ISSN 1019-3316, Herald of the Russian Academy of Sciences, 2012, Vol. 82, No. 2, pp. 111–119. © Pleiades Publishing, Ltd., 2012.
Original Russian Text © V.P. Dymnikov, V.N. Lykosov, E.M. Volodin, 2012, published in Vestnik Rossiiskoi Akademii Nauk, 2012, Vol. 82, No. 3, pp. 227–236.

Review

The main tool in the study of both the climate system as a whole and processes within it is mathematical (numerical) simulation, based on a hierarchy of models, from the core global models of the general circulation of the atmosphere and ocean to large-scale models of geophysical turbulence. This is discussed in the article below.

**DOI:** 10.1134/S1019331612020037

# Modeling Climate and Its Changes: Current Problems

**V. P. Dymnikov, V. N. Lykosov, and E. M. Volodin***

Among the greatest problems of the contemporary stage of science development is climate change prediction. According to the estimates of the Intergovernmental Panel on Climate Change (IPCC), human intervention has been a large contributor to this change in recent decades [1]. Natural climate variability is equally important. The most significant manifestations of the inherent variability of the earth's climate system are the El Niño–Southern Oscillation (ENSO), the North Atlantic Oscillation (NAO), and the Arctic Oscillation (AO).

Mathematically, climate is defined as a statistical ensemble of states taken on by the climate system during a sufficiently large time interval [2]. According to the World Meteorological Organization (WMO), the classical averaging period is 30 years. In the general case, an ensemble is understood not only as a set of states but also as a certain probabilistic measure assigned over this set that determines the probability of the system to stay within a certain subset of the given set [1].

We should consider in detail the problem of climate definitions since numerous discussions on climate are caused by different definitions of climate. The above definition of climate implies that climate characteristics are any parameters averaged by a probability measure (statistical characteristics). In particular, if we view a weather forecast as the calculation of a system trajectory at the final time interval and understand trajectory predictability as the characteristic time of convergence of the localized initial distribution of dots in a phase space, characterizing errors of the initial state into an equilibrium (climatic) distribution, this characteristic, averaged across the whole ensemble of initial data, will be a climatic characteristic. The characteristic time determines the system's sensitivity to small external impacts. In this sense, the notions *climate model* and *weather prediction model* should coincide. At present, under natural limitations associated with a large difference in integration time scales, models differ greatly in their accuracy of space–time approximation (the choice of spatial and temporal resolution) and, as a consequence, in their description of subgrid-scale processes.

Problems of climate change reproduction and prediction, unlike classical problems of physics, have the following specific feature: they do not allow for direct physical experiment. Moreover, owing to specific characteristics of the climate system (for example, the atmosphere and ocean are thin films), laboratory experiments are also very problematic. For a detailed investigation of the real climate system, there is only a limited set of parameters of the system's trajectory of several decades long, during which more or less full-sized measurements were conducted.

## A BRIEF BACKGROUND

Mathematical modeling in atmospheric physics started with stating and solving the problem of hydrodynamic numerical weather prediction. For the first time, a weather forecast problem like that of mathematics and mechanics was formulated in V. Bjerknes' article, published in 1904 [3], where it was treated as a problem with initial conditions for the equations of the dynamics of a baroclinic fluid. In the early 1920s, L.F. Richardson proposed a method of numerical weather report [4]. Since information about the real state of the whole atmospheric column is necessary as the initial condition, he designed instruments for atmospheric measurements at altitudes of several kilometers above the underlying terrain. When constructing the theory of numerical weather prediction, Richardson studied the turbulence of the atmospheric boundary layer, radiation processes, and atmospheric thermodynamics.

