Gap soliton dynamics in a nonuniform resonant structure

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A model for the propagation of coherent pulses along a one-dimensional, resonantly absorbing Bragg grating that includes localized inhomogeneous population inversion at its center is presented. The long-range coupling between the optical field and resonant atoms allows for controllable trapping of a gap soliton by the local inversion, thus opening new opportunities for control of signal transmission and localization of light.

Recent successful fabrication of periodic solid-state structures on a 10-nm length scale, coupled with developments in laser pulse shaping techniques, promises a new generation of optoelectronic devices. Combining such nanostructuring with a nonlinear material promises a rich environment in which to conduct experiments. For example, a one-dimensional photonic-bandgap (PBG) structure that consists of a periodic dielectric stack with a nonlinear response, offers nonlinear pulse propagation within a linear Bragg bandgap that is known as a gap soliton.1–5 Such gap solitons retain many of the properties, such as a constant shape and velocity, of their traditional non-linear Schrödinger counterparts. However, unlike a spatially uniform medium, the PBG structure is capable of trapping, or even of periodically changing, the amplitude and the direction of propagation of the soliton.6–11 The gap soliton can also be localized on a defect inside a PBG structure with cubic nonlinearity.12

In a resonant PBG structure the defect can be created not only by means of a periodicity breakup but also by a spatially localized population inversion. Unlike in a nonresonant PBG structure, for which the strength of the defect remains unchanged, this type of defect can be modified by the propagating gap soliton. This ability to control the strength of the defect by manipulating the incident field opens the possibility of a range of potentially useful optical switching effects. In this Letter we consider propagation of a sechlike pulse through a resonantly absorbing PBG structure with a population inversion created at its center by an incoherent pump.

We consider a bidirectional quasi-monochromatic scalar field propagating through a one-dimensional resonant PBG structure that consists of periodic thin layers containing two-level atoms separated by a transparent dielectric material. The period of structure d is equal to field wavelength λ, which, in turn, is resonant with the dipole transition of the atom. In this case the evolution of the amplitudes of the backward and forward waves $E^\pm(x,t)$, dimensionless complex atomic polarization $P(x,t)$, and inversion population $n(x,t)$ obey the following set of equations6,10,11:

$$\Omega_t^\pm + \Omega_x^\pm = P, \quad P = n(\Omega^+ + \Omega^-),$$

$$n_t = -\text{Re}[P^*(\Omega^+ + \Omega^-)],$$

where $\Omega^\pm = 2\tau_c\mu E^\pm/\hbar$, $\tau_c = (8\pi e T_1/3c\rho\lambda^2)^{1/2}$ is the cooperative time that characterizes the mean photon lifetime in the medium, $T_1$ is the excited-level lifetime, $c$ is the dielectric constant of the medium, $\rho$ is the density of resonant atoms, $\mu$ is the transition matrix element, $c$ is the speed of light, and a subscript means the corresponding derivative. For a numerical solution we adopt the dimensionless time and propagation coordinate variables $x = x/c\tau_c$ and $t = t/\tau_c$. As input values we use the following conditions: $\Omega^+(x = 0, t) = \Omega_0 \text{sech}(t - t_0)/\tau_p$, $\Omega^-(x = L, t) = 0$, $\Omega^-(x, t = 0) = 0$, $n(x, t = 0) = -1 + \nu_n \text{sech}(x - x_0)/L_n$, $P(x, t = 0) = 0$, where $\tau_p$ is the normalized duration of the incident pulse, $L$ is the structure length, $\nu_n$ is the amplitude of the inversion, and $x_0$ and $L_n$ are its center and its width, respectively. The dimensionless equations (1) are solved by means of a characteristic method.

Consider first a situation in which there is a complete inversion in the middle of the sample, that is, $\nu_n = 2$ and $x_0 = L/2$, and the amplitude of the input pulse $\Omega_0$ is chosen to form a gap $2\pi$ pulse. If the width of the inversion is small, $L_n/L \approx 0.01$, the input pulse evolves into a steady-state shape after losing some of its energy to nonlinear resonant reflection at the input boundary [Fig. 1(a)]. Although its velocity grows because of the amplification in the center of the sample, the propagation retains all the features that are pertinent to a nonexcited structure.6 Increasing the width of the inversion perturbation displays a more-complex resonant PBG dynamic that had not previously been predicted. Figure 1(b) shows that the pulse becomes unstable at $L_n/L \approx 0.1$ and oscillates about the center of the structure with decreasing amplitude. Eventually, the pulse is trapped by the defect created by the population inversion. The defect breaks the trapped pulse into a double-humped structure; the width of each hump is almost equal to $L_n$. The positions of the peaks correspond to the turning points of the trapped pulse. As the trapped pulse oscillates back and forth across the defect it continually extracts energy, leaving
Fig. 1. Spatiotemporal profile of the inversion population of the resonant PBG structure and gap soliton: $\Omega_0 = 2.7$, that either (a) traverses a defect with $l_n/L = 0.01$ or (b) is captured there if the width of the defect is increased, $l_n/L = 0.1$. Note the decay of the soliton oscillations and the defect breakup as a result of the solitons’ inelastic interaction.

