Available online at www.sciencedirect.com
SCIENCE DIRECT®

ELSEVIER

Scripta Materialia 50 (2004) 291–294

Scripta MATERIALIA

www.actamat-journals.com

Negative stiffness of a layer with topologically interlocked elements

Y. Estrin [a,*], A.V. Dyskin [b], E. Pasternak [a,b], S. Schaare [a], S. Stanchits [c], A.J. Kanel-Belov [d]

[a] Institut für Werkstoffkunde und Werkstofftechnik, Technische Universität Clausthal, Agricolastrasse 6, D-38678 Clausthal-Zellerfeld, Germany
[b] School of Civil and Resource Engineering, The University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia
[c] Dept. 3.2, GeoForschungsZentrum Potsdam, Telegrafenberg D425, 14473 Potsdam, Germany
[d] School of Engineering and Science, International University of Bremen, P.O. Box 750561, 28725 Bremen, Germany

Received 31 July 2003; received in revised form 18 September 2003; accepted 22 September 2003

Abstract

Unusual mechanical response of a structure with topologically interlocked elements is discussed. Under point loading, it behaves pseudo-plastically, with a negative stiffness in part of the unloading curve. This effect is inherent in the structure and is attributed to changing contact conditions due to rotation of the elements.

Keywords: Topological interlocking; Structural behavior; Mechanical properties

1. Introduction

It has been found recently [1] that identical elements having the shape of one of the five platonic bodies (tetrahedron, cube, octahedron, dodecahedron and icosahedron) allow topological interlocking. That is to say, such elements can be assembled in a layer that maintains its integrity despite the absence of any binder phase or connectors between elements. This is ensured by the specific arrangement of the elements in which removal of any element from the structure is prevented by its neighbors. Possible practical applications of topological interlocking were discussed earlier [1–3]. The case of cube shaped elements is illustrated in Fig. 1. The absence of physical connection between the interlocked elements prevents crack propagation between them and thus gives rise to enhanced fracture toughness [2,3]. This type of interlocking also determines the resistance of the assembly to point loading (indentation) and its dependence upon the magnitude of the lateral pressure exerted by peripheral constraint.

As the elements in the interlocking structures are not connected to each other mechanically, they retain the degrees of freedom associated with the movement of a solid body: three translational and three rotational ones. When the elements are assembled in an interlocking structure, their degrees of freedom become interdependent, owing to their non-round shape and to interlocking. One can expect that this interdependence may lead to unusual mechanical properties, in particular when a load is applied. In this paper we report an anomaly in the behavior of assemblies of interlocked cubes found in unloading following a point loading test.

2. Topological interlocking of cube shaped elements

Cube shaped elements can be arranged in a monolayer assembly to produce an interlocked structure [1]. This is done by arranging their hexagonal sections to form a honeycomb pattern covering the plane. (The regular hexagon section of a cube is the one that passes through its center and the centers of its edges.) A fragment of such an assembly is shown in Fig. 1. Interlocking is ensured by kinematical interaction between the neighbors. For instance, a reference block, denoted R in Fig. 1, is prevented from being displaced downwards by neighbors 1, 3 and 5, while neighbors 2, 4 and 6 prevent its upward displacement. Of course, conditions are different for elements located at the periphery of the assembly, but they are the only ones that need to be constrained, which is then sufficient to hold the entire

* Corresponding author. Tel.: +49-5323-722004; fax: + 49-5323-723148.
E-mail address: juri.estrin@tu-clausthal.de (Y. Estrin).

1359-6462/$ - see front matter
doi:10.1016/j.scriptamat.2003.09.053

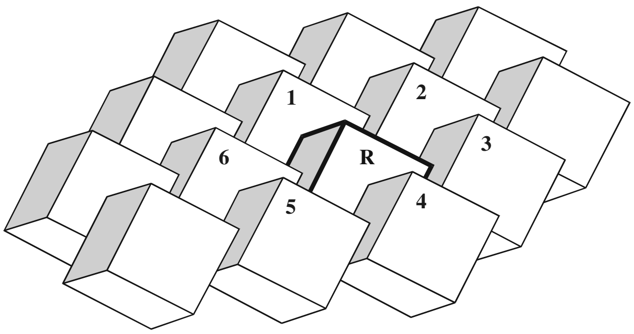

Fig. 1. Interlocking of cube shaped elements in a layer. The neighboring blocks numbered 1, 3 and 5 block downward displacements of a reference element (R), while elements 2, 4 and 6 block its upward displacements.