However, the attempt at practical weather prediction for one day (May 20, 1910, for the Nuremberg–Augsburg region in Germany) by the numerical

* The authors work at the RAS Institute of Numerical Mathematics. Academician Valentin Pavlovich Dymnikov is a chief research fellow. RAS Corresponding Member Vasilii Nikolaevich Lykosov is a chief research fellow. Evgenii Mikhailovich Volodin, Dr. Sci. (Phys.–Math.), is a leading research fellow.

method (with the then computing aids, such as slide rules and mechanical calculators) failed: the calculated surface pressure increased by 145 GPa in 6 hours, which exceeded almost 50 times the observed trend. The causes were the following: only the land-based data of a small network of meteorological stations in Europe were used as the initial conditions; the finite-difference method that Richardson used turned out to be computationally unstable (the Courant–Friedrichs–Lewy condition for the correlation of spatial and temporal steps was established later [5]); the equations of atmospheric hydrodynamics used in the prediction scheme, along with relatively slow motions responsible for synoptic processes, also described all sorts of "noises," such as acoustic and gravitational waves, the filtration of which had not been done at the initial moment.

One of the causes of Richardson's failure—the presence of noises in synoptic motions—was eliminated by I.A. Kibel' [6]. He proposed the basic principle of simplifying the equations of atmospheric hydrodynamics (asymptotic "quasi-geostrophic decomposition"), which allowed him to develop a filtration procedure from the solutions of equations of weather-insignificant meteorological noises and served as the basis for creating the hydrodynamic theory of short-term weather forecasts [7]. The advent of electronic computers made the first "practical" numerical weather prediction possible in the early 1950s (a model based on the barotropic vorticity equation [8]). This was the implementation of the first stage of the plan of the Meteorological Research Group at the Institute for Advanced Study (Princeton, United States) to create a series of models that would reproduce step-by-step and better and better the real state of the atmosphere. The principal possibility to solve operationally the problem of weather prediction by numerical methods with improved computers (in particular, by excluding nonarithmetical operations) was an important conclusion.

The central problem of climate theory, stated in the first half of the 20th century, was the reproduction of the main characteristics of atmospheric circulation with mathematical models [9]. This work understood the theory of general circulation as the possibility to describe it with the equations of geophysical hydrodynamics. In 1956, the results of the first numerical experiment reproducing the main characteristics of general atmospheric circulation with a two-layer, quasi-geostrophic, "hemispheric" model were published [10], and, in the early 1960s, the first nine-level model based on primitive (nonsimplified) equations appeared [11]. Models of the ocean's general circulation [12, 13] were developed simultaneously with atmospheric models. In 1969, the results of numerical experiments with the first coupled model of general circulation of the atmosphere and ocean were published [14].

In 1973, in Russia, on G.I. Marchuk's initiative, the Division of Oceanology, Atmospheric Physics, and Geography of the USSR Academy of Sciences decided to create mathematical models of climate based on the models of general circulation of the atmosphere and ocean. One such model, whose computation was based on the conservation laws and implicit decomposition methods, was built at the Computer Center of the Siberian Branch of the USSR Academy of Sciences. For the first time, the problem of general circulation of the atmosphere and ocean was discussed in all its aspects, from physical experimentation and its mathematical formulation to computation [15].

The limited capabilities of the computers of that time and the insufficiently detailed parameterization of physical processes in the atmosphere and ocean made it impossible to obtain the quality of climate reproduction that would meet the state-of-the-art requirements (in particular, the so-called climate "drift" appeared, the growing deviation of predicted characteristics from the observed ones). Nevertheless, the above works laid the foundation for the further development of climate simulation both in the world and in Russia. Moreover, it was demonstrated that progress in computer development allowed us to build more precise models of specific physical processes and, thus, not only to improve climate models and weather prediction techniques but also to formulate new tasks and new requirements on computer systems.

At present, climate models are being intensively developed due to improvements in computers. This "parallelism" in development is necessary to understand the mechanisms responsible for the reproduction of various climate characteristics. The processing of the results of numerical experiments aimed at creating a model of contemporary climate within international programs has shown that the main characteristics obtained with various models and then averaged over the whole set of models turn out to be closer to the observed reality than the characteristics obtained with individual, although the best, models.

