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Abstract. Degenerative-dystrophic lesions of large joints arethe most common diseases. Up to now, the problem of hip arthroplasty with dysplastic coxarthrosis has not been completely resolved. The most interesting is the modeling of the contact between the cavity and the neck of the thigh. This paper consists of verification algorithm for acetabular component model. Particularly contact displacements and stresses fields are considered. The main idea of this method is comparison between analytical solution and solution which we get by means of the finite element method. This comparison is used to verify contact elements and contact settings between contact and target area. For doing that the nonstationary contact problem with moving boundary conditions had been solved analytical. Algorithm of the verification of this problem is described below.

1. Introduction

Degenerative-dystrophic lesions of large joints arethe most common diseases. The main method of treating patients with dysplastic coxarthrosisremains hip replacement, reconstructive operational interventions are effective only in the early stages of the disease and give a positive result only for 5-10 years [1-5]. Up to now, the problem of hip arthroplasty with dysplastic coxarthrosis has not been completely resolved. Nowadays computer aided methods are popular to solve this problem [6-8]. The most interesting is the modeling of the contact between the cavity and the neck of the thigh. There are numbers of methods to simulate the contact [9-11]. More than changes of the mechanical parameters of the bone can influence on the quality of the surgery [12, 13]. To solve this problem it is necessary to obtain a reference solution. This solution should be compared with solutions obtained through finite element method (FEM). The general algorithm for constructing this solution is presented below. The solutions of the nonstationary contact problem given below are suitable for arbitrary smooth-shaped stamps. Planar nonstationary problems for an elastic half-space and absolutely rigid stamps are considered [11, 14]. We use Cartesian rectangular coordinate system. The axis Ox is directed along the unperturbed boundary of the half-space z=0, and the Oz axis is directed into the interior of the half-space. At the initial instant of time, the half-space is in an unperturbed state, and the impactor bounded by the surface z=*f*(x) moves along the axis at a certain speed V (see Figure 1). Suppose that the contact between the striker and the half-space occurs under conditions of free slippage:

$$\sigma_{xz}\big|_{z=0} = 0,\ x \in (-\infty, \infty)$$

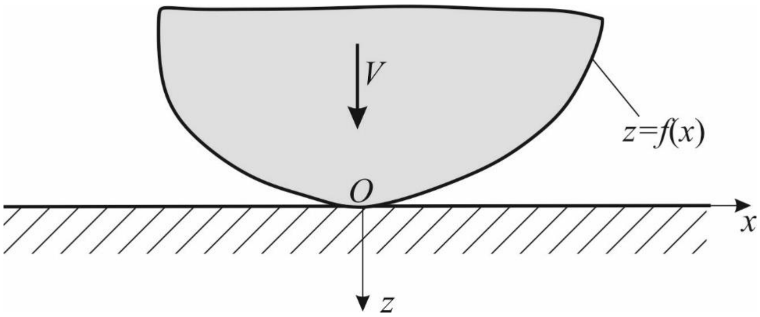

Figure 1. Impact scheme.

2. Materials and Methods

The mathematical formulation of these problems includes:

equations of motion in potentials:

$$\Delta\varphi = \ddot{\varphi},\ \Delta\psi = \eta^2\ddot{\psi},\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2};$$

- connection of displacements with potentials:

$$u = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z},\ w = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x};$$

- Cauchy relations:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z},\ \varepsilon_{zz} = \frac{\partial w}{\partial z} = \frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z},$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{\partial^2\varphi}{\partial x\partial z} + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right);$$

