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An Analytical Model for the Propagation of Thermal Runaway Electrons in Solar Flares

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Abstract—The nature of the hard X-ray emission from solar flares is well known. The observed emission in both the corona and the chromosphere consists of two components: nonthermal and thermal. The nonthermal and thermal components are attributable to the bremsstrahlung of accelerated electrons and heated plasma electrons, respectively. Since the nonthermal and thermal hard X-ray emission spectra partially overlap, their proper interpretation directly depends on the accuracy of the kinetic models describing the propagation of thermal and nonthermal runaway electrons in the solar atmosphere. The evolution of the distribution function for the latter, i.e., the electrons accelerated in the magnetic reconnection region, is accurately described in the approximation of present-day thick-target models with a reverse current. Here we consider a model for the thermal runaway of electrons and find an analytical solution of the corresponding kinetic equation in which the Coulomb collisions are taken into account. The degree of polarization of the emission has been estimated to be no greater than $\sim$5%. The derived distribution function can also be used to calculate the thermal X-ray emission spectrum and, as a consequence, to interpret the observations of the thermal component in the X-ray spectrum of a solar flare.

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1. INTRODUCTION

According to the historically first, but largely correct and fundamental theoretical views of the solar flare mechanism (Giovanelli 1948; Parker 1957; Sweet 1958, 1969; Syrovatskii 1962, 1966), strong magnetic fields in the solar atmosphere are the flare energy source. A key role of the peculiar redistribution of magnetic fluxes that changes their topological connectivity, magnetic reconnection, was shown in the mentioned classic papers. As a result of the magnetic reconnection effect, the energy of interacting magnetic fluxes is converted into the kinetic energy of charged particles and fast directed MHD plasma flows, jets.

Despite the great variety of physical conditions under which magnetic reconnection is realized as a fundamental mechanism of primary energy release in solar flares, the overall picture of the flare and its scenario are currently believed to be understandable and well known (Priest and Forbes 2000; Somov 2012, 2013; Krucker et al. 2008; Zharkova et al. 2011; Emslie et al. 2012). The conditions necessary for fast magnetic reconnection are formed in the solar corona before the beginning of the most powerful, so-called impulsive phase, which lasts from several seconds to tens of seconds. During such reconnection the electrons, protons, and other ions are accelerated by an electric field inside the high-temperature reconnecting current layer to energies much greater than the thermal energies of particles in the solar corona and chromosphere (Hudson and Ryan 1995; Somov 2000; Aschwanden 2002; Miroshnichenko 2015).

A typical flare scenario is shown schematically, but with the sequence of physical processes and their relative positions in Fig. 1. From the solar corona the plasma with a frozen-in strong magnetic field flows into the reconnecting current layer (RCL) with a relatively low velocity $\mathbf{v}_0 \sim 10$ km s^{-1}. Inside the current layer the freezing-in conditions are violated and the reconnected magnetic field lines together with the "super-hot" (an electron temperature $T_e \gtrsim$ 30 MK), almost collisionless plasma move from the super-hot current layer in opposite directions (predominantly upward and downward) with velocities $\mathbf{v}_1 \sim 10^3$ km s^{-1} (Somov 2013). The bremsstrahlung of super-hot plasma electrons and accelerated elec-

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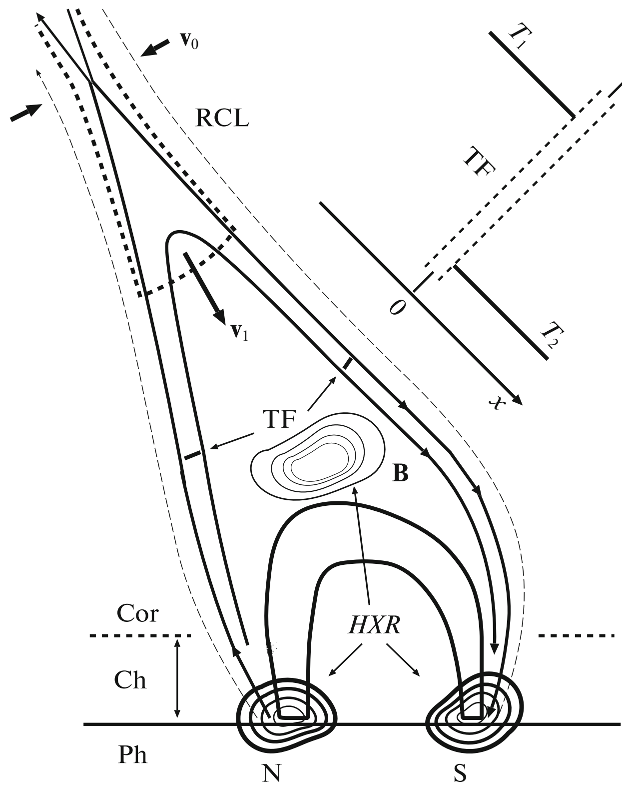

Fig. 1. The most significant fragment of the classical picture of a solar flare. The energetic electrons run away from a reconnecting current layer (RCL) with temperature T_1 through a turbulent front (TF) into a less hot (colder) flare-loop plasma with temperature T_2.

trons produces a source of hard X-ray emission moving in the corona. Gradually cooling, the super-hot plasma becomes visible in less hard X-ray emission. Figure 1 shows only the most prominent fragment of the extended region of the entire flare, namely the reconnected magnetic field lines $\mathbf{B}$ moving from the current layer with velocity $\mathbf{v}_1$ downward, toward the chromosphere (Ch) and the photosphere (Ph); N and S are a pair of photospheric magnetic field sources, for example, sunspots.

Penetrating into the chromosphere, where the plasma density is much higher, the energetic electrons rapidly lose their kinetic energy through Coulomb collisions. Here, as in the corona, they generate hard X-ray bremsstrahlung, quite often the most intense one. The sources of this emission lie at the footpoints of the tubes of reconnected magnetic field lines, the so-called flare loops, and the tube footpoints collectively form the "flare ribbons." The latter are accessible to the most comprehensive study through ground-based and space observations. The contours in Fig. 1 indicate the coronal and chromospheric sources of hard X-ray emission.