International programs, such as the Atmospheric Model Intercomparison Project (AMIP) and the Coupled Model Intercomparison Project (CMIP), are targeted at comparing models developed by research groups from different countries both with other models and with observation data. This helps study systematic errors in the reproduction of contemporary climate and assess the range of climate changes, predetermined, for example, by the anthropogenic impact. In Russia, in particular, such climate models are created at the RAS Institute of Numerical Mathematics and the Main Geophysical Observatory [16]. The coupled model of the atmosphere and ocean [17], built by

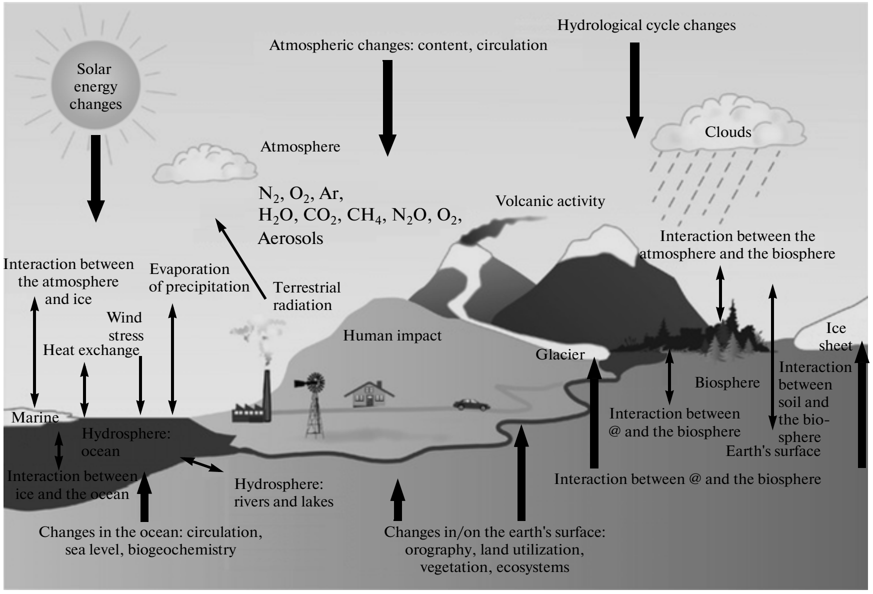


**Fig. 1.** Schematic of the climate system's components, their interactions, and main processes.

the RAS Institute of Atmospheric Physics for the study of long-term climate changes, belongs to the class of models of intermediate complexity.

## PRINCIPLES OF CONSTRUCTING CONTEMPORARY CLIMATE MODELS

According to the WMO's definition, the following interacting components form the earth's climate system: the atmosphere, i.e., the earth's gaseous shell of a complex composition (oxygen, nitrogen, carbon dioxide, water vapor, ozone, etc.), which affects the transfer of solar radiation, coming to the atmosphere's upper boundary, to the earth's surface and which is the most variable component of the system under consideration; the ocean, i.e., the main water reservoir in the system, consisting of the salt waters of the world ocean and the seas adjacent to it, absorbing the main part of solar radiation that comes to its surface, and representing, owing to the high heat capacity of the water, a powerful energy accumulator; the land, i.e., the surface of the continents with its hydrological system (inland water bodies, swamps, and rivers) and the soil (including ground waters); the cryosphere, i.e., continental and marine ice, mountain glaciers, the snow cover, and the cryolithozone (permafrost); and the biota, i.e., the vegetation on land and in the ocean, as well as organisms in the air, sea, and land, including man (Fig. 1).

