge into strong one, which is forgotten significantly slower; 2) nonlinear dependence of the efforts (F) made by a pupil in one unit of time from his/her lag D of teacher's requirements; 3) reduction of the transmission coefficient of the communication channel "teacher–pupil" K with the growth in the speed (υ) of the new material presentation. According to the principle of plurality of the description, any complex system can be modelled by a large number of ways. By changing initial data and parameters of the model, it is possible to investigate ways for the system development and to define its state at the end of training. This is the advantage of this approach in comparison with the method of the qualitative analysis. Therefore, researching various mathematical and computer models of the training process has a great importance for the development of didactics.

1. Multi–component modelling of training at school (The model 1).

It is known that the process of mastering (assimilation) and remembering of any given information consists in establishing associative links between new and existing knowledge. As a result, the acquired knowledge becomes stronger and is forgotten much slower (Mayer, 2014; Mayer, 2013). The process of increasing the durability of acquired knowledge used by a pupil in everyday activities is the cornerstone for the formation of abilities (know–hows) and skills that remain for a long time. A pupil's total knowledge Z includes weak knowledge of the first category (or type) Z_1, stronger knowledge of the second category (know–how or ability) Z_2, and very strong knowledge of the third category (skills) Z_3: $Z = Z_1 + Z_2 + Z_3$. In the course of training ($k=1$) at first, the information given by a teacher turns into knowledge of the first category, and then, as a result of its use while performing educational tasks, into knowledge of the second and third categories (Firgue 1). So, the durability of the acquired material increases gradually. The speed of the transformation (or transition) of weak knowledge into the category of stronger knowledge is characterized by coefficients of assimilation α, α_1, and α_2.

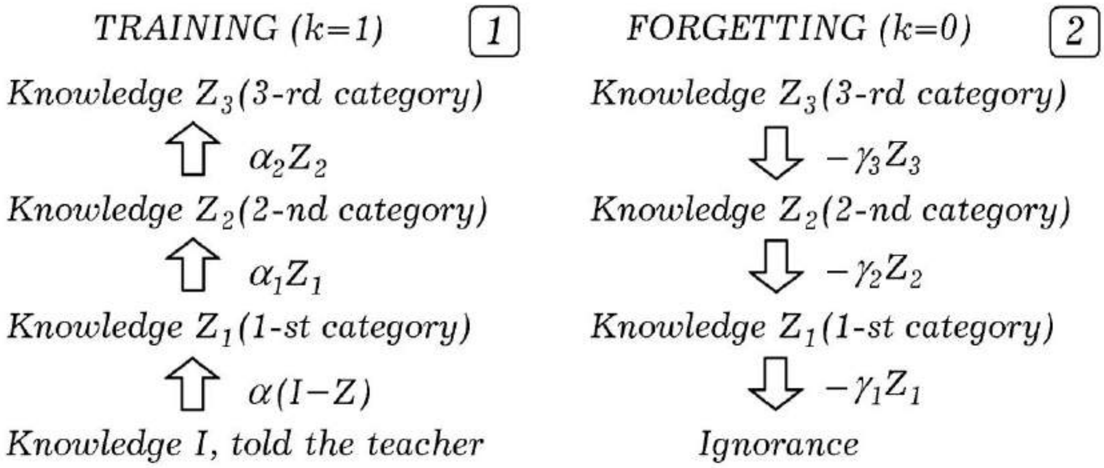

Figure 1. Changes in the durability of acquired knowledge during training and forgetting.

With no training ($k=0$) there is a back transition observed (firgue 1). A part of strong knowledge of the third category gradually becomes less strong knowledge of the second category, then it partially turns into the category of weak knowledge of the first category and becomes forgotten. The speed of the ransformation of strong knowledge into weak one and into ignorance while forgetting is characterized by the coefficients of forgetting γ_1, γ_2, and γ_3. So, the following principles are the cornerstone of the offered model:

1) In the course of training, a pupil operates with the information which is available for him/her, performing various educational tasks. Thus, the knowledge provided by a teacher at first is acquired as weak or fragile (become knowledge of the first category), then in the process of its revision and use, it becomes stronger, turning into knowledge of the second category, and finaly it becomes strong, i.e. knowledge of the third category.
2) The speed of the increase of a pupil's weak knowledge in the course of training is proportional to the difference between the level of teacher's requirements L (the quantity of the reported knowledge) and a pupil's total knowledge $Z = Z_1 + Z_2 + Z_3$; it also may be equal to $\alpha(L-Z)$.

3) While training, the speed of transformation of weak knowledge Z_i into stronger knowledge Z_{i+1} is proportional to the quantity of weak knowledge Z_i and is equal to $\alpha_i Z_i$ ($i=1$, 2).
4) With no training, there is the process of forgetting observed: a pupil's knowledge becomes less strong, and then turns into ignorance. The speed of transformation of a strong pupil's knowledge Z_i into less strong knowledge Z_{i-1} or into ignorance is proportional to the quantity Z_i and is equal to $-\gamma_i Z_i$ ($i=1$, 2, 3).