- Hooke's law:

$$\sigma_{xx} = \varepsilon_{xx} + \left(1 - 2\eta^{-2}\right)\varepsilon_{zz} = \Delta\varphi - 2\eta^{-2}\left(\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}\right) =$$

$$= \left(1 - 2\eta^{-2}\right)\Delta\varphi + 2\eta^{-2}\left(\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right) =$$

$$= \left(1 - 2\eta^{-2}\right)\ddot{\varphi} + 2\eta^{-2}\left(\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right),$$

$$\sigma_{zz} = \varepsilon_{zz} + \left(1 - 2\eta^{-2}\right)\varepsilon_{xx} = \Delta\varphi - 2\eta^{-2}\left(\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right) =$$

$$= \ddot{\varphi} - 2\eta^{-2}\left(\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}\right),$$

$$\sigma_{xz} = 2\eta^{-2}\varepsilon_{xz} = \eta^{-2}\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2} + 2\frac{\partial^2\varphi}{\partial x\partial z}\right) =$$

$$= \eta^{-2}\left(-\Delta\psi + 2\frac{\partial^2\psi}{\partial x^2} + 2\frac{\partial^2\varphi}{\partial x\partial z}\right) = -\ddot{\psi} + 2\eta^{-2}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\varphi}{\partial x\partial z}\right);$$

- initial conditions:

$$\varphi|_{\tau=0} = \dot{\varphi}|_{\tau=0} = \psi|_{\tau=0} = \dot{\psi}|_{\tau=0} = 0;$$

- boundary conditions:

$$w|_{z=0} = w_T,\ x \in \Omega_T;\ \sigma_{zz}|_{z=0} < 0,\ x \in \Omega_T;$$

$$\sigma_{zz}|_{z=0} = 0,\ x \notin \Omega_T;\ \sigma_{xz}|_{z=0} = 0,\ x \in (-\infty, \infty);$$

$$\varphi = O(1),\ \psi = O(1),\ r \to \infty,\ r = \sqrt{x^2 + z^2},$$

Where Ω_T - the contact area, $w_T = w_T(x,\tau)$ - moving the boundary of the stamp (see Figure 2).

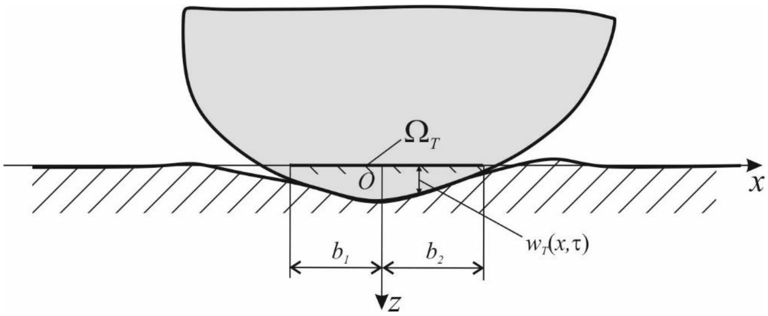

Figure 2. Scheme of movement of contact area.

The solution of this problem was obtained with the use of the method of functionally invariant solutions. Passing through the intermediate calculations, let us consider the final relations for the stresses in the region of contact and displacements in the region of contact. It is considered that there is no friction in the contact region (contact is made with slippage).

$$u(x,z,\tau) = -\mathrm{Re}\frac{\partial^{m-1}}{\partial\tau^{m-1}}\left[\int_0^{\theta_1}\Phi(\theta)\theta d\theta - \int_0^{\theta_2}\Psi(\theta)\sqrt{\eta^2-\theta^2}d\theta\right],$$

$$w(x,z,\tau) = -\mathrm{Re}\frac{\partial^{m-1}}{\partial\tau^{m-1}}\left[\int_0^{\theta_1}\Phi(\theta)\sqrt{1-\theta^2}d\theta + \int_0^{\theta_2}\Psi(\theta)\theta d\theta\right],$$

$$\sigma_{zz}(x,z,\tau) = \eta^{-2}\,\mathrm{Re}\frac{\partial^{m}}{\partial\tau^{m}}\left[\int_0^{\theta_1}(\eta^2-2\theta^2)\Phi(\theta)d\theta + \int_0^{\theta_2}2\theta\sqrt{\eta^2-\theta^2}\Psi(\theta)d\theta\right],$$

where

$$\theta_k = \frac{x\tau + iz\sqrt{\tau^2-\eta_k^2 r^2}}{r^2} = \frac{\hat{x} + i\hat{z}\sqrt{1-\eta_k^2\hat{r}^2}}{\hat{r}^2},$$

$$\hat{x} = x/\tau,\ \hat{z} = z/\tau,\ r^2 = x^2 + z^2,\ \hat{r}^2 = \hat{x}^2 + \hat{z}^2.$$

$$\Phi(\theta_k) = \varphi'^* = \frac{\partial\varphi^*}{\partial\theta_k},\ \Psi(\theta_k) = \psi'^* = \frac{\partial\psi^*}{\partial\theta_k}$$