...a greater depletion at the center of the defect than at either edge. Mathematically, the double-peaked profile of the trapped soliton corresponds to the dark-soliton solution of the Maxwell–Bloch equations in the inverted medium, and physically it is reminiscent of the oscillatory regime and self-organized feedback growth in a superradiant laser.\(^1\) The effects of the incoherent pump intensity, or strengths of the defect, are presented in Fig. 2, which again shows the projection of the population inversion onto the $(x, t)$ plane. These results demonstrate how the pump can influence the dynamics of the resonant PBG structure. When complete inversion is not reached, $\nu_n = 1.75$ [Fig. 2(a)], the defect is still capable of capturing the gap soliton, although the amplitude of the oscillations becomes larger and, correspondingly, the capture occurs on a longer time scale. Lowering the pump level to $\nu_n = 1.5$ [Fig. 2(b)] decreases the potential barrier produced by the population inversion, and the pulse traverses the structure, although with some delay caused by the attractive action of the potential.

The next question to ask is how the presence of the attractive potential modifies the propagation of a pulse with amplitude smaller than that needed to generate a stable gap $2\pi$ soliton. It was shown previously that, in this case, the pulse propagation becomes unstable and a delayed reflection can occur.\(^10,11,16\) This effect is explored in Fig. 3, where the projection of the population inversion onto the $(x, t)$ plane is shown for the four decreasing values of $\Omega_0$. Initially, a moderate decrease of the input amplitude leads to a readily observable slowing down of the pulse that is due to the intense nonlinear generation of the backward-propagating wave between the input facet of the structure and the inversion range. This is followed by the capture of the pulse by the defect, with subsequent disintegration of the inversion into a double-humped structure [Fig. 3(a)]. Further decreasing the input amplitude leads to the pulse’s stopping on the edge of the inversion range and to a delayed reflection with some amount of the pulse energy left in the form of residual inversion [Fig. 3(b)]. The value of the delay exceeds more than twice the time that the pulse with higher amplitude needs to traverse the non-excited structure (see below). The effect of the resonant nonlinear reflection becomes more pronounced at smaller values of the input amplitude [Figs. 3(c) and 3(d)]. It can be seen that decreasing the amplitude slightly will reduce both the level of energy left across the structure in the form of the residual inversion and the delay time that is the time during which the pulse escapes from the structure in the backward direction.

To elucidate the physical mechanism behind the results observed above, it is worth recalling the importance that the exact value of the input amplitude has on the stability of the pulse propagation through a resonant PBG structure. In this case a decrease of the input amplitude causes an increase of the nonlinearly reflected wave that creates a short-range repulsive potential on the front edge of the pulse and eventually provides a blockage mechanism whose efficiency grows as the input amplitude decreases.\(^10,11\) If there is an inversion at the center of the structure, it will create a long-range attractive potential that is able to overcome...
the blocking action of the noninverted structure. This is so even when the input amplitude is too small to provide significant penetration into the nonexcited structure. This speculation can be confirmed by the fact that the pulse of the input amplitude as in Fig. 3(a) is reflected by the resonant PBG structure, provided that all the atoms are initially in the ground state.

In conclusion, we have numerically studied the propagation of gap solitons in a resonant PBG structure that contains a defect in the form of population inversion in the center of the sample. In the absence of population inversion, pulse propagation was previously shown to be arrested by the delayed resonant nonlinear reflection. The presence of an inversion that may be due to the incoherent pump is shown to lift this restriction and to result in new and unexplored phenomena. The case examined here can be extended to a more practical application if one considers a distributed reflector surrounding a local gain region in a semiconductor laser. However, a more detailed description of the effects intrinsic to semiconductors will be required before a III–V semiconductor-based device can be properly designed.

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