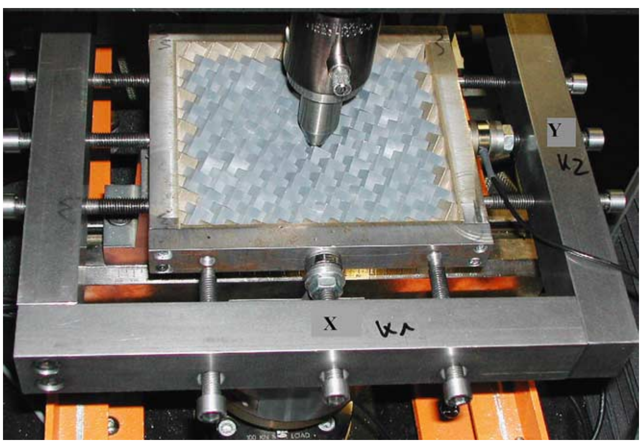

Fig. 2. Point load (indentation test). The point load is transmitted by a ball situated in a pit formed by the indenter and a group of three neighboring cubes. The lateral constraint is provided by the inner frame. The calibrated lateral load is provided by the outer frame through bolts marked X and Y connected in series with loading cells.

structure together. In particular, in the point load (indentation) test shown in Fig. 2 this is accomplished by a special steel frame that provides controllable lateral loading. The point load (indentation) tests were conducted on assemblies of aluminum cubes (Young's modulus, $E = 70$ GPa) and PVC cubes (Young's modulus, $E = 3$ GPa) in displacement-controlled regime under different lateral loads (total force in each direction equals 1, 1.5, 2, 2.5 and 4 kN). In all cases, the same element size (12.25 mm edge size) was used. The point loading was conducted through a steel ball 12.7 mm in diameter situated beneath the indenter in a 'pit' formed by a group of three neighboring cubes.

3. Results of point load tests

Some results of point load testing are shown in Figs. 3 and 4. The point load was applied using a 100 kN Instron loading frame (tests presented in Figs. 3(a) and 4)

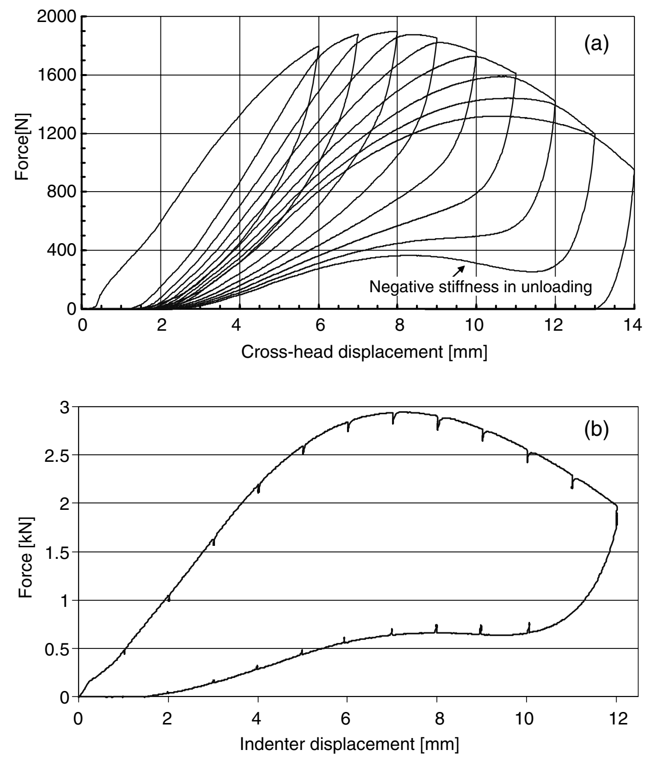

Fig. 3. Loading–unloading response of a layer of interlocked aluminum cubes: (a) lateral force 1.5 kN; (b) lateral force 2.5 kN. The vertical strokes spaced at 1 mm correspond to pauses in the loading/unloading (arrests of the cross-head of the testing machine).