In some flares (for example, with a bright coronal source) allowance for the primary acceleration in the reconnecting current layer turns out to be insufficient. The scenario for the double successive acceleration of electrons of one population (the acceleration in the current layer plus the succeeding acceleration of the same electrons in a collapsing magnetic trap) was proposed by Somov and Kosugi (1997) and was called double step acceleration. This effect should not be confused with the so-called two-phase acceleration (Wild 1963), when the primary accelerated electrons during the impulsive flare phase (several seconds) or, possibly, quite different electrons are accelerated to relativistic energies (as was suggested previously, at shock waves) considerably later, during the second flare phase (with a delay from several minutes to hours; Sakai and de Jager 1996).

The hard X-ray emission spectrum of a solar flare is formed by both thermal and nonthermal runaway electrons. A complete model description of the propagation of energetic electrons in the solar atmosphere (heated and accelerated ones) includes both the coronal collapsing magnetic traps accelerating both thermal and nonthermal particles (Somov and Bogachev 2003; Bogachev and Somov 2005, 2007) and the reverse-current effect (Gritsyk and Somov 2014). The thermal model proposed by us is an inseparable part of the complete description, because it takes into account the interaction of thermal or, more precisely, super-hot electrons with the corona and the chromosphere. Allowance for the emission generated by them is needed to interpret the hard X-ray emission of a flare both in the corona and, especially, in the chromosphere, where the target is thick.

The nonthermal and thermal components of the hard X-ray emission partially overlap not only in photon energy, but also in space. An additional difficulty in interpreting the observed spectra and spatial distributions of the hard X-ray emission from solar flares stems from the fact that the energetic super-hot electrons run away from the super-hot plasma along reconnected magnetic field lines through a thermal turbulent front (TF). The latter moves with a velocity higher than the super-hot plasma velocity $\mathbf{v}_1$ (see Fig. 1). Thus, the total emission in the region under the turbulent layer is the sum of two components, nonthermal and thermal.

The thermal component in the hard X-ray emission spectrum at low energies dominates in the case of a chromospheric source. However, in the corona, near the turbulent front (TF), the picture is somewhat "smeared" by the contribution from the thermal bremsstrahlung of the super-hot plasma located near the reconnection region. Furthermore, the presence of a collapsing magnetic trap can produce additional plasma heating in the corona. Despite the great variety and complexity of physical processes in solar flares (see, e.g., Somov 1992), a turbulent front will inevitably be present between the super-hot plasma and the less hot background plasma of the solar atmosphere in any case and within any realization of a specific flare.

At present, there is no doubt that the observed hard X-ray emission with a photon energy $\mathcal{E}_{h\nu} \gtrsim 20$ keV is attributable to the bremsstrahlung of electrons accelerated by an electric field in the magnetic reconnection region. The boundary distribution function of such particles is in the form of a power law, while its change with depth is accurately described in terms of the thick-target model with a reverse current (Litvinenko and Somov 1991). In Gritsyk and Somov (2014, 2016, 2017) this approximation was successfully used to interpret the highly accurate satellite observations of the December 6, 2006 and July 19, 2012 solar flares. The model description of the propagation of thermal electrons running away from the reconnection region and generating X-ray bremsstrahlung with a photon energy $\mathcal{E}_{h\nu} \lesssim 20$ keV (the thermal component in the spectra of the coronal and chromospheric sources) is of great practical importance for interpreting the total X-ray spectrum of a flare.

The goal of this paper is an investigation and model description of the thermal runaway electron propagation processes during solar flares. This model is needed to interpret the thermal component in the X-ray emission spectra and is particularly topical in connection with a significant increase in the accuracy of present-day space observatories and, as a consequence, the need for using more accurate kinetic models for the propagation of electrons during flares. The corresponding kinetic problem is formulated in Section 2. Its analytical solution is presented in Section 3. In Section 4, based on the solution obtained, we estimate the polarization of the hard X-ray emission. The necessary conclusions are contained in the Conclusions.

2. FORMULATION OF THE PROBLEM

(A) Physical formulation of the problem. As has been noted in the Introduction, the thin reconnecting current layers located in the regions of interaction of the magnetic fluxes in the solar atmosphere, predominantly in the corona, are the energy source in solar flares. During flares in the current layers the magnetic energy is converted into the thermal and kinetic energies of the plasma and accelerated particles. At the same time, the accelerated particles excite plasma turbulence that heats the plasma electron component in the current layer to huge temperatures: $T_1 \gtrsim 10^8$ K (Somov 1981). This layer is commonly called the *super-hot* turbulent current layer (Somov 2013). The super-hot turbulent current layer replenish the "reservoir" of super-hot electrons. The Maxwellian distribution of these electrons within our problem formulation is characterized by temperature T_1.

The present-day "thermal models" somehow take into account the interaction of high-temperature electrons with the ion-acoustic turbulence excited by the reverse current. This interaction produces a front of anomalous heat conduction (the turbulent front TF in Fig. 1) propagating along the flare loop (see Fig. 1.2.6 in Somov (1992)) into a colder ($T_2 \gtrsim 10^6$ K in the corona, $T_2 \lesssim 10^4$ K in the chromosphere) plasma with the ion sound speed. As a result, in contrast to the thick-target model (see, e.g., Syrovatskii and Shmeleva 1972; Somov and Syrovatskii 1976), the emitting region gradually fills the entire volume of the magnetic flux tube as the front advances toward the chromosphere.

The turbulent layer of anomalous heat conduction is thin with respect to the super-hot electrons and cannot change their Maxwellian distribution. If this layer were thick, then, in principle, it could affect the anisotropy of runaway electrons. However, as our calculations will show, the angular distribution of super-hot electrons remains almost isotropic down to significant depths in the chromosphere. Within our problem formulation $T_2 \lesssim 10^4$ K is the Maxwellian temperature of the background plasma. Therefore, there is nothing to prevent us from using the Landau collision integral to describe the Coulomb interaction of runaway electrons with the plasma.

