THE APPLICATION OF CLIFF DEGRADATION MODELS FOR ESTIMATION OF THE INITIAL HEIGHT OF RAMMED-EARTH WALLS (POR-BAJIN FORTRESS, SOUTHERN SIBERIA, RUSSIA)*

G. L. ALFIMOV and G. V. NOSYREV
National Research University of Electronic Technology, Zelenograd, Moscow, Russia

V. PANIN
Faculty of Geography, Moscow State University, Moscow, Russia

A. ARZHANTSEVA
Institute of Ethnology and Anthropology, Russian Academy of Sciences, Moscow, Russia

and G. OLEAGA†
Departamento de Matemática Aplicada, Facultad de Ciencias Matemáticas, Universidad Complutense, Madrid, Spain

The main objective of this study is to recover the initial geometry of earthen walls from the shape of wall remains. The original parameters of the walls have been estimated by fitting the field-measured profiles with the theoretical shape predicted by the model. We estimate: (i) the initial wall height (between 2 and 3 m); (ii) their shape (vertical or nearly vertical); and (iii) the time for complete degradation (between 250 and 400 years, depending on wall orientation). We show that this approach yields the best results when the main degradation factor is the temperature gradient, as it is for south-oriented wall faces.

KEYWORDS: RAMMED-EARTH CONSTRUCTION, EARTHEN ARCHITECTURE, HANGTU, SEMI-ARID CLIMATE, SLOPE DEGRADATION MATHEMATICAL MODELS, SLOPE MASS MOVEMENT, MEDIEVAL, UIGUR, TUVA

INTRODUCTION
Slope degradation processes have long been a focus for geologists and geomorphologists because of their theoretical implications in landscape development and their applied value in construction (engineering prospects) and agriculture (erosion). In archaeology, slope degradation has been studied in attempts to assess the rates of denudation and burial of archaeological structures. An example is the field experiment with the filling of a chalk ditch by denudation of its banks and barrow (Ashbee and Jewell 1998). The results of this experiment were used to test existing cliff degradation theories (Hutchinson 1998) and to develop models describing the siltation of different morphological types of archaeological ditches (Hutchinson and Stuart 2003). This approach can be applied to other kinds of archaeological objects exhibiting steep faces that suffer from

*Received 20 January 2012; accepted 17 July 2012
†Corresponding author: email goleaga@gmail.com
© University of Oxford, 2012
gravity-induced destruction. Old structures made of unconsolidated materials have experienced considerable transformation since their erection, and special consideration is required to establish their initial geometry. Objects of earthen architecture that occur widely across the world are of particular interest (Rael 2009). A mass-balance approach has been successfully applied to reconstruct the initial height of mud-brick buildings via volumetric estimations of decay products distributed along walls of known width (Scheglov 1982). Unfortunately, this approach requires a priori information about the initial shape of the wall, which is available for mud-brick walls (with a rectangular cross-section) but may be more complex for other construction types; for example, the rammed-earth technique. Apart from that, it is desirable that independent methods are used to verify the results based on the mass-balance approach.

In this paper, we explore the potential of geomorphological models for steep-slope degradation to recover the initial geometry of rammed-earth walls. As a case study we use the Por-Bajin fortress, an early medieval site with Chinese-style earthen architecture in the mountains of Tuva, southern Siberia. We focus on the evolution of the trapezoidal internal walls of the fortress and offer a mathematical model for the wall degradation process. A comparison of theoretical predictions for the shape of wall remains with those found during excavations allows us to estimate the initial height and shape of the walls, as well as the timescale of the degradation and destruction.

GENERAL CHARACTERISTICS OF THE SITE

Natural conditions
The archaeological site of Por-Bajin, or Por-Bazhyn (‘Clay House’ in Tuvinian), 50°36′54″N, 97°23′06″E, is located on a 6 ha island in the southwestern part of Lake Terekhol (Fig. 1). The climate is characterized as temperate semi-arid, with long cold winters (mean January temperature −29°C) and warm summers (July +14°C). Average annual precipitation is 320 mm, of which 70–80% falls as rain and 20–30% as snow. The average period of snow cover is 200 days, from the beginning of November to the middle of April. During the warm season, anticyclone weather conditions prevail, with hot days accompanied by cold nights, resulting in high daily temperature ranges (15–20°C). In June to August, precipitation is 55–70 mm monthly. Given the hot daylight conditions, this quantity is limiting for vegetation. Surface exposure is important in terms of local humidity and vegetation cover: plains and south-facing slopes are occupied by steppe-grass communities; more shadowed north-facing slopes are covered by taiga forests (mainly larch). Similar local variability of soil moisture and vegetation types is observed within the fortress due to shadowing from artificial walls.