The result of training is characterized by the total level of acquired pupil's knowledge Z and by the durability coefficient $K_D=(Z_2/2+Z_3)/Z$. If all knowledge is acquired by a pupil during studies is weak ($Z_1=Z$, $Z_2=Z_3=0$), the durability coefficient $K_D=0$. It is necessary to aspire to a situation, when all acquired knowledge is strong ($Z_3=Z$, $Z_1=Z_2=0$), then $K_D=1$. Studying one theme very long, the knowledge level Z increases to L, along with it there is a share increase of strong knowledge Z_3/Z, the durability K_D also grows, tending to 1.

The offered three–component model of training is expressed by the system of the equations (when training is $k=1$; while forgetting is $k=0$):

$$dZ_1/dt=k(\alpha(L-Z)-\alpha_1 Z_1)-(1-k)(\gamma_1 Z_1-\gamma_2 Z_2),$$
$$dZ_2/dt=k(\alpha_1 Z_1-\alpha_2 Z_2)-(1-k)(\gamma_2 Z_2-\gamma_3 Z_3),$$
$$dZ_3/dt=k\alpha_2 Z_2-(1-k)\gamma_3 Z_3,\ Z=Z_1+Z_2+Z_3.$$
$$\alpha=(0.003+0.01\cdot(1-\exp(-Z/100)))\cdot(1-0.07\cdot S_j),$$
$$\alpha_1=\alpha/e,$$
$$\alpha_2=\alpha_1/e\ ,\ \gamma_1=0.001,\ \gamma_2=\gamma_1/e,\ \gamma_3=\gamma_2/e,\ e=2.72....$$

To solve this system of the equations, using the numerical method, there is a special computer program. It contains the cycle on time, in which the quantity of different types of a pupil's knowledge is defined in the following time moment $t+\Delta t$, and the result is displayed in a text or graphic formats. Therefore, it is possible to simulate training at 11–year school. The table (Fig. 2.1) shows tentative (or estimated) values of the level of eacher's requirements L_j ($j=1$, 2, …, 11) for each class; the complexity of training material S_j is given as $S_j=0.07j$. It is considered that a pupil studies for 275 days within a year and has a rest during 90 days in the form of a summer vacation. Coefficients of assimilation and forgetting are selected so that the graph of total knowledge would approximately correspond to a rather successful pupil who acquires 70–90 percent of the required information (Fig. 2.2). The abscissa axis shows the time in days from the moment of pupil's arrival at school in the first form. It is obvious that eventually the quantity of total knowledge $Z(t)$ and the levels of the formation of abilities (know–how) and skills increase. After the course of training, the process of forgetting begins; first of all, a pupil loses weak knowledge which is not demanded in practice. Failures in graphics $Z(t)$ correspond to the three–month vacation.

1

Class	L
1 class	10
2 class	20
3 class	33
4 class	49
5 class	68
6 class	90
7 class	115
8 class	143
9 class	174
10 class	208
11 class	245

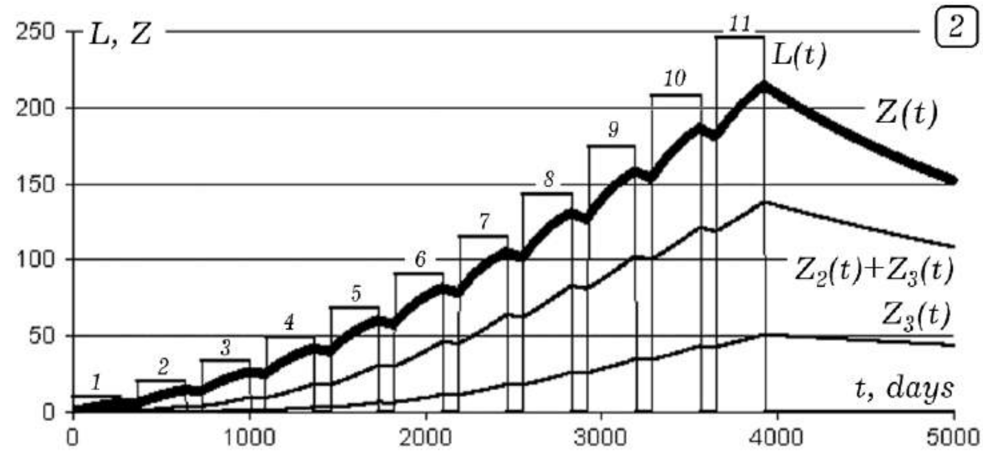

Figure 2. Results of imitating modelling of training at school.