Thus, we assume that the electron temperature drops rapidly along the magnetic flux tube under consideration to the surrounding plasma temperature ($T_2 \sim 10^4$–10^6 K). To be more precise, we assume that the characteristic length scale at the temperature gradient is small compared to the mean free path of super-hot electrons. In this case, the energy transfer from the reservoir into the hard X-ray emission source (the low-temperature part of the flare loop; see Fig. 1) occurs non-diffusively. One way of such energy transfer is the thermal runaway of electrons. This effect ensures the propagation of the directed flow of energetic electrons into the emission source. In this paper we do not consider the excitation of ion-acoustic oscillations in the surrounding plasma by the beam and related phenomena.

(B) Mathematical formulation of the problem. Let, for simplicity, in the frame of reference associated with the front the super-hot and cold plasmas occupy two half-spaces, $x < 0$ and $x > 0$, separated by a flat thin turbulent layer TF at $x = 0$ (Fig. 1). The essence of the thermal runaway of super-hot electrons through the turbulent front TF (Fig. 1) into the colder background plasma is that the mean free path of an electron in the plasma increases with its kinetic energy $\mathcal{E}$:

$$l_{\mathcal{E}} = l_T(\mathcal{E}/k_\mathrm{B}T_1)^2, \tag{1}$$

where l_T is the mean free path of thermal electrons with temperature T_1 in the super-hot plasma and k_B is the Boltzmann constant. As a result, in the presence of a temperature gradient (in general, any gradient), some flow of fast electrons with a mean free path greater than the characteristic length scale on which the temperature varies, i.e.,

$$l_{\mathcal{E}} > \lambda \equiv T_1/|\nabla T_1|, \tag{2}$$

emerges, in addition to the diffusion of thermal electrons.

This causes the number of fast electrons in the cold plasma to increase. By analogy with the runaway of electrons in an electric field, this phenomenon was called the *thermal runaway* of electrons and was investigated, for example, by Gurevich and Istomin (1979). To find the distribution function of runaway electrons $f_\mathbf{v}$ in the cold plasma, we used the kinetic equation with the Landau collision integral calculated for a fully ionized Maxwellian plasma (for more details, see, e.g., Somov 2012):

$$\frac{\partial f_\mathbf{v}}{\partial t} + v\cos\theta\frac{\partial f_\mathbf{v}}{\partial x} - \frac{eE}{m_e}\frac{\partial f_\mathbf{v}}{\partial v} = St_\mathrm{L}(f_\mathbf{v}), \tag{3}$$

where

$$\begin{aligned} &St_\mathrm{L}(f_\mathbf{v}) \\ &= \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\nu_\mathrm{coll}(v)\left(\frac{k_\mathrm{B}T_2}{m_e}\frac{\partial f_\mathbf{v}}{\partial v} + vf_\mathbf{v}\right)\right] \\ &+ \nu_\mathrm{coll}(v)\frac{\partial}{\partial\cos\theta}\left(\sin^2\theta\frac{\partial f_\mathbf{v}}{\partial\cos\theta}\right). \end{aligned} \tag{4}$$

Expression (4) implies that the velocities of energetic electrons $v \gg (2k_\mathrm{B}T_2/m)^{1/2}$, i.e., a linearized Landau collision integral is considered. We assume that the cold plasma is composed of electrons and protons with a constant temperature T_2.

The plasma parameters are also assumed to change only along one coordinate x, θ is the angle between the electron velocity vector $\mathbf{v}$ and the x axis; in our formulation of the problem E is the reverse-current electric field strength (see Somov 2012). The subscript "$\mathbf{v}$" indicates that the sought-for function $f_\mathbf{v} = f_\mathbf{v}(x, v, \theta)$ is the electron distribution function in velocity vector $\mathbf{v}$.

In the collision integral (4) the collision frequency of energetic electrons with thermal electrons and protons in the cold plasma is

$$\nu_\mathrm{coll}(v) = \frac{4\pi n_2 e^4}{{m_e}^2 v^3}\ln\Lambda, \tag{5}$$

where n_2 is the electron number density in the cold plasma and $\ln\Lambda$ is the Coulomb logarithm.

Gurevich and Istomin (1979) made two simplifications. First, on time scales of the order of the Coulomb collision time the injection of electrons in the cold target plasma may be considered as a stationary process, while their distribution in the target, i.e., in the half-space $x > 0$ (Fig. 1), may be considered as a steady-state one. For this reason, the term $\partial f_\mathbf{v}/\partial t$ in the kinetic equation (3) may be neglected. Second, the reverse-current electric field strength E was set equal to zero.

(C) Dimensionless form of the equations. Let us introduce dimensionless variables:

$$\mu = \cos\theta,$$

$$s = \frac{\pi e^4 \ln\Lambda}{(k_\mathrm{B}T_1)^2}\int\limits_0^x n_2(x')dx'$$

is the ratio of the penetration depth of energetic electrons into the cold plasma (the so-called target) to the mean free path,

$$z = m_e v^2/2k_\mathrm{B}T_1$$

is the ratio of the kinetic energy of energetic electrons to the thermal energy of super-hot plasma particles,

$$\tau = T_2/T_1$$

is the ratio of the cold plasma temperature to the super-hot plasma temperature.