Structure and building techniques of the fortress
The site is dated to the mid-eighth century AD (Vainshtein 1964) and is a rectangle (215 × 160 m), with its long side running approximately west–east (Fig. 2). The interior is taken up by a central palace complex, two large courtyards and a chain of small courtyards along the ramparts. Three types of walls can be distinguished (Fig. 2 (b)): first, massive external walls (ramparts) 8–10 m high and about 10 m thick at their base; second, internal courtyard walls, the remains of which are 1.5–1.8 m high and about 2 m thick at their base; and, third, walls of dwellings within the small courtyards, the remains of which are 0.6–0.9 m high and 0.5–0.6 m thick. Walls of courtyards and dwellings are completely buried under weathering products.
During the 2007–8 seasons, large-scale fieldwork was undertaken at the fortress. Apart from archaeological excavations, work included the complex study of the island and adjacent areas and their geomorphological processes. Prior to excavation, the entire island and its archaeological remains were laser-scanned, resulting in a complete topographical plan (1:200) and a digital 3D elevation model. This model served as the basis for all further work at the site.

All walls were made of lacustrine clays and built using the Chinese hangtu technique (Knapp 2000; Rael 2009). Hangtu construction works by putting down wet clay in a wooden frame and compacting it with wooden rams or blocks to produce a hard uniform layer that is 12–15 cm thick. The process is repeated until the required height of the wall is achieved (Knapp 2000). Building materials were extracted from a shallow quarry (limited by permafrost to a maximum depth of 2 m) located at the southern shore of the island and now covered by the lake (Fig. 2 (b)).

Given the orientation of the main axis of the fortress, the wall faces are almost exactly due north, south, east and west (in what follows, called N-faces, S-faces, E-faces and W-faces). Thus, a wall running north–south has an E-face and a W-face; a cross-section through such a wall would run east–west (in what follows, called an EW-profile). Correspondingly, a wall running east–west has a S-face and an N-face, and its cross-section runs south–north (in the following, called a SN-profile).

A MATHEMATICAL DESCRIPTION OF DEGRADATION PROCESSES

Basic factors causing the degradation of clay constructions

Since the site is located in a remote region with a low population density, the degradation of clay constructions can be assumed to have been caused mainly by natural processes. Seismic shocks...
Figure 2  The Por-Bajin fortress: (a) an aerial view from the north-west (July 2007); (b) a shaded elevation model obtained by a terrestrial laser scanning technique. Visible wall remains: 1, external walls (ramparts); 2, courtyard walls (2a, higher ones; 2b, lower ones); 3, walls of buildings. P, Central palace; G, main gate; V, two buildings excavated by S. I. Vainshtein in the late 1950s.
have been found to be responsible for repeated collapses of the outer ramparts (Panin 2011). Seismic activity has also affected the interior walls; however, in general, their destruction has been a slow and continuous process caused predominantly by gravitation, supplemented by water and wind actions. In the semi-arid conditions of the Terekhol Basin, another important factor of degradation is the vegetation. These factors will now be discussed in turn, in order to justify the assumptions for our mathematical model.

From a geomorphic viewpoint, hangtu walls may be seen as slopes of high gradient. Their weathering is similar to that taking place on natural steep slopes: detachment and falling down of individual particles, or small aggregates, and talus growing at the base. Processes similar to the destruction of hangtu walls have been described on sides of gullies cut into loess in the piedmont zone of western Tien-Shan (Golosov and Panin 1988). Loess slopes 8–10 m long and 35–50° steep erode most actively between January and March. Weathering rates demonstrate a close correlation with the weather conditions, which are governed by alternating 5–10 day long cyclone/anticyclone phases. Destruction of loess slopes occurs during dry and sunny anticyclonic phases, being a weathering-limited process (according to Carson and Kirkby 1972) driven by a combination of factors: loess moistening by melting snow, night frosts and high temperature gradients in the ground caused by intensive morning insolation. Sediment yield from slopes begins with the morning rise of the surface temperature between 8 a.m. and 10 a.m. and peaks between 10 a.m. and 2 p.m., along with the highest rates of surface temperature rise. It stops after all the surface material loosened by the night frost has been used up.