2. Dependence of understanding from speed of receipt of information (The model 2).

The simplified information–cybernetic model of didactic system (Fig. 3.1) consists of a source of information (a teacher) and a receiver of information (a pupil), which are connected by the direct communication channel (from a teacher to a pupil) and the return communication channel (feedback from a pupil to a teacher). Let us assume the studied theme includes N elements of learning material (ELM), which are connected with each other, and a teacher demands assimilation of all studied information; his/her requirements level L is equal to amount of the knowledge Z_0. We consider that the complexity of i–th ELM S_i is proportional to expenses of time and pupil's efforts, which are required for assimilation of these ELM. Then for simplest ELM $S=1$, and for more difficult ELM–s S is more than 1. The level of requirements imposed by a teacher $L = S_1 + S_2 + ... + S_N$. If all N ELM–s have the complexity 1, then $L = N$. The speed of information transfer is equal to the quantity of the knowledge reported by a teacher in the conventional unit of time (CUT): $\upsilon = dI/dt = dL/dt$; it is measured in CUT^{-1}. It depends on the level of requirements L, that is on the number of ELM–s N, and their complexity is S_i, $i = 1,2,...N$.

The result of training depends on the degree of understanding the studied material. The person understands information, if he/she is able to correlate it to the own categorial system of concepts. In his/her consciousness, there is an on-going transcoding of the arriving speech or text information, its "laying" in own conceptual system and memorization. The more complicated the statement of a teacher (i.e. the speed υ), the more mental actions should be made by a pupil in prder to understand it. If a teacher states the difficult material immediately, jumping through the elementary reasonings, which are difficult for a pupil, a pupil may not be able to connect the new information with own system of concepts, understanding all carried-out reasonings.

If we neglect the forgetting, the speed of increase in a pupil's knowledge is proportional to the efforts F spent in one unit of time, i.e. $dZ/dt = \alpha F$. The motivation for training and the efforts F spent by a pupil: 1) at small $D = L - Z$ (L slightly exceeds Z), it is proportional to the volume D; 2) at large D, it is reduced to 0. It is possible to write down the3 following: $F(D) = 0.0127D/(1+\exp((D-150)/50)$. From the graph $F(D)$ (Fig. 3.2), it is clear that there is an optimum difference $D = L - Z$, at which the speed of the increase of knowledge is maximum. The value $C = 1/(1+\exp(r(D-\theta))$ allows taking into account that at big D a pupil reduces his/her efforts as he/she understands that the inability to quickly understand and acquire the reported materials.

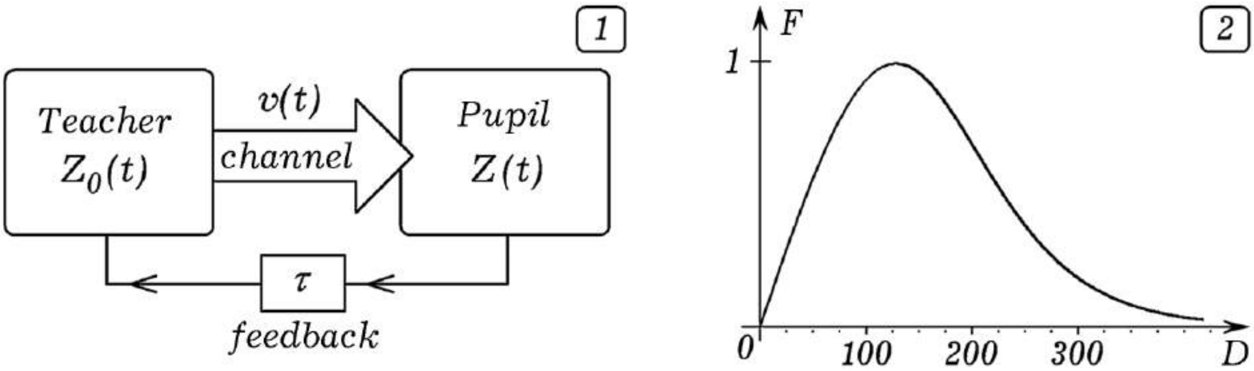

Figure 3. The cybernetic system of training. The graphs of dependence $F = F(D)$.

During the training, weak knowledge is transformed into strong (abilities and skills are formed), which may be forgotten slower. We consider the three-component model:

$$\frac{dZ_1}{dt} = \frac{k\alpha D}{1+\exp(r(D-\theta))} - k\alpha_1 Z_1 - \gamma_1 Z_1, \quad \frac{dZ_2}{dt} = k\alpha_1 Z_1 - k\alpha_2 Z_2 - \gamma_2 Z_2,$$

$$\frac{dZ_3}{dt} = k\alpha_2 Z_2 - \gamma_3 Z_3, \quad Z = Z_1 + Z_2 + Z_3, \quad D = L - Z, \quad r = 0.02, \quad \theta = 150.$$

Here Z_1, Z_2 and Z_3 are the amounts of weak knowledge, abilities and skills (that is strong knowledge) of a pupil. They differ in the durability of assimilation and have forgetting coefficients $\gamma_1 = 2 \cdot 10^{-3}$, $\gamma_2 = 2 \cdot 10^{-4}$, $\gamma_3 = 2 \cdot 10^{-5}$ CUT^{-1}. The assimilation of coefficients $\alpha_1 = 0.17$, $\alpha_2 = 5 \cdot 10^{-3}$, $\alpha_3 = 1.7 \cdot 10^{-3}$ CUT^{-1} characterize the speed of assimilation of a pupil's knowledge and its transition from weak knowledge to strong one. So far, there is a training $k = 1$ and $k = 0$ when it stops. The result of training is characterized by the total level of acquired knowledge $Z = Z_1 + Z_2 + Z_3$.