In these new variables the kinetic equation (3) takes the form

$$z^2\mu\frac{\partial\varphi}{\partial s} - 2z\frac{\partial\varphi}{\partial z}\left(1 - \tau\frac{\partial\varphi}{\partial z}\right) \tag{6}$$
$$- 2\tau z\frac{\partial^2\varphi}{\partial z^2} + 2\mu\frac{\partial\varphi}{\partial\mu} + \left(1-\mu^2\right)\left(\frac{\partial\varphi}{\partial\mu}\right)^2$$
$$- \left(1-\mu^2\right)\left(\frac{\partial^2\varphi}{\partial\mu^2}\right) = 0.$$

Instead of the distribution function $f_{\mathbf{v}}$, a new function is introduced here:

$$\varphi = -\ln f_{\mathbf{v}}. \tag{7}$$

In contrast to our formulation of the problem, the following parameter is small in Gurevich and Istomin (1979):

$$\gamma = \frac{l_T}{T_1}\left|\frac{dT_1}{dx}\right|_{\max} \ll 1, \tag{8}$$

where

$$l_T = \frac{(k_{\mathrm{B}}T_1)^2}{\pi e^4 n_1 \ln\Lambda}$$

is the mean free path of thermal electrons in the super-hot plasma, $(dT_1/dx)_{\max}$ is the maximum value of the temperature gradient, and n_1 is the electron number density in the super-hot plasma. The condition (8) implies that the temperature gradient is fairly small. Thus, Eq. (6) was in the form

$$z_\gamma{}^2\mu\frac{\partial\varphi}{\partial s_\gamma} - 2z_\gamma\frac{\partial\varphi}{\partial z_\gamma}\left(1 - 2\tau\gamma^{1/2}\frac{\partial\varphi}{\partial z_\gamma}\right) \tag{9}$$
$$- 4\gamma^{1/2}\tau z_\gamma\frac{\partial^2\varphi}{\partial z_\gamma{}^2} + 2\mu\frac{\partial\varphi}{\partial\mu} + \left(1-\mu^2\right)\left(\frac{\partial\varphi}{\partial\mu}\right)^2$$
$$- \left(1-\mu^2\right)\left(\frac{\partial^2\varphi}{\partial\mu^2}\right) = 0,$$

where the variables $s_\gamma = 4\gamma s$ and $z_\gamma = 2\gamma^{1/2}z$. This allowed a solution of Eq. (9) to be sought in the form of a series in powers of the small parameter γ, i.e.,

$$\varphi = \frac{1}{\gamma^{1/2}}\varphi_0 + \frac{1}{\gamma^{1/4}}\varphi_1 + \varphi_2 + \ldots. \tag{10}$$

In this case, the main distribution function φ_0 turned out to be spherically symmetric, while the correction to it φ_1, on the contrary, was highly directional at high energies. The analysis performed by the authors showed that the non-diffusive flow of electrons, which depends exponentially on the temperature gradient, exceeds the diffusive one for $\gamma > 10^{-2}$. However, as has been noted in the Introduction when formulating the model under consideration as applied to solar flares, the electron temperature gradient is great, i.e., $\gamma = l_T/\lambda \geq 1$, where λ is the characteristic length scale of the gradient. Thus, the distribution function (10) derived by Gurevich and Istomin (1979) cannot be used in our model, because its derivation essentially suggests the condition (8).

3. SOLVING THE PROBLEM OF THE THERMAL RUNAWAY OF SUPER-HOT ELECTRONS

In applications to solar flares, not γ but another parameter, τ, is small in Eq. (9). This allows us to find a solution for the super-hot electrons in the cold plasma of the solar atmosphere during a flare. The parameter γ will not be assumed to be small. On the contrary, bearing in mind the physical conditions in a solar flare, especially during its "hot" or "main" phase (Somov 1992), we will set

$$\gamma = 1. \tag{11}$$

Furthermore, we will first seek a solution in a small neighborhood of the axis of the runaway electron flux, i.e., near the point $\mu = 1$. Neglecting the last two terms that contain the factor $\left(1-\mu^2\right)$ in Eq. (9), we obtain the equation

$$z_\gamma{}^2\mu\frac{\partial\varphi}{\partial s_\gamma} - 2z_\gamma\frac{\partial\varphi}{\partial z_\gamma}\left(1 - 2\tau\frac{\partial\varphi}{\partial z_\gamma}\right) \tag{12}$$
$$- 4\tau z\frac{\partial^2\varphi}{\partial z_\gamma{}^2} + 2\mu\frac{\partial\varphi}{\partial\mu} = 0.$$

Note that the dependence on s_γ and μ enters into (12) as the combination

$$\eta = s_\gamma/\mu, \tag{13}$$

which corresponds to a simple kinematic effect. If the angular diffusion is neglected, then the energetic electrons moving at an angle θ to the x axis traverse a distance equal to $x/\cos\theta = x/\mu \sim s_\gamma/\mu$. For this reason, the solution of Eq. (12) must depend on the variable η. Substituting (13) into (12), we obtain the equation

$$z_\gamma{}^2\frac{\partial\varphi}{\partial\eta} - 2\eta\frac{\partial\varphi}{\partial\eta} \tag{14}$$
$$- 2z_\gamma\frac{\partial\varphi}{\partial z_\gamma}\left(1 - 2\tau\frac{\partial\varphi}{\partial z_\gamma}\right)$$
$$- 4\tau z_\gamma\frac{\partial^2\varphi}{\partial z_\gamma{}^2} = 0.$$

Given the smallness of the parameter τ, we will seek a solution of this equation in the form

$$\varphi(\eta, z_\gamma) = \Phi_0(\eta, z_\gamma) + \Phi_1(\eta, z_\gamma), \qquad (15)$$

where $\Phi_1(\eta, z_\gamma) = 0$ at $\tau = 0$. The function $\Phi_0(\eta, z_\gamma)$ satisfies a simple linear equation:

$$z_\gamma{}^2 \frac{\partial \Phi_0}{\partial \eta} - 2\eta \frac{\partial \Phi_0}{\partial \eta} - 2z_\gamma \frac{\partial \Phi_0}{\partial z_\gamma} = 0. \qquad (16)$$

This equation can be solved by the method of characteristics (see Section 3.3 in Vladimirov (1981)). As a result, we obtain the general solution

$$\Phi_0(\eta, z_\gamma) = F\left(\frac{z_\gamma}{2} + \frac{\eta}{z_\gamma}\right), \qquad (17)$$

where F is an arbitrary function.

Since the Coulomb collisions of high-energy electrons with particles of the colder background plasma are weak, we will seek a solution that satisfies the boundary condition

$$\Phi_0(\eta, z_\gamma) \to \frac{z_\gamma}{2} \quad \text{when} \quad z_\gamma \to \infty \qquad (18)$$

at any fixed η. In other words, in view of the large mean free path (1), the very fast electrons hardly interact with the plasma, while their distribution function may be deemed to be the same as it was initially.