The similarity of the physical properties of compacted lacustrine silts (hangtu) and loess, as well as comparable climatic conditions—that is, seasons with daily temperature oscillations around the zero point—make the above process a probable mechanism for the destruction of the interior walls at Por-Bajin. Due to gravitation, weathered material of a wall falls down and accumulates near the wall base to form debris. The debris fan buries the wall base and prevents further weathering there. At the time of our fieldwork, almost all of the internal walls had already been completely buried under debris and thus conserved for an unknown time period.

The roles of water and wind transport are important for modelling, since both can transport wall material over large distances. The crucial question is whether they have contributed significantly to erosion. We argue that the effects of water and wind are much less than the effect of gravitation. First, no evidence has been found for water erosion, neither morphological (rills, fans) nor sedimentological (thinly graded lamination). Due to the very short slope length and the highly permeable debris, surface run-off will probably not occur even during high-intensity rainfall. Second, in an attempt to estimate the role of the aeolian factor, we cut a trial trench within the palace courtyard at a fair distance from any wall. This means that any sedimentation detected here would clearly be attributable to aeolian transport. Buried pre-fortress soil was found at a depth of only 3–5 cm. But even this small amount of aeolian deposition was derived mainly from the high outer ramparts, implying that there should be no need to include water- and wind-caused losses of material in our mathematical model of the destruction of interior walls.

The role of vegetation is twofold. On the one hand, roots of plants and shrubs destroy clay constructions. On the other hand, vegetation consolidates the surface layer and protects it from further weathering. The difference in insolation between S-faces and N-faces of walls makes the vegetation cover heterogeneous: S-faces are dry, and therefore vegetation is poor and corresponds to semi-desert assemblages. As a consequence, the organic content in the debris of S-faces is approximately equivalent to that in the initial construction material. By way of contrast, N-faces remain wet during the entire summer; vegetation there is richer and corresponds to steppe assemblages. Loss-on-ignition analysis shows that the organic content in debris on N-faces is...
twice that on S-faces. Moreover, dense shrub communities effectively catch sediment particles, leading to a steeper debris slope and therefore to more rapid burial and less denudation of the wall face.

The overall effect of vegetation is difficult to account for. It is included in our model indirectly, since it determines (together with other factors) the angle $\alpha$ that the debris surface forms with the level surface. We show that the model yields the best results for the faces where the effect of vegetation is minimal.

Mathematical models for wall degradation

For a rough estimate of wall height, one may use equations based on the balance of masses of wall and debris and on simple geometrical considerations (the ‘geometric’ approach). Such a method has been used, for instance, for estimating the height of adobe walls in the northern Black Sea region (Scheglov 1982). Let us assume that the wall is long enough and its initial cross-section is a trapezoid with base $w$, height $H$ and base angle $\beta$, and that the material of the wall was deposited on both sides of the wall remains. Let us denote the cross-section area of the wall remains by $S_0$, the area of left-hand side debris by $S_1$ and of right-hand side debris by $S_2$, and let us introduce factors $c_1$ and $c_2$ that describe the change of volume when the wall material is deposited into the right-hand and left-hand debris, respectively. Then the height $H$ can be calculated as follows:

$$H = \frac{1}{2\cot\beta} \left( w - \sqrt{w^2 - 4S \cot\beta} \right),$$

where $S = S_0 + S_1c_1 + S_2c_2$. The values of $S_{0,1,2}$, $c_{1,2}$, $\beta$ and $w$ can be measured in the field.

However, the practical application of the ‘geometric’ approach shows that the result is rather sensitive to the value of the angle $\beta$, which is not known exactly for the faces of walls constructed using the hangtu technique. For instance, the estimation of $H$ for the walls in Por-Bajin conditions by means of equation (1) yields a spread in values from 2.5 m to 5 m, as the angle $\beta$ varies between 70° and 90°. So, other methods for estimating $H$ are highly desirable.