The properties of the contact assignment in the finite element model of the acetabulum component are selected in accordance with the solutions obtained above. As a form of the stamp, a hemisphere was taken (the model of the neck of the thigh). This form of the stamp is smooth and the function describing the law of the introduction of the sphere has an order of homogeneity m equal to 2. But in considered there is no need to take into account to study behavior of the hole model. Particular part of acetabular component where contact characteristics have been studied are presented on Figure 3. The computational mesh of the acetabular component model (see Figure 3) has a hexagonal structure, in order to obtain the most correct results consistent with the solution given above. The mesh has a regular structure, there were no problems with the disagreement of linear dimensions during the study.

The results of the calculation are in good agreement with the analytical solution given above with the introduction of the stamp (femoral neck) into the acetabular component no more than 5% of the radius of the stamp (the radius of the stamp is introduced here as a dimensionless parameter.The mechanism of transition to dimensionless parameters is given above). In the case of further introduction of edge effects due to the complex geometry of the model, the convergence with analytical solutions is significantly reduced.

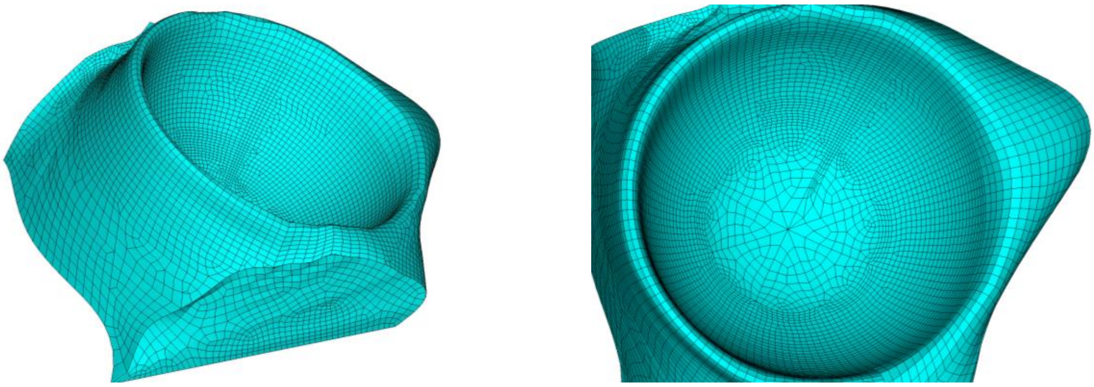

Figure 3. Hexagonal mesh of acetabular component.

3. Discussion

In this paper, the verification algorithm for the computational model of the acetabular component in the part of the contact interaction is considered. Analytical dependences for the distribution of contact stresses inside the region of the contact and displacements outside the contact region are obtained. This approach is also suitable for working out other tasks, where a very careful analysis is required in assessing contact stresses. Due to the fact that the contact interaction problem is solved in a fairly general form, its solutions can be used for bodies with strongly pronounced anisotropy.

4. Conclusion

Often, the settings for contact interaction in FEM programs pursue the goal of numerical convergence of the solution obtained. It is by no means always obvious to what exactly the solutions obtained should converge. In the pursuit of numerical convergence, physical aspects of contact interaction are often lost. In this paper, a reference solution is presented, in accordance with which the corresponding contact settings are selected. Numerical convergence is achieved by constructing a regular sufficiently thick hexagonal grid. In many applied problems this is often difficult to achieve. However, one must understand that the distribution of contact displacements and stresses is very sensitive to the choice of contact elements and the settings of their interaction.

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