Given the condition (18), for the solution (17) we find

$$\Phi_0(\eta, z_\gamma) = \frac{z_\gamma}{2} + \frac{\eta}{z_\gamma}. \qquad (19)$$

Let us now take into account the nonlinear terms in Eq. (14). Let

$$\Phi_1(\eta, z_\gamma) = \psi_1(\eta, z_\gamma)\tau + \psi_2(\eta, z_\gamma)\tau^2 + ... \qquad (20)$$

Substituting the sought-for solution in the form (19), (20) into Eq. (14) and neglecting the high powers of $1/z_\gamma$, we obtain an equation for $\psi_1(\eta, z_\gamma)$:

$$z_\gamma{}^2 \frac{\partial \psi_1}{\partial \eta} - 2\eta \frac{\partial \psi_1}{\partial \eta} - 2z_\gamma \frac{\partial \psi_1}{\partial z_\gamma} + z_\gamma - \frac{4\eta}{z_\gamma} = 0. \qquad (21)$$

Similarly to the solution of Eq. (16), we find the general solution of Eq. (21) by the method of characteristics:

$$\psi_1(\eta, z_\gamma) = R\left(\frac{z_\gamma}{2} + \frac{\eta}{z_\gamma}\right) - 2\left(\frac{z_\gamma}{2} + \frac{\eta}{z_\gamma}\right)\ln z_\gamma + \frac{3}{2}z_\gamma, \qquad (22)$$

where R is an arbitrary function. We will require that

$$\psi_1(\eta, z_\gamma) \to 0 \quad \text{when} \quad z_\gamma \to \infty \qquad (23)$$

and at fixed η. Choosing the form of the function R in Eq. (22), given (23), we then obtain

$$\psi_1(\eta, z_\gamma) = -\eta/z_\gamma. \qquad (24)$$

Thus, in the first order in small parameter τ and in $1/z_\gamma$ the sought-for solution (15) is

$$\varphi(s_\gamma, z_\gamma, \mu) = \frac{z_\gamma}{2} + (1-\tau)\frac{s_\gamma}{\mu z_\gamma}. \qquad (25)$$

Since the temperature T_1 of the super-hot plasma is much higher than the temperature T_2 of the colder one (see Section 2), the small parameter τ may be neglected in Eq. (25):

$$\varphi(s_\gamma, z_\gamma, \mu) = \frac{z_\gamma}{2} + \frac{s_\gamma}{\mu z_\gamma}. \qquad (26)$$

This means that, here and below, the energy diffusion in Eq. (12) may be neglected within the chosen approximation. The regular energy losses of the flow of fast electrons during their Coulomb collisions with cold plasma particles rather than the energy diffusion are of greatest importance. However, it should be remembered that the energy diffusion may turn out to be significant at low energies, i.e., when $z_\gamma \to 2$ (or $z \to 1$).

Recall that the solution (26) is valid only in a small neighborhood of the point $\mu = 1$. In order to find the distribution function for the electrons with the remaining pitch angles, let us turn to Eq. (9). However, given the results obtained, we will set $\tau = 0$ in it; furthermore, as has already been noted, $\gamma = 1$. We obtain the equation

$$z_\gamma{}^2 \mu \frac{\partial \varphi}{\partial s_\gamma} - 2z_\gamma \frac{\partial \varphi}{\partial z_\gamma} + 2\mu \frac{\partial \varphi}{\partial \mu} + (1-\mu^2)\left(\frac{\partial \varphi}{\partial \mu}\right)^2 - (1-\mu^2)\left(\frac{\partial^2 \varphi}{\partial \mu^2}\right) = 0. \qquad (27)$$

At a given distance x from the transition layer TF (see Fig. 1) a larger number of electrons with a fixed energy moving at a small angle θ, i.e., electrons that traversed the smallest distance while undergoing a minimum number of scatterings, is to be expected. In other words, the fact that the distribution function of energetic electrons is sharply anisotropic with its maximum at $\mu = 1$ is to be expected. These qualitative considerations are also confirmed by the solution obtained by Gurevich and Istomin (1979). Thus, it makes sense to seek a solution of Eq. (27) in the form of a series in powers of the small parameter $(1 - \mu)$:

$$\varphi(s_\gamma, z_\gamma, \mu) = \varphi_0(s_\gamma, z_\gamma) + \varphi_1(s_\gamma, z_\gamma)(1-\mu) + \varphi_2(s_\gamma, z_\gamma)(1-\mu)^2 + ..., \qquad (28)$$

where, in view of (26),

$$\varphi_0(s_\gamma, z_\gamma) = \varphi(s_\gamma, 1, z_\gamma) = \frac{z_\gamma}{2} + \frac{s_\gamma}{z_\gamma}. \qquad (29)$$

Substituting (28) into (27) and equating the terms with identical powers of $(1-\mu)$, we obtain a chain of equations for the functions $\varphi_i(s_\gamma, z_\gamma)$. The first two equations of this chain are

$$z_\gamma{}^2 \frac{\partial \varphi_0}{\partial s_\gamma} - 2z_\gamma \frac{\partial \varphi_0}{\partial z_\gamma} - 2\varphi_1 = 0, \tag{30}$$

$$-z_\gamma{}^2 \frac{\partial \varphi_0}{\partial s_\gamma} + s_\gamma{}^2 \frac{\partial \varphi_1}{\partial s_\gamma} - 2z_\gamma \frac{\partial \varphi_1}{\partial z_\gamma} + 2\varphi_1 + 2\varphi_1{}^2 - 8\varphi_2 = 0. \tag{31}$$

Neglecting the highest powers of $1/z_\gamma$, from (29)–(31) we find

$$\varphi_1(s_\gamma, z_\gamma, \mu) = s_\gamma/z_\gamma, \tag{32}$$

$$\varphi_2(s_\gamma, z_\gamma, \mu) = \frac{1}{2} s_\gamma/z_\gamma. \tag{33}$$

Thus, returning to the variables s and z, we obtain the sought-for distribution function

$$\varphi(s, z, \mu) = z + 2\frac{s}{z} + 2\frac{s}{z}(1-\mu) + \frac{s}{z}(1-\mu)^2. \tag{34}$$

Let us return to the function $\varphi_0(s, z)$. At any fixed penetration depth s it has a minimum in energy (or velocity v) on the curve $z = \sqrt{2s}$. Consequently, the distribution function $f_{\mathbf{v}0} = \exp(-\varphi_0)$ corresponding to φ_0 reaches a maximum in velocity on the same curve, implying the presence of an instability that leads to the excitation of plasma oscillations. This, in turn, means that the derived expression for $\varphi_0(s, z)$ is not valid for all s and z, as follows from the derivation.