An alternative approach to describe wall weathering can be adopted from geomorphological studies. It is based on the classic model for the degradation of a rock cliff called the Fisher–Lehmann (FL) model. This model was proposed by Fisher (1866) for vertical chalk cliff weathering and later generalized by Lehmann (1933). The basic assumptions of the FL model are as follows:

![Figure 3](image-url)  

Figure 3  
A cross-section of an originally trapezoidal-sectioned wall after completion of the weathering cycle: dark grey, wall remains (area $S_0$); light grey, debris composed basically of derivates from wall destruction (areas $S_1$ and $S_2$).
The ground surface is horizontal. The surface of the slope is rectilinear and forms angle $\alpha$ with the horizontal surface. Products of slope degradation accumulate at the foot of the slope, forming debris. The weathering process does not affect the lower part of the slope, which is covered by the debris. Weathering of the upper part of the slope not covered by debris occurs uniformly, with constant velocity $v$.

The material of the destroyed wall is distributed along the debris surface instantly and uniformly, and its volume increases by the constant factor $1/c$, where $0 < c \leq 1$.

A schematic illustration of the process is shown in Figure 4. The evolution of the point $A$ of the slope/debris contact can be described by the following differential equation:

$$
(H - y) \cdot (dx - dy \cot \beta) = c \left( dy - \frac{dx}{\cot \alpha} \right) y \cot \alpha.
$$

(2)

This differential equation is based on the simple balance relation between volumes $V_R$ and $V_D$ (for a clear derivation, see Scheidegger 1970). Here, $(x,y)$ are Cartesian coordinates with the origin at the point O at the slope foot before the degradation (Fig. 4). Equation (2) does not include $t$ explicitly. However, infinitesimal values of $dt$, $dx$ and $dy$ satisfy the relation

$$
v \ dt = dx - \cot \beta \ dy.
$$

The exact solution of equation (2) is as follows:

$$
x = \frac{cH(\cot \alpha - \cot \beta)}{(1 - c)^2} \ln \left( \frac{H}{H - (1 - c)y} \right) - \frac{(c \cot \alpha - \cot \beta)y}{1 - c}.
$$

(3)

Figure 4  A schematic illustration of the Fisher–Lehmann model.
It corresponds to the theoretical curvilinear surface, \( x = x_{FL}(y) \), of the wall remains that lie under the debris—*the FL curve*. Time \( t \) is related to \( y \) by the formula

\[
t = \frac{c(\cot \alpha - \cot \beta)}{v(1-c)} \left( \frac{H}{1-c} \ln \left( \frac{H}{H - (1-c)y} \right) - y \right).
\]  

The FL model also admits various generalizations (see, e.g., Van Dijk and Le Heux 1952; Scheidegger 1970; Obanawa and Matsukura 2006).

Typically, the FL model has been applied to long mountain slopes. However, it has recently been shown that this model properly describes the evolution of short slopes such as faces of artificial ditches (Hutchinson 1998; Hutchinson and Stuart 2003). The theoretical profiles agreed well with the results of a field experiment carried out in south-west England (Ashbee and Jewell 1998).

Let us now discuss ways to adopt the FL model for a description of wall degradation. Assume that the initial cross-section of the wall is a rectangle or isosceles trapezoid. Consider the following possibilities:

(i) **Scenario 1**: symmetrical destruction of a ‘rectangular’ or ‘trapezoidal’ wall (Fig. 5). Assume that the weathering process runs symmetrically on both faces of the wall obeying the FL model, with the same density coefficient \( c \). The surface of the wall remains on either side can then be approximated by the FL curve given in equation (3). However, this scenario implies that at the last stage of the process, a narrow crest is formed on the top of the wall (Fig. 5). If the crest thickness \( w_{cr} \) is much smaller than its height \( h_{cr} \), it becomes unstable. The evolution of this unstable crest can hardly be described by the FL model. This imposes a restriction for the maximum height of the wall: the volume of the crest should be small when compared with the total volume of the removed wall material. One can illustrate this reasoning using the following example. Let a ‘rectangular’ wall be 2 m wide, with \( \alpha = 22^\circ \) and \( c = 0.75 \), which are typical values in Por-Bajin conditions (see below). Let the initial height of the wall be 2.5 m. Then the value of \( h_{cr} \) according to the FL model is about 1 m. For a crest of width \( w_{cr} = 0.5 \) m (which still remains stable), the cross-section area is about 0.5 m\(^2\), which is small when compared to the cross-section area of the removed material, of about 2.9 m\(^2\). Therefore, we conclude that the FL model is applicable in this case. However, we should note that any crest destruction scenario affects the shape of the FL surface only *locally*, near its top.