On the basis of the equations, which are written down, the above computer model of training was created. Let a pupil has to understand and remember sequence of the interconnected reasonings in an interval from 0 to 400 CUT (for example, a formula conclusion, or the solution of a complex problem). At the time of $t' = 400$ CUT, corresponding to the end of training, the control of his knowledge Z is exercised. During the training, the speed υ of receiving the information from a teacher remains to a constant, and at a very moment, a teacher demands knowledge of all previous material. Thus, the level of requirements of a teacher grows under the law: $L = \upsilon \cdot t$. We define the amount of knowledge $Z(t')$ acquired by the pupil and the training effectiveness ratio $K = Z(t')/L$ at the various υ.

The computer model allows to receive graphs $Z_1(t)$, $Z_2(t)$, $Z_3(t)$, and $Z(t)$ for various values of speed υ of the information transfer. If it is too great (υ>12 CUT^{-1}) , at first a pupil still has time to follow the teacher's reasoning and assimilates almost all the elements of the educational material, but after some time, this pupil "comes off" a teacher, ceases to understand and acquire his/her reasonings. The gap D between the level of requirements $L(t)$ and knowledge of a pupil $Z(t)$ increases, motivation to learning and the efforts made by him/her F decrease. Therefore, the amount of acquired knowledge Z and the efficiency coefficient K at the end of lessons are significantly lower. After the training, because of forgetting, Z, Z_1, Z_2, and Z_3 also decrease. At a less speed υ, a pupil understands almost all the studied material. Similar graphs $Z(t)$, $Z_2(t) + Z_3(t)$ and $Z_3(t)$ corresponding to the increase L are in Fig. 4.1.

We carry out a series of numerical experiments, varying υ from 1 to 20 CUT^{-1} with the duration of training 400 CUT. We obtain the graphs of dependences knowledge Z and a pupil's learning efficiency K from the data transfer rate υ (pic. 4.2). From them it is clear the following: 1) at υ < 11 CUT^{-1}, the amount of acquired knowledge Z increases in direct proportion, learning efficiency K is high and remains constant; 2) if υ >11 CUT^{-1}, the amount of acquired knowledge Z and the learning efficiency sharply fall; 3) there is an optimum speed of submission of information (about 11 CUT^{-1}), at which the amount of acquired knowledge reaches its maximum. The received results are consistent with the second Shannon's theorem, from which follows that if productivity of a source exceeds the communication channel capacity with noise, then there is no method of coding that allows transferring the error-free information.

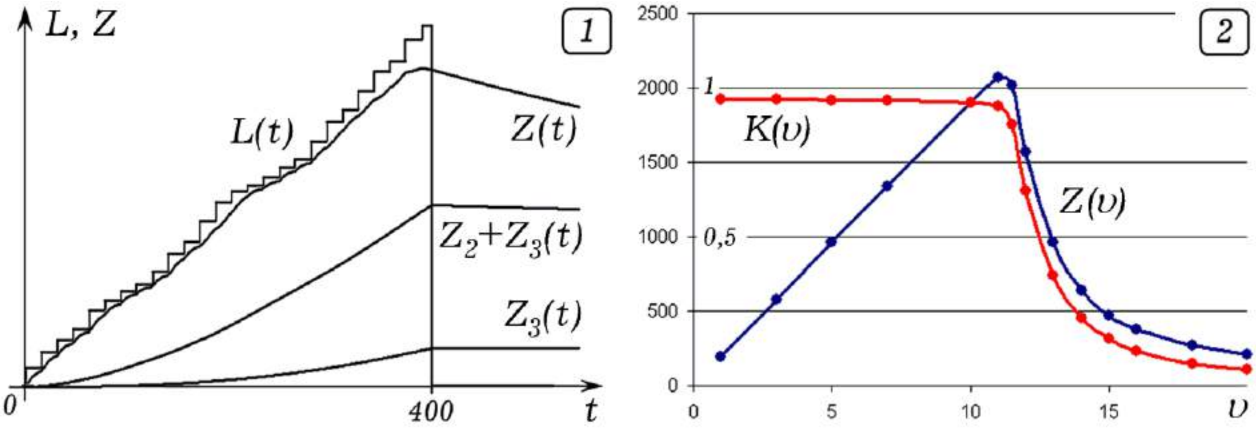

Figure 4. The graphs of dependence $Z = Z(\upsilon)$ and $K = K(\upsilon)$ at $t' = 400$ CUT.