The mean free path of fast electrons in the superhot plasma (1) or the corresponding dimensionless penetration depth is defined by the formula

$$s(z) = z^2. \tag{35}$$

For this reason, in the range of energies $z > \sqrt{s}$ at a distance s from the transition layer there are many electrons that underwent virtually no scatterings, which creates the directivity of the flow of fast electrons. At the same time, for $z < \sqrt{s}$ the distribution function differs little from the isotropic one. We will assume, for simplicity, (and this is sufficient for our analysis) that in the range of energies $z < \sqrt{s}$ the distribution function is isotropic and is

$$\varphi_0(s, z) = z + 2s^{1/2}. \tag{36}$$

It follows from Eq. (34) that the electron distribution function decreases exponentially with penetration depth into the cold plasma. Its directivity can then be estimated as follows. The number of electrons with energy z arriving an angle θ at point s is

$$N_e \sim \exp(-\xi/\xi_z) = \exp\left[-\frac{\xi}{\xi_T}\left(\frac{1}{z}\right)^2 \frac{1}{\mu}\right], \tag{37}$$

where $\xi = \int_0^x n_2(x')dx'$ and $\xi_T = l_T n_1$.

Consider the electrons with an energy $\mathcal{E}$ in the range

$$k_\mathrm{B}T_1\left(\frac{\xi}{\xi_T}\right)^{1/2} \le \mathcal{E} \le k_\mathrm{B}T_1\left(\frac{\xi}{\xi_T}\right)^{1/2} + k_\mathrm{B}T_1. \tag{38}$$

They constitute the bulk of all the electrons described by the function (34), because $N_e \sim e^{-\mathcal{E}/k_\mathrm{B}T_1}$. It can be seen from (37) that most of the electrons will arrive at point ξ within the angle

$$\frac{\xi}{\xi_T}\left(\frac{k_\mathrm{B}T_1}{\mathcal{E}}\right)^2 \lesssim \mu. \tag{39}$$

Having estimated $(k_\mathrm{B}T_1/\mathcal{E})^2$ from (38), we obtain an estimate of the directivity of the function (34):

$$\mu \gtrsim \frac{\xi}{\left(\xi^{1/2} + \xi_T{}^{1/2}\right)^2}. \tag{40}$$

Relation (40) suggests that the expansion (28) is legitimate and, consequently, the solution (34) is valid for $\xi \gg \xi_T$.

In the region $0 \le \xi \le \xi_T$ adjacent to the transition layer TF (Fig. 1) the distribution function has a different form. The electrons described by this distribution function interact weakly with cold electrons. Therefore, neglecting the explicit dependence on μ in Eq. (27), we obtain the sought-for distribution function in the region $0 \le \xi \le \xi_T$:

$$\varphi(s, z) = z + 2\frac{s}{z}. \tag{41}$$

However, this distribution function cannot be isotropic. Its anisotropy can be taken into account if the function (41) is assigned only to those electrons that have not yet interacted with the cold plasma, i.e., those flying at such an angle that μ

$$1 > \mu > \frac{\xi}{\xi_T}\left(\frac{k_\mathrm{B}T_1}{\mathcal{E}}\right)^2. \tag{42}$$

Finally, let us now rewrite the distribution function for energetic electrons in dimensional variables:

$$f_{\mathbf{v}}(\xi,\mathcal{E},\theta)=\begin{cases}K\exp\left[-\left(\frac{\mathcal{E}}{k_{\rm B}T_1}+2\frac{\xi}{\xi_T}\frac{k_{\rm B}T_1}{\mathcal{E}}\right)\right], \quad \cos\theta>\frac{\xi}{\xi_T}\left(\frac{k_{\rm B}T_1}{\mathcal{E}}\right)^2,\\ \xi<a\xi_T, \quad \mathcal{E}>k_{\rm B}T_1\left(\frac{\xi}{\xi_T}\right)^{1/2}\\ K\exp\left[-\left(\frac{\mathcal{E}}{k_{\rm B}T_1}+2\frac{\xi}{\xi_T}\frac{k_{\rm B}T_1}{\mathcal{E}}(2-\cos\theta)+\frac{\xi}{\xi_T}\frac{k_{\rm B}T_1}{\mathcal{E}}(1-\cos\theta)^2\right)\right],\\ \cos\theta\gtrsim\frac{\xi}{\left(\xi^{1/2}+\xi_T{}^{1/2}\right)^2}, \quad \xi>a\xi_T, \quad \mathcal{E}>k_{\rm B}T_1\left(\frac{\xi}{\xi_T}\right)^{1/2}\\ K\exp\left[-\left(\frac{\mathcal{E}}{k_{\rm B}T_1}+2\left(\frac{\xi}{\xi_T}\right)^{1/2}\right)\right], \quad \mathcal{E}<k_{\rm B}T_1\left(\frac{\xi}{\xi_T}\right)^{1/2}.\end{cases} \tag{43}$$

Here, K is the normalization constant. For example, in Gritsyk and Somov (2014) this constant is determined from the condition for the normalization to the energy flux density transferred by energetic electrons. Using the free model parameter a we can specify the penetration depth of high-energy thermal electrons into the cold plasma starting from which the Coulomb collision cross section for such particles increases noticeably. Of course, this will affect significantly the polarization of the X-ray emission in this energy range.