(ii) **Scenario 2**: non-symmetrical destruction of a ‘rectangular’ or ‘trapezoidal’ wall (Fig. 6). Assume that the wall was initially symmetrical, of rectangular or trapezoidal section. Let one of the faces (for instance, the left one) retreat much faster than the other one, and let its destruction follow the FL scheme. The weathering of the right face may or may not obey the FL scenario. The

![Figure 5](image_url)  
*The destruction of a rectangular-sectioned wall, leading to the formation of a curvilinear FL surface under the debris (D).*

stages of the process are shown in Figure 6. The left part of the wall remains, which is covered by the debris, should also match the FL surface—probably except for its top part, which is affected by the destruction of the crest and the opposite face of the wall. So, in both scenarios a large part of the section obeys the FL curve. The comparison of the theoretical FL curves calculated for various parameters $H$ and $\beta$ with excavated sections of the wall remains will then yield an estimate for the wall characteristics.

DATA FOR THE FL MODEL: A FIELD AND LABORATORY STUDY

Interior wall profiles

During fieldwork, 11 profiles of interior wall remains were obtained (for their locations, see Fig. 7). Six of them cut south–north walls (giving EW-profiles), while the other five profiles cut east–west walls (giving SN-profiles). The profiles of the wall remains and the corresponding
Debris profiles were digitized using an electronic tachymeter with centimetre accuracy (Fig. 8). It is noteworthy that the SN-profiles (the right-hand column in Fig. 8) display several common features: they are not symmetrical, with both the wall contour and the debris contour having a steeper north side. By way of contrast, all EW-profiles (the left-hand column in Fig. 8) are quite symmetrical.

Evaluation of angle $\alpha$

The FL model assumes that the angle $\alpha$ formed by the debris surface with the horizontal level is a constant value. In a rough approximation, $\alpha$ can be identified with the angle of repose of the debris material. Being one of the basic characteristics of granular substance, the angle of repose is identified with the maximum angle that ensures static equilibrium of the material. However, real conditions are far from laboratory ones due to the heterogeneity of the wall material, variable humidity and vegetation. Therefore, the angle $\alpha$ has been calculated by approximating the debris profile by a straight line and measuring the angle between this line and the horizontal axis. This straight-line approximation is correct for the part of debris adjacent to the wall, but it becomes questionable at some distance from the wall. In order to verify this approximation, we evaluated the volume of debris for several selected wall faces, both directly and using the straight-line approximation. The discrepancy was about 10% and was regarded as admissible.

Multiple measurements made using the digital elevation model (Fig. 2) show that the angle $\alpha$ is different for wall faces of different orientations. For S-faces, this angle is (on average) about 22°, for E- and W-faces it is about 25°, and for N-faces it is about 29°. The increase in angle...
values reflects the stabilizing role of grass cover and debris humidity. Both soil humidity and density of vegetation cover are minimal on sun-exposed S-faces, and maximal on shaded N-faces.

The composition of wall and debris material and the computation of coefficient c

The material of SN-profile 4 and EW-profile 6 was subjected to sampling and laboratory study. Samples of unit volume were taken from uncovered walls (15–18 samples) as well as from debris sections (25–35 samples at different distances from the wall). The wall material was sampled in such a way that there were two samples from each hangtu layer (12–14 cm). For each sample, its mass and density were measured. The material was then dried at 105°C in a muffle furnace. The mass change corresponds to the mass of water in a volume unit of wall material. Then the samples were annealed at 500°C. The mass of organic substances in a volume unit of wall material is equal to the mass lost while annealing. The same procedure was repeated for the debris samples. The coefficient $c$, a volumetric ratio of the hangtu wall material to its derivates (debris), was then calculated by means of the following formula:

$$c = \frac{\rho_2 \gamma_2}{\rho_1 \gamma_1}.$$  

Here, $\rho_{1,2}$ are the densities of the wall and the debris, respectively, and $\gamma_1$ and $\gamma_2$ are the portions of pure clay, without water and organic substances, per unit of volume of the wall and the debris. The value of $c$ was used for the numerical calculation of the FL profile, in equation (3).