3. Dependence of the training result from alternation of the educational material (Model 3).

We consider the dependence of transmission coefficient of the communication channel from the speed of information reporting by a teacher. The investigation of another digital model of a pupil is based on the following principles:

1. If to neglect the forgetting, the speed of increasing of a pupil's knowledge dZ/dt is proportional to his/her efforts F spent in one unit of time, which depends on the difference D between the level of teacher's requirements L and the pupil's knowledge Z.
2. The motivation to learning and the efforts F spent by a pupil at a small lag $D = L - Z$ increases at first, reaches its maximum, and decreases at big D, aspiring to some limit $b = 0.1–0.3$ (because a pupil realizes that he/she cannot acquire the demanded material).
3. The communication channel "teacher–pupil" has a certain capacity. At the small speed of presenting new materials by a teacher $\upsilon = dL/dt$, the coefficient of transmission K of the communication channel is equal to 1; at big υ, a pupil does not manage to apprehend, understand, and acquire the teacher's reasoning; therefore, K decreases to 0.
4. The pupil's condition in each time moment is defined by the amount of weak knowledge Z_1, along with the quantity of abilities Z_2 and skills Z_3 (strong knowledge). Weak (or poor) knowledge is forgotten quicker than strong knowledge.
5. In the course of training ($k = 1$) the amount of pupil's weak knowledge Z_1 increases, and part of weak knowledge transforms

into stronger knowledge (at first, it happens with abilities Z_2 and than with skills Z_3).

6. In the absence of training ($k=0$), there is the forgetting, i.e. strong knowledge (skills) gradually turns into less strong, and the quantity of weak knowledge Z_1 decreases under the exponential law. The offered mathematical model of a pupil is reduced to the following system of the equations:

$$dZ_1/dt = k\alpha \cdot K(\upsilon)F(D) - k\alpha_1 Z_1 - \gamma_1 Z_1 + \gamma_2 Z_2,$$

$$dZ_2/dt = k\alpha_1 Z_1 - k\alpha_2 Z_2 - \gamma_2 Z_2 + \gamma_3 Z_3, \quad dZ_3/dt = k\alpha_2 Z_2 - \gamma_3 Z_3,$$

$$Z = Z_1 + Z_2 + Z_3, \quad K(\upsilon) = 1/(1+\exp(0.25\upsilon - 3)), \quad D = L - Z, \quad F(D) =$$

$$1.65k(1-\exp(-0.01D)) \cdot \left(0.15 + \frac{0.85}{1+\exp(0.02D-4)}\right).$$

Here the forgotten coefficients are equal to $\gamma_1 = 10^{-3}$, $\gamma_2 = \gamma_1/2.72$, $\gamma_3 = \gamma_2/2.72$ CUT^{-1}, the assimilation coefficients: $\alpha_1 = 14$, $\alpha_2 = 5\cdot 10^{-3}$, $\alpha_3 = \alpha_2/2.72$ CUT^{-1}. On the basis of those equations, the computer program for modeling training is created.

Situation 1. During a lesson, a teacher presents the material with some constant speed υ, so that $L(t) = \upsilon \cdot t$. Results of modeling of training at $\upsilon \approx 9$ CUT^{-1} are presented in Fig. 5.1. At a small speed υ, a pupil acquires all information given by a teacher. If the speed of transfer of new knowledge is large, a pupil is not able to follow his/her teacher on time, his/her lag D increases, and at some moment t', he/she "comes off" or gets behind the teacher, understanding only part of the studied material. If speed υ is even more, a pupil "comes off" the teacher earlier, acquiring even less.

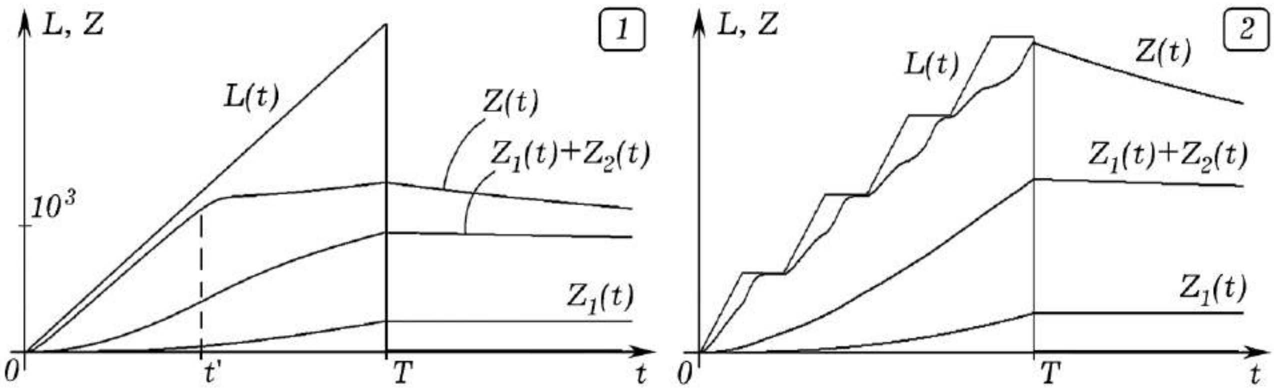

Figure 5. Continuous and stepwise ($s=4$) increase in the level of requirements L.