The distribution function (43) takes into account the diffusion in angle θ, i.e., the scattering of the flow of energetic electrons during their Coulomb collisions with thermal electrons of the cold plasma. This can be made sure by looking at Fig. 2. Indeed, near the layer TF, i.e., at the boundary ($\xi_1=0$), the function (43) is isotropic. As the thermal runaway electrons penetrate into deeper plasma layers (ξ_2,ξ_3,ξ_4 in Fig. 2), their distribution function decreases due to Coulomb collisions. However, for higher-energy particles and particles with a pitch angle $\cos\theta\simeq 1$ this process is slower, which creates only a slight directivity (anisotropy) of the distribution function for super-hot electrons.

Using (43) and the expressions for the bremsstrahlung cross section, we can calculate the spectrum and polarization of the hard X-ray bremsstrahlung generated by the flow of thermal runaway electrons during a flare. Calculating the polarization and comparing the estimates with observational data are of crucial importance in checking whether the model description of the runaway electron propagation processes in the solar atmosphere is correct. The polarization estimates obtained within the thermal model proposed here are given in the next section.

4. POLARIZATION OF THE HARD X-RAY EMISSION

We will use the distribution function (43) to calculate the polarization of the hard X-ray bremsstrahlung of thermal runaway electrons. Let $I_{||}$ and $I_{\perp}$ be the corresponding fluxes from the source under study with a polarization parallel and perpendicular to the plane formed by the line of sight (the direction from the emission source toward the observer) and the magnetic field near the Earth. Then, according to the formulas derived in Nocera et al. (1985), for the hard X-ray flux we have

$$\begin{aligned} I_{\perp}+I_{||} &= \kappa\Bigg[8\int\limits_{h\nu}^{\infty}AC\left(\int\limits_0^{\infty}L_0d\xi\right)zdz \\ &+\frac{8}{3}\int\limits_{h\nu}^{\infty}BC\left(\int\limits_0^{\infty}L_0d\xi\right)zdz \\ &+\frac{12\sin^2\psi-8}{15}\int\limits_{h\nu_\nu}^{\infty}BC\left(\int\limits_0^{\infty}L_2d\xi\right)zdz\Bigg]. \end{aligned} \tag{44}$$

The quantity

$$I_{\perp}-I_{||}=-\frac{4}{5}\kappa\sin^2\psi\int\limits_{h\nu_\nu}^{\infty}BC \times\left(\int\limits_0^{\infty}L_2d\xi\right)zdz. \tag{45}$$

Here, ψ is the angle between the line of sight and the direction perpendicular to the magnetic field; A, B, and C are the differential bremsstrahlung cross sections (Elwert and Haug 1970); $L_0=L_0(z,\xi)$ and $L_2=L_2(z,\xi)$ are the coefficients of the expansion of the distribution function (43) into a series in Legendre polynomials. The use of ξ here is very convenient, because it allows us to obviate the need to make assumptions about the plasma density distribution in the target and the extent of the X-ray source. In (44) and (45) the upper limit of integration over

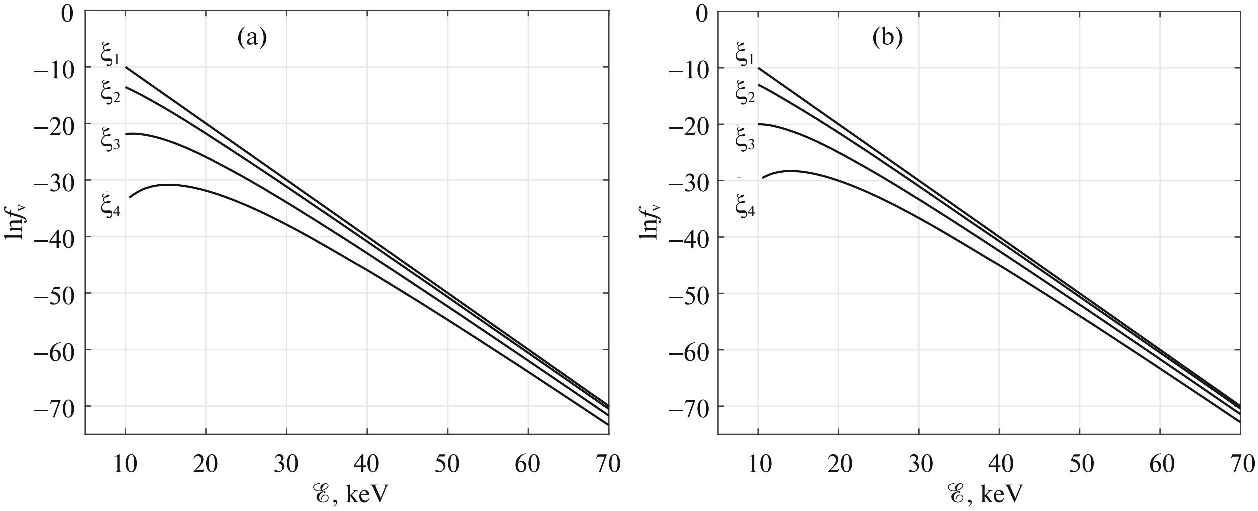

Fig. 2. Distribution functions for thermal runaway electrons at various penetration depths into the cold plasma: $\xi_1 = 0$, $\xi_2 = 15\xi_T$, $\xi_3 = 50\xi_T$, and $\xi_4 = 100\xi_T$. The calculations were performed by assuming that the temperature of the source of energetic electrons $T_1 = 10^8$ K, the parameter $a = 10$, and the normalization constant $K = 1$. The following pitch angles were considered: $\cos\theta = 0.8$ (a) and 1 (b).