The results of the laboratory study are shown in Table 1. For the EW-profiles, the debris material is similar on both sides of the walls and at different distances from the walls. For the SN-profiles, the debris material differs significantly on the north and south sides. On the south side, the debris material is denser close to the wall, and lighter and more friable away from the wall, due to the increasing organic content. On the north side, the debris material is lighter and richer in organic remains.

The excavations of 1957: a micro-model of wall degradation

During the 2008 fieldwork, we inspected the remains of a building excavated in 1957 by S. Vainshtein (Fig. 7). The excavated wall remains were about 1 m high and 0.6–0.8 m wide at the base. As they were not protected by any conservation measures, they have undergone weathering, with derivates accumulating at the wall base. Two typical profiles of the wall remains with debris, an SN-profile and a EW-profile, are shown in Figure 9. One should note that the SN-profile is

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Average physical properties of wall and debris material: for the debris, averaging has been done over the parts adjacent to the wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side</td>
<td>Density (kg m$^{-3}$)</td>
</tr>
<tr>
<td></td>
<td>Wall</td>
</tr>
<tr>
<td>Profile 4, south side</td>
<td>1100</td>
</tr>
<tr>
<td>Profile 4, north side</td>
<td>1100</td>
</tr>
<tr>
<td>Profile 6, east side</td>
<td>1150</td>
</tr>
</tbody>
</table>

asymmetrical, with a much flatter and more rectilinear south slope, as is typical of all SN-profiles (Fig. 8). We collected and weighed material accumulated along 1 m of wall length at walls facing in various directions. The result was 73.7 kg for the S-face, 8.4 kg for the N-face, 35.6 kg for the E-face and 49.6 kg for the W-face. This means that the degradation process for an S-face is much faster than for all other faces.

As the time of degradation since the excavation of this building is known (about 50 years), the degradation rate can be calculated. Given the quantities of accumulated weathering products, the retreat rate of the wall faces can be estimated at 0.004–0.006 m per year for S-faces and 0.002–0.003 m per year for E- and W-faces.

RESULTS

Comparison of wall profiles with FL curves

All the digitized wall profiles were compared with theoretical FL curves calculated for various values of $\beta$ (between 75° and 90°) and $H$. The ‘best-fit’ curve for a fixed value of $\beta$ is the FL curve that passes above the wall profile in such a way that the area $D_S$ between the wall profile and the FL curve is minimal. Some illustrations for the comparison procedure are presented below.

SN-profiles. Three S-faces (profiles 4, 7 and 8) are shown in Figure 10 against some FL profiles calculated for two values of angle $\beta$ ($\beta = 75^\circ$ and $\beta = 90^\circ$) and various values of initial height $H$. It follows from Figure 10 that for both $\beta$ values there is an FL profile that fits the S-face of the wall contour well. For $\beta = 75^\circ$, the height of the walls varies from 2.5 m to more than 4 m, whereas for $\beta = 90^\circ$, the corresponding height lies between 2 m and 2.5 m.

In Figure 11 three N-faces (profiles 4, 7 and 8) are approximated by some FL profiles calculated for two values of angle $\beta$ ($\beta = 75^\circ$ and $\beta = 90^\circ$) and various values of initial height $H$. It follows from Figure 11 that the match is extremely bad for $\beta = 75^\circ$, and doubtful for $\beta = 90^\circ$.

EW-profiles. It turns out that the approximation of E- and W-faces by the FL curves is worse than for S-faces. Figure 12 shows the fit of both sides for three EW-profiles (2, 6 and 11) by FL curves. Thus, assuming $\beta = 75^\circ$, the initial height of the wall can be estimated to about 4 m for profiles 6 and 11, and to more than 4 m for profile 2. Assuming $\beta = 90^\circ$, one obtains heights of about 2.5–3.0 m for all three profiles.