Situation 2. Half of the training time is spent on studying the theoretical material (L grows), and another half is on remembering by repetition and fulfilling of practical tasks (L remains to a constant). A teacher divides the theoretical material on $s=4$ parts and alternates them to practical tasks, so that the total values of time of studying the theory t_T and the practice t_P are identical and equal to $T/2$. Graphs in Fig. 5.2 correspond to the maximum speed of presenting the new material υ_m, at which a pupil still acquires practically all information. The model also shows that with increasing in quantity of portions s, the maximum speed υ_m, at which a pupil is still capable to acquire all new material, becomes more; also, total knowledge provided by the teacher and acquired by the pupils at the end of training (at the moment $t=T$) grows. This result can be interpreted as follows: if 25 pupils have various coefficients of assimilation α, then the alternation of the theory and practice at the same speed υ of the statement of the theoretical material provides the assimilation of provided knowledge with a large number of pupils.

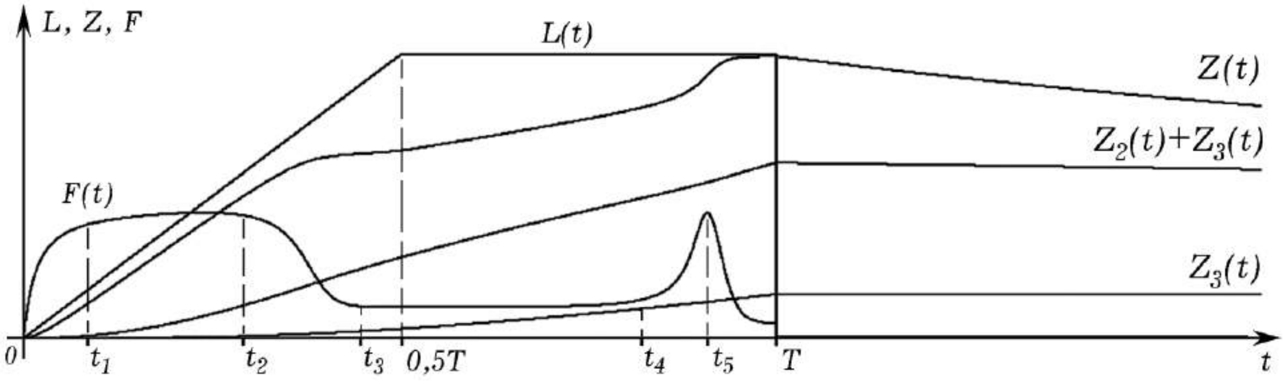

Figure 6. The results of the simulation study ($\upsilon=9.5$ CUT^{-1}, $s=1$).

Situation 3. A teacher does not divide the training material into parts; he/she T presents theoretical issues with a constant speed for one half of the training time, so that $L(t)=\upsilon\cdot t$, $\upsilon=9.5$ CUT^{-1}, $s=1$. After that he/she organizes the process of repetition, i.e. $L=const$ (Fig. 6). It is seen that at first with the increasing gap D, the pupil's force F increases (from 0 to t_1) and then remains high (from t_1 to t_2) as D is close to the optimal value. The speed of presentating the new material is still too high, so the pupil's gap D is increasing so that he/she breaks away from the teacher (from t_2 to t_3) and begins to make less efforts F. As a result, a pupil acquires the necessary training material badly (time from t_3 to t_4). From the moment $t=T/2$, a teacher organizes the repetition ($L=$

const, $\upsilon = 0$), and a pupil makes practical tasks. During the period from t_4 to t_5 the lag D is reduced, the efforts F are increasing sharply, reaching a peak. There is a jump: in a short time the total knowledge of the pupil Z increases almost to the level of the teacher's requirements L. After the end of training ($t > T$), the pupil's efforts F turn into zero, forgetting weak pupil's knowledge Z_1 and rapidly decreasing, and solid knowledge (i.e. skills) is reduced significantly slower.