Contemporary climate models are based on several principles. It is assumed that the equations of classical equilibrium thermodynamics are true locally and that the Navier–Stokes equations for a compressible fluid are true for the description of the dynamics of the atmosphere and ocean. Since the contemporary models, owing mainly to computing capacities, use the Reynolds equations, averaged by some spatial and temporal scales of the Navier–Stokes equation, it is assumed that there exists a principal possibility of their closure. The closure procedure assumes that the effects of subset-scale processes (scales smaller than the averaging scale) can be expressed through the characteristics of large-scale processes. The latter include (shortwave and longwave) radiation transfer; moisture phase transfer and local sedimentation; convection; turbulence in boundary layers (some characteristics of these layers are described explicitly); small-scale orographic disturbances; wave resistance (interaction between small-scale gravitational waves and the main flow); small-scale dissipation and diffusion; and the transfer of heat, moisture, methane, and other gases in the land's active layer, including that in the presence of water bodies. Finally, the hydrostatic approximation that the vertical pressure gradient is counterbalanced by gravity is true for the description of large-scale atmospheric and oceanic motions. The use of this approximation requires additional simplifications (the earth's constant radius and the neglect of the Coriolis force's components with a vertical speed component), so that the system of equations in the absence of external energy sources and dissipation obeys the energy preservation law. The equations of the thermodynamics of the atmosphere and ocean, the closures of subset-scale processes, and marginal conditions were considered in detail in [18].

Obviously, if the initial data are random, it is impossible to obtain analytical solutions to complex nonlinear equations in the hydrothermodynamics of the atmosphere and ocean; so, their approximate values are attempted with various finite-dimensional approximations. Approximations should be constructed so that the preservation law, an analog of the initial law, should be observed in the absence of dissipation and energy sources. This law automatically leads to computational stability of the solution of a difference problem if stability is understood as the continuous dependence of the solution norm on the right-side norm and the initial-data norm.

At the same time, the above requirement is insufficient for constructing difference schemes for climate models. It is important to note that, unlike weather prediction problems, where it is necessary to reproduce the solution at the final interval, climate simulation problems need the approximation of the initial model's attractor as a plurality and a measure on it or a statistical stationary solution. The proof of the solubility of finite-dimensional climate models and of the existence of a global attractor for them presents few problems [2]. One problem is in proving the convergence of attractors of finite-dimensional approximations to the initial model's attractor if the approximation parameters tend to zero. The difficulty here is also in the choice of the metric in which convergence is studied. Constructive assessments of the above convergence in "useful" (Hausdorffian) metrics are currently absent, which is an important and interesting problem of computational mathematics. Since there are no convergence theorems, climate-system modeling uses an approach related to the approximation of the most significant physical processes that take part in climate formation. Here are a few examples.

Since the atmosphere and ocean are quasi-two-dimensional, spectral energy transfer in these media depends on the laws of a 2-D fluid. As is known, the ideal incompressible 2-D fluid has two quadratic invariants, energy and enstrophy (the vorticity square). In addition, energy distribution in the inertial interval of scales, in which energy dissipation and generation are practically absent and the main process is spectral energy transfer, depends, in fact, on enstrophy transfer toward high wave numbers. To observe this condition in a numerical model, it is necessary to construct finite-dimensional analogs so that the finite-dimensional analogs of energy and enstrophy, which would be invariants in the absence of dissipation and sources, would also exist in the 2-D asymptotic.

Note, however, that measurements made in recent decades (see, for example, [19]) show that the atmosphere has principal features that distinguish its evolution from the behavior of a quasi-two-dimensional fluid. Energy generation in the atmosphere occurs on synoptic scales owing to the implementation of baroclinic instability. At scales exceeding synoptic ones, the inertial interval is absent and spectral energy distribution depends on the relative ratio of the characteristic time of energy dissipation in the boundary layer to the characteristic time of nonlinear interactions. On scales smaller than synoptic ones, the inertial interval exists, and there, according to the theory of 2-D turbulence, energy distribution takes the form of $k^{-3}$ ($k$ is a spatial wave number). However, starting with a scale of about 800 km, the energy distribution follows the $k^{-5/3}$ law, as in the Kolmogorov 3-D turbulence, although the atmosphere is probably quasi-two-dimensional on these scales