The results of approximation of the actual profiles by FL curves are collated in Table 2. The following comments may be added:

- The results are given for five EW-profiles and four SN-profiles. Two of 11 profiles were excluded: EW-profile 1, with the highest wall remains (1.95 m), and SN-profile 10, with the
lowest wall remains (1.3 m). In both cases, the destruction process was influenced by the destruction of neighbouring building features and therefore the results may not be representative.

- The results for N-faces were not included in Table 2 because the correspondence between the theoretical and actual profiles in these cases is bad (Fig. 11).

Figure 10  Fitting S-faces of profiles 4 (a, b), 7 (c, d) and 8 (e, f) to calculated FL profiles: upper row, base angle $\beta = 75^\circ$; lower row, base angle $\beta = 90^\circ$. The heavy lines show the best fit. The H values denote the initial wall heights (m) corresponding to particular FL profiles.

Figure 11  Fitting N-faces of profiles 4 (a, b), 7 (c, d) and 8 (e, f) to calculated FL profiles: upper row, base angle $\beta = 75^\circ$; lower row, base angle $\beta = 90^\circ$. The H values denote the initial wall heights (m) corresponding to particular FL profiles.
Figure 12  Fitting profiles 2 (a, b), 6 (c, d) and 11 (e, f) to calculated FL profiles: upper row, base angle $\beta = 75^\circ$; lower row, base angle $\beta = 90^\circ$. The $H$ values denote the initial wall heights (m) corresponding to particular FL profiles.

Table 2  Reconstruction of the wall heights for nine profiles obtained in fieldwork. Legend: W, west face of profile; E, east face of profile; S, south face of profile. Numbers in bold indicate the cases in which $\Delta S$ is less than 10% of the area of the approximated face. See other comments in the text.

<table>
<thead>
<tr>
<th>Profile</th>
<th>Height of wall remains (m)</th>
<th>Estimates of initial wall height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>‘Geometric’ approach, $\beta = 90^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta = 75^\circ$</td>
</tr>
<tr>
<td><strong>SN-profiles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 (S)</td>
<td>1.6</td>
<td>2.6</td>
</tr>
<tr>
<td>4 (S)</td>
<td>1.5</td>
<td>2.3</td>
</tr>
<tr>
<td>7 (S)</td>
<td>1.5</td>
<td>–</td>
</tr>
<tr>
<td>8 (S)</td>
<td>1.4</td>
<td>–</td>
</tr>
<tr>
<td><strong>EW-profiles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (W)</td>
<td>1.7</td>
<td>2.5</td>
</tr>
<tr>
<td>2 (E)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>5 (W)</td>
<td>1.6</td>
<td>–</td>
</tr>
<tr>
<td>5 (E)</td>
<td>–</td>
<td>$=3.5$</td>
</tr>
<tr>
<td>6 (W)</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>6 (E)</td>
<td>–</td>
<td>$=4$</td>
</tr>
<tr>
<td>9 (W)</td>
<td>1.6</td>
<td>–</td>
</tr>
<tr>
<td>9 (E)</td>
<td>–</td>
<td>$&gt;4$</td>
</tr>
<tr>
<td>11 (W)</td>
<td>1.6</td>
<td>2.1</td>
</tr>
<tr>
<td>11 (E)</td>
<td>–</td>
<td>$&gt;4$</td>
</tr>
</tbody>
</table>
• In all calculations, the coefficient $c$ was taken as 0.87 for S-faces and 0.77 for E- and W-faces (Table 1). The angle $\alpha$ was calculated by means of the procedure described above. The value of $\alpha$ was taken as 22° for S-faces and 25° for E- and W-faces.
• The cases of particularly good correspondence between FL curves and real profiles ($\Delta S$ smaller than 10% of the area of the approximated face) are marked in bold. This is the case mainly for S-faces, with $\beta = 80°$ and $\beta = 90°$.
• For the profiles where volumes of debris are known for both sides, the heights calculated by means of equation (1) are also given (in the column ‘Geometric’ approach).
• For the base angle $\beta = 75°$, the majority of the calculated $H$ values are above 4 m. In our opinion, this value is doubtful. Thus, the FL model suggests that the base angle $\beta$ was greater than 75°.