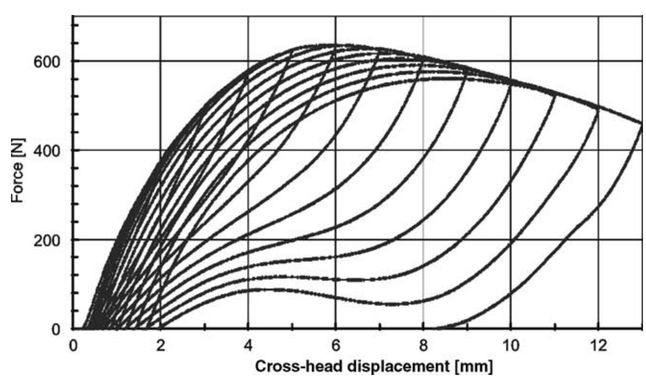

Fig. 4. Loading–unloading response of a layer of interlocked PVC cubes (lateral force: 1.5 kN).

and a 4600 kN MTS loading frame (test presented in Fig. 3(b)). The loading was cyclic, with increasing maximum deflection. The indenter (cross-head) displacement rate was 1 mm/min. It is seen that the loading response is highly non-linear. (Note, in particular, the large residual deflections observed.) The unloading response at cycles with large maximum deflections is most remarkable: it shows an *increase* in the loading force with *decreasing* deflection near the end of the unloading curve. This *anomalous unloading response*, characterized by a negative effective stiffness of the structure, was observed under different lateral loads, for elements made from two entirely different materials (aluminum and PVC) and with two different loading machines.

4. Proposed mechanism of stiffness increase in unloading

Visual examination of the assemblies during and after loading revealed considerable rotations of the elements, both in-plane and out-of-plane, cf. Fig. 5. These rotations, caused by the elements being squeezed out of the structure under loading, lead to an alteration of the contact conditions of the elements. In particular, due to loading, a face-to-face contact between the cubes eventually gives way to a vertex-to-face contact, Fig. 6. The corresponding drastic reduction in the contact area results in a considerable decrease in the effective bending stiffness of the layer. This manifests itself first in a pseudo-plastic behavior and, eventually, when sufficiently large deflections are reached, in post-peak softening. Conversely, in unloading upon such large deflections, the character of contact changes from vertex-to-face type back to the face-to-face type. This is accompanied with an increase of the contact area, resulting in increased bending stiffness, Fig. 6. If the increase of the bending stiffness is strong enough to outstrip the force reduction caused by the decrease in deflection, the anomalous unloading response with the negative effective stiffness is observed.

The change of contact involves local indentations (cf. insert on Fig. 5), the cube vertices acting as indenters leading to local plastic flow, which is time dependent. This is, indeed, supported by Fig. 3(b) where the vertical strokes spaced at 1 mm correspond to pauses in the loading/unloading when the cross-head of the testing machine was arrested. The load drops after pre-loading are a signature of stress relaxation due to local plastic flow. Interestingly, vertical strokes associated with intermittent arrests of the cross-head during unloading point to stress increments rather than relaxation. This can be seen as an additional indication that a negative stiffness in unloading can be attributed to changing contact conditions.