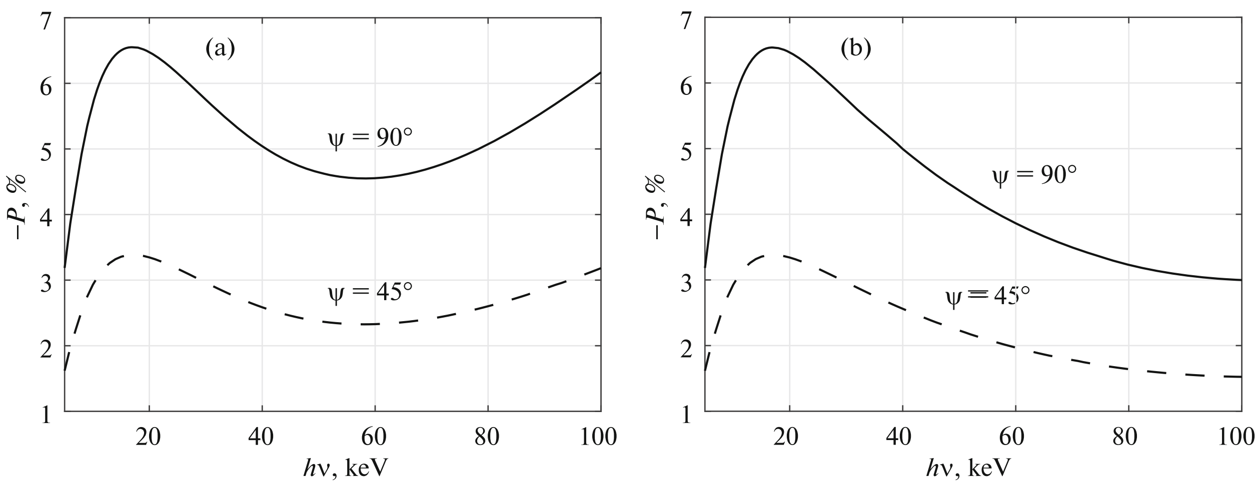

Fig. 3. Polarization of the hard X-ray emission generated by thermal electrons versus photon energy $h\nu$ for various angles ψ. The calculations were performed by assuming that the temperature $T_1 = 10^8$ K and the parameter $a = 5$ (a) and 10 (b).

the penetration depth ξ of the source is infinite. The constant $\kappa \sim KS_{HXR}/R^2$, where S_{HXR} is the area of the emission source and R is the distance from the Earth to the Sun. Obviously, the polarization

$$P = \frac{I_\perp - I_{||}}{I_\perp + I_{||}} \qquad (46)$$

does not depend on specific values of this constant.

The results of our polarization calculations for various angles ψ are presented in Fig. 3. The degree of polarization within the thermal model proposed here is small, because the anisotropy of the distribution function (43) is comparatively small. Indeed, it was shown in Section 3 that low-energy ($\mathcal{E} < k_\mathrm{B}T_1(\xi/\xi_T)^{1/2}$) electrons lose their energy very rapidly through Coulomb collisions, while their distribution function differs little from the isotropic one. Such particles generate an almost unpolarized hard X-ray emission. In contrast, energetic electrons with energies $\mathcal{E} > k_\mathrm{B}T_1(\xi/\xi_T)^{1/2}$ penetrate to great depths into the cold plasma, almost without scattering, and provide some anisotropy of the distribution function.

In Fig. 3a the emission polarizations at high energies exceed noticeably those in Fig. 3b. The estimates were made for $a = 5$ and 10 in the former and latter cases, respectively (see Eq. (43)). This result is easy to explain, because the low values of a imply that the energetic particles begin to scatter comparatively rapidly (i.e., at small penetration depths), while their distribution function becomes increasingly anisotropic ($\cos\theta \approx 1$). In the latter case, an analogous effect is achieved at greater penetration depths of hot electrons into the cold plasma, where, however, the number of energetic particles decreases significantly. It should be noted that the interpretation of the observations of the thermal component in the X-ray spectrum of a flare depends little on the choice of a, because at energies $\mathcal{E}_{h\nu} \lesssim 20$ keV the predicted degrees of polarization of the emission do not depend on a either (cf. Figs. 3a and 3b).

It is fundamentally important that the distribution function of thermal runaway electrons remains an almost equilibrium and almost isotropic one in the chromospheric part of the target. This manifests itself in a very low degree of polarization of the hard X-ray bremsstrahlung and confirms the original assumptions of our model.

5. CONCLUSIONS

Present-day space and ground-based observations of solar flares in various ranges of the electromagnetic spectrum with high temporal, spatial, and spectral resolutions are an excellent basis both for investigating separate physical processes and for understanding the mechanism of the entire flare. This complex electrodynamic phenomenon in a high-conductivity plasma with a strong magnetic field is accompanied by the acceleration of charged particles, intense plasma heating to very high temperatures, and, as a consequence, the thermal runaway of the so-called super-hot electrons.

The kinetic description of the propagation of accelerated electrons that is based on the approximation of the thick-target model with a reverse current and that takes into account the additional acceleration in collapsing magnetic traps (Gritsyk and Somov 2017) allows the hard X-ray observations of solar flares to be interpreted with an accuracy corresponding to the accuracy of present-day space experiments. The mentioned self-consistent nonthermal model is successfully used to describe the propagation of accelerated electrons with energies $\mathcal{E} \gtrsim 20$ keV, which form a power-law hard X-ray spectrum at high energies. In contrast, thermal super-hot electrons with energies $\mathcal{E} \lesssim 20$ keV generate less hard X-ray emission with a spectrum that closely resembles a Maxwellian distribution.

For this reason, here we considered the propagation of super-hot electrons running away along magnetic field lines from the heating region, a super-hot reconnecting current layer (Somov 2013) and collapsing magnetic traps, into the solar atmosphere, a region of less hot background plasma. We formulated and investigated the corresponding kinetic equation, in which the Coulomb collisions of thermal runaway electrons with plasma particles were taken into account, and found its analytical solution. Based on the distribution function of thermal runaway electrons found, we estimated the polarization of the hard X-ray emission, which turned out to be very low ($P \sim -5\%$). An experimental confirmation of such low polarizations remains a question of future space observations. However, the results obtained here can be used already now to interpret the thermal X-ray bremsstrahlung spectra of flares. The most careful and complete kinetic description of solar flares is needed primarily from the viewpoint of planned space observations of solar flares (see, e.g., Grefenstette et al. 2016).

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