**SUMMARY AND DISCUSSION**

The weathering of *hangtu* (compacted clay) interior walls in the Por-Bajin fortress is driven by the same mechanisms that act in semi-arid climates on steep slopes composed of natural clay or loess. Most of the weathered material has been found to accumulate at wall bases. This allows for the application of cliff degradation models developed in theoretical geomorphology. We suggest a mathematical model based on the Fisher–Lehmann approach (Fisher 1866; Lehmann 1933) to study the processes of destruction of artificial walls. Comparison of the uncovered profiles of wall remains with the predictions of the Fisher–Lehmann model allows estimation of the initial wall height. In this study, the walls were assumed to be initially trapezoidal in cross-section. The base angle of the trapezoid was considered as an unknown parameter, with possible values between 75° and 90°. The lowest dispersion of estimated wall heights was found for walls with vertical (90°) faces. The reconstructed height values were in the range of 2–3 m. If the base angle of the trapezoid is assumed to be about 75°, the correspondence between the theoretical and actual wall profiles is, in general, poor. We therefore conclude that, initially, the wall faces were vertical or near-vertical.

The SN-profiles of walls (i.e., profiles facing north and south) are asymmetrical. We assume that their destruction follows Scenario 2 (see above). The best correlation between the predictions of the model and the observed profiles of wall remains has been found at southern wall faces. The characteristic features of these faces are high temperature gradients in daily weathering cycles and poor vegetation. Degradation of these faces is faster than on other faces. For northern wall faces, the correlation between the theoretical and actual profiles was very bad. The Fisher–Lehmann model is probably not applicable in this case, because of the significant role of the vegetation in stabilizing debris fans. North faces of walls should complete their weathering cycle much faster due to the formation of a protective cover of grass and shrubs in this shaded and therefore relatively humid location.

The temperature gradient mechanism probably also predominates in the cases of east and west faces. Wall degradation in EW-profiles (i.e., profiles facing east and west) proceeds symmetrically. We assume that their destruction follows Scenario 1. However, the process runs more slowly, probably due to lower heating gradients than in the case of south faces. The correlation between the actual and computed profiles is worse for east and west faces than for south faces. The Fisher–Lehmann model may be improved in this case by taking into account non-uniform weathering rates at various wall levels. However, assuming that the model still gives a rough approximation of the wall degradation process, we chose predicted profiles that best fitted the actual profiles, and found good agreement for south–north profiles: assuming initially near-vertical faces, the estimates for the initial wall heights are between 2 and 3 m.
The discovery of a building excavated in 1957 and subjected to weathering over 50 years allows us to calculate the rate of wall degradation. Rates of weathering of steep hangtu walls have been estimated at 2–3 mm per year for east and west faces, 4–6 mm per year for south faces, and about an order of magnitude less for north faces. Thus, the Fisher–Lehman model yields an estimate of the time for complete wall degradation. Consider an interior wall with an SN-profile, of rectangular cross-section, and 2 m wide by 2.5 m high, and assume that degradation follows Scenario 2. Then, according to equation (4), the Fisher–Lehmann process passes through three-quarters of the wall thickness from the south in a period of between 250 and 375 years. This means that within this interval, the remains of the wall will become completely buried under the products of wall degradation. In the case of the EW-profile, Scenario 1 can be assumed. A Fisher–Lehmann process on a wall of the same geometry as assumed before passes through four-fifths of the wall thickness in a period of between 275 and 400 years. The residual wall top is a ‘crest’ 1 m high and 40 cm wide, which is unlikely to survive for long. Therefore, a period of between 250 and 400 years seems to be a plausible estimate for the whole cycle of complete wall degradation.

ACKNOWLEDGEMENTS

The authors are grateful to the Por-Bajin Fortress Cultural Foundation for financial support. The contributions of A.V.P. and I.A.A. were supported in part by the Russian Foundation for Basic Research (project no. 09-05-00351). G.O. was partially supported by the Spanish project MTM2007-61755. We appreciate the assistance from a number of colleagues in the field and in computer data processing, and we offer special thanks to M. Bronnikova, N. Kosevich and E. Seleznева. We would like to express special gratitude to Professor H. Härke for his invaluable help in the correction of English and for his useful comments.

REFERENCES

Knapp, R. G., 2000, China’s old dwellings, University of Hawai‘i Press, Honolulu.