The model offered above allows to make a number of computer experiments at various speeds υ of a statement of material and number s of portions in order to study their influence on amounts of acquired knowledge and the pupil's efficiency coefficient. In Fig. 7 graphs of the following dependences are shown: 1) amounts of the acquired knowledge $Z(T)$ and the pupul's efficiency coefficient $K = Z(T)/L(T)$ from the speed υ of reporting of information at various $s = 1$, 2, 4, 8, 16 (Fig. 7.1 and 7.2); 2) the maximum quantity of acquired knowledge Z_m and the corresponding speed of the statement υ_m from s (pic. 7.3). From Fig. 7.1, it is also clear that at $s = 1$ values $Z_m \approx 1700$ and $\upsilon_m \approx 9$ CUT^{-1}, but at $s = 16$ values $Z_m \approx 2600$ and $\upsilon_m \approx 12$ CUT $^{-1}$. From graphs $Z(\upsilon)$ and $K(\upsilon)$ it is also clear: 1) the values $Z(\upsilon)$ and $K(\upsilon)$ sharply decrease at $\upsilon > \upsilon_m$: a pupil ceases to acquire information; 2) at the increase in s, the greatest possible amount Z_m of acquired knowledge, the value and the corresponding speed of a statement υ_m grow, striving for limit values. It is corresponding to the second theorem of Shannon: a pupil acquires information if the speed of its transfer not exceeds the communication channel capacity.

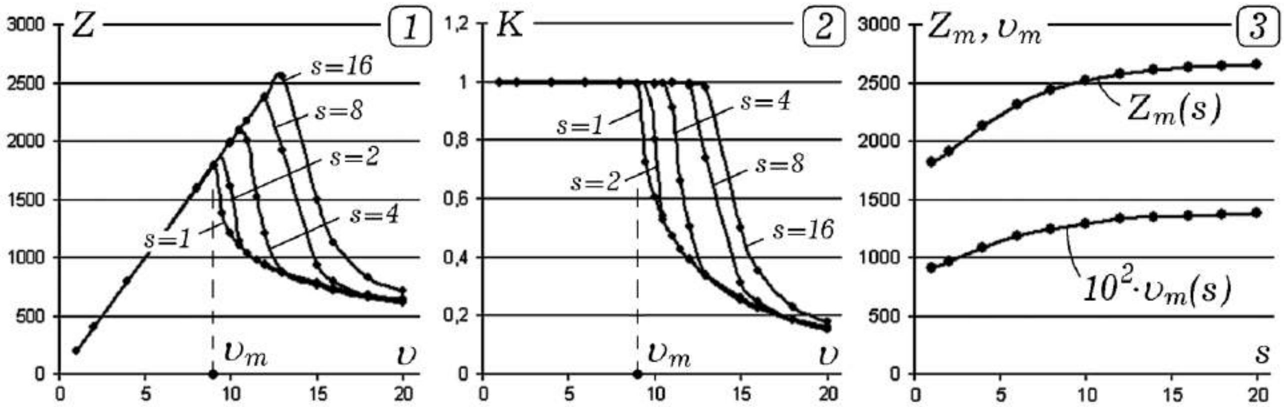

Figure 7. Graphs $Z(\upsilon)$, $K(\upsilon)$ at the various s; graphs $Z_m(s)$, $\upsilon_m(s)$.

The sharply reduced character of the assimilation level of the educational material in dependence from the speed of its statement near critical value υ_m (graphs $Z(\upsilon)$ and $K(\upsilon)$) corresponds to the border between two pupil's states, when he/she understands and acquire the studied material, and when he/she cannot make it. So, the offered computer model of a pupil allows to prove that a teacher has to alternate the statement of theoretical material to the performance of practical tasks and consideration of examples of the studied theories, using the concrete cases.

Conclusion.

In this article the new approach to the modeling of training processes, which is the further development of the already known methods (Atkinson, Baujer, & Kroters, 1969; Dobrynina, 2009; Dorrer & Ivanilova, 2007; Itelson, 1964; Ivashkin & Nazojkin, 2011; Kudrjavcev, Vashik, Strogalov, Alisejchik & Peretruhin, 1996; Leontev, & Gohman, 1984; Mayer, 2014; Mayer, 2013; Roberts, 1986; Solovov & Menshikov, 2001; Firstov, 2011), is offered. The developed computer model of a pupil considers the following: 1) the speed of increase in pupil's knowledge is proportional to his/her efforts spent in the unit of time, which depends on the difference between the level of teacher's requirements and the level of pupil's knowledge; 2) at increase in lag from teacher's requirements, a pupil's motivation to learning, along with the spent efforts, increases at first, reaching its maximum, and then decreases; 3) at increase in the speed of the statement of new materials, the transmission coefficient of the "teacher–pupil" channel is equal to 1 at first and then smoothly decreases to 0; 4) the condition of a pupil in each moment of time is defined by the amount of weak (poor) and strong (solid) knowledge; weak knowledge is forgotten quicker than strong knowledge is; 5) during the training, the amount of weak pupil's knowledge increases, and part of weak knowledge transforms into stronger one; 6) after the end of the training, a pupil begins to forget information, because strong knowledge gradually turns into less strong, and the amount of weak knowledge decreases on the exponential law.

By methods of imitating modeling, it is found: 1) An increase of pupil's knowledge is observed during the training at school; 2) there is the change in pupil's knowledge and the made efforts in a case when a teacher gives new information during one half of the training time, and the other half of time is spent for fixing, repeating and performance of practical tasks; 3) there is the growth of pupil's knowledge in the case when a teacher divides the theoretical material into several parts and alternates them with some practical tasks; 4) there is the dependence of quanty of acquired knowledge and the pupil's efficiency coefficient from the speed of information reporting at various number of portions; 5) there is the dependence of the maximum quantity of acquired knowledge and the corresponding speed of the statement from the number of portions of the training material.

During the research, it is also found: 1) there is the critical value of the speed of teacher's reporting information beyond which a pupil cannot acquire the material; 2) at the increase in the number of portions of the studied material, the maximum quantity of acquired knowledge and the corresponding speed of the material statement increase, striving to the limit values. At the increase in speed of providing information on a new theme, there is the sharp falling of the pupil's efficiency coefficient; it corresponds to the border between two states when a pupil acquire the studied questions and when cannot make it. The results of modeling confirm that when studying a new theme, a teacher has to alternate the statement of theoretical material to the performance of practical tasks and the consideration of examples , used for studying theoretical positions on a numberof different concrete cases. At the increase in quantity of portions (the number of portions into which the theoretical material is divided), the efficiency of learning grows, aspiring to some limit.

References

Atkinson, R., Baujer, G., & Kroters, Je. (1969). *Vvedenie v matematicheskuju teoriju obuchenija* [Introduction to the mathematical theory of training]. Moscow: Mir. (In Russian).

Dobrynina, N. F. (2009). Matematicheskie modeli rasprostranenija znanij i upravlenija processom obuchenija studentov [Mathematical models of the spread of knowledge and learning management students]. *Fundamentalnyye issledovaniya, 7*, 7-9. (In Russian).

Dorrer, A. G., & Ivanilova, T. N. (2007). Modelirovanie interaktivnogo adaptivnogo obuchajushhego kursa [Modeling of interactive adaptive course of learning]. *Sovremennyye problemy nauki i obrazovaniya, 5,* 53-61. (In Russian).

Firstov, V. E. (2011). *Matematicheskie modeli upravlenija didakticheskimi processami pri obuchenii matematike v srednej shkole na osnove kiberneticheskogo podhoda* [Mathematical models of controlling the didactic process of teaching mathematics in secondary schools on the basis of the cybernetic approach] (Doctoral Dissertation). St. Petersburg. (In Russian).

Itelson, L. B. (1964). Matematicheskie i kiberneticheskie metody v pedagogike [Mathematical and cybernetic methods in pedagogy], Moscow: Prosveshhenie. (In Russian).

Ivashkin, Ju. A., & Nazojkin, E. A. (2011). Multiagentnoe imitacionnoe modelirovanie processa nakoplenija znanij [Multi-agent simulation of the process of accumulation of knowledge]. Retrieved from http://simulation.su/files/immod2011/material/98.pdf. (In Russian).

Kudrjavcev, V. B., Vashik, K., Strogalov, A. S., Alisejchik, P. A., & Peretruhin, V. V. (1996). Ob avtomatnom modelirovanii processa obuchenija [On the automaton model of the learning process]. *Discrete Mathematics and Applications, 6(6)*, 533–539. doi: 10.4213/dm554. (In Russian).

Leontev, L. P., & Gohman, O. G. (1984). *Problemy upravlenija uchebnym processom: Matematicheskie modeli* [Problems Training Management: Mathematical model]. Riga. (In Russian).

Mayer, R. V. (2013). Osnovnaja zadacha matematicheskoj teorii obuchenija i ee reshenie metodom imitacionnogo modelirovanija [Main problem of mathematical theory of training its decision with using method of imitating modeling]. *Perspective of Science and Education, 5*, 95–100. (In Russian).

Mayer, R. V. (2014). *Kiberneticheskaja pedagogika: Imitacionnoe modelirovanie processa obuchenija: monografija* [Cybernetic pedagogics: Imitating modeling of training process: monograph]. Glazov: Glazov. gos. ped. in–t. Retrieved from: http://maier–rv.glazov.net. (In Russian).

Roberts, F. S. (1986). *Diskretnye matematicheskie modeli s prilozhenijami k social'nym, biologicheskim i jekologicheskim zadacham* [Discrete mathematical models with annexes to social, biological, and ecological tasks], Moscow: Nauka. (In Russian).

Solovov, A. V., & Menshikov, A. A. (2001). *Diskretnye matematicheskie modeli v issledovanijah processov avtomatizirovannogo obuchenija* [Discrete mathematical models in the study of processes automated training]. *Educational Technology & Society, 4*, 205 – 210. (In Russian).