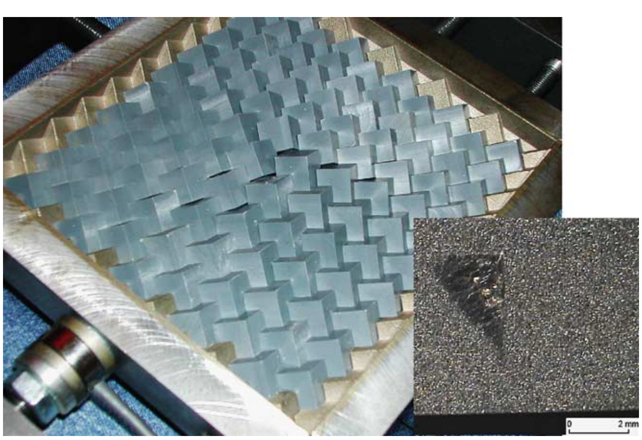

Fig. 5. The assembly after unloading. The residual rotations of the elements are clearly seen. The insert shows an indentation mark on a cube face due to vertex-to-face contact between the cubes.

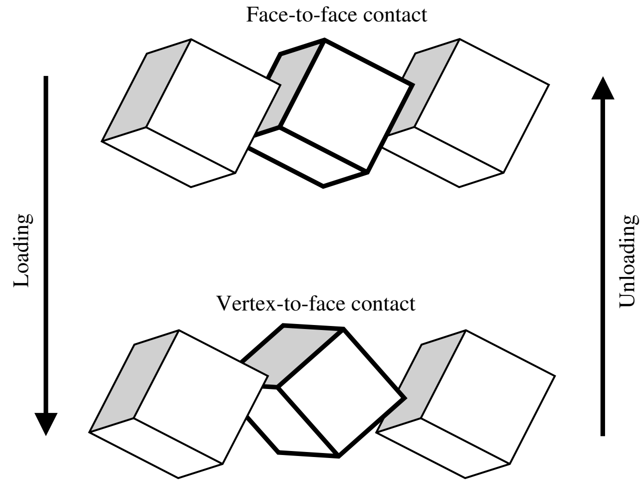

Fig. 6. The face-to-face contact in the initial arrangement of the cube shaped elements in the assembly is transformed to a vertex-to-face contact due to rotation of the elements under loading. In unloading, the elements rotate back gradually reverting the character of contact back to the face-to-face one.

The proposed mechanism qualitatively explains the occurrence of global negative stiffness exhibited by an interlocked structure. It should be noted that it is different from other known mechanisms of anomalous stiffness behavior, such as the one based on local negative stiffness of structural elements of a composite suggested in [4,5]. It would be interesting to see how a combination of the interlocking principle and local negative stiffness of the 'building blocks' can influence the mechanical (and acoustical) properties of assembled structures.

5. Conclusion

Anomalous (negative) stiffness was observed in the unloading parts of point load tests of interlocking assemblies of cube shaped elements. As the cubes were made from different materials and the assemblies were tested in different loading apparatuses and under various lateral loads, this phenomenon is seen as an intrinsic property of the structure of interlocked cube shaped elements. It is believed to be associated with the rotational degrees of freedom and the variation of the character of the contact between the cubes under unloading.

Acknowledgements

The authors wish to thank Mr. M. Bosse, Dr. T. Lebedkina and Mr. D. Trenke for help with tests involving PVC elements. Useful discussions with Prof. G. Ziegmann and Prof. N. Müller are appreciated. Support from the Australian Research Council through Discovery Grant DP0210574 (2002–2004) and Linkage International Grant LX0347195 (2002–2004) as well as from DFG through Grant ES-74/10-1 is acknowledged. One of the authors (E.P.) acknowledges support through an Alexander von Humboldt Research Fellowship 2002–2003.

References

[1] Dyskin AV, Estrin Y, Kanel-Belov AJ, Pasternak E. Philos Mag Lett 2003;83:197.
[2] Dyskin AV, Estrin Y, Kanel-Belov AJ, Pasternak E. Scr Mater 2001;44:2689.
[3] Dyskin AV, Estrin Y, Kanel-Belov AJ, Pasternak E. Adv Eng Mater 2001;3:885.
[4] Lakes RS, Lee T, Wang YC. Nature 2001;410:565.
[5] Lakes RS. Phys Rev Lett 2001;86:2897.