**Interlocking of convex polyhedra: towards a geometric theory of fragmented solids**

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**Abstract**

The article presents arrangements of identical platonic solids with very special and curious properties. Namely, the solids are situated in a sort of a layer and are interlocked in the sense that no one of them can be moved out without disturbing others. This situation cannot happen in the plane. First examples of this sort (composed of irregular convex polyhedra) were complicated and were constructed in a non regular way: [1]. The examples presented here were constructed in framework of applied studies [3–8] and were not described in mathematical publications.

Figure 1 shows a fragment of such arrangement of regular tetrahedra (the set of them should be extended along the horizontal plane in the obvious way): [2].

![Figure 1. Fragment of interlocking set of tetrahedra.](image)

The intersection of the tetrahedral with the “central” horizontal plane is the tiling of the plane with equal squares. A.J. Kanel-Belov considered another regular tiling - the hexagonal (honeycomb) one and found a new type of interlocking solids - cubes. The hexagons are intersections of the cubes with the plane (see Figure 2). Hexagonal sections exist also for
Figure 2. Hexagonal tiling of the plane and the associated platonic solids with their respective hexagonal middle sections. Orientations of arrows indicate the inclinations of faces of the modified hexagonal prisms that provide their interlocking.

Figure 3. Fragment of interlocking set of cubes.

The above examples cover all types of platonic bodies, except for the icosahedron. It was later found by A.J. Kanel-Belov that icosahedra could be put in an interlocking arrangement if a decagonal middle cross-section of the icosahedron is used as a basis. The decagons can be
arranged on a plane and arrows showing inclination of faces can be assigned to their sides in a way, Fig. 6, producing an interlocking arrangement of icosahedra, Fig. 7. It should be noted that not all faces of an icosahedron are in contact with its neighbors. As the dodecahedron

Figure 4. Fragment of interlocking set of octahedra.

Figure 5. Fragment of interlocking set of dodecahedra normal to the symmetry axis of 3\textsuperscript{rd} order.
Figure 6. Decagonal tiling of plane and the associated dodecahedron and icosahedron. Only arrows that correspond to contacting faces are shown.

Figure 7. Fragment of interlocking set of icosahedra.

possesses a decagonal middle cross-section as well, a similar decagon-based arrangement is possible with dodecahedra, Figs. 6, 8. Thus, dodecahedra permit two interlocking arrangements
normal to their symmetry axes of 3rd and 5th order.

Some discussions about possible use of interlocking structures can be found in [9–11].

The technicalities related to the subject of this paper (in particular, the interlocking criteria, the relations between rotational and translational interlocking, etc.) are posted at arxiv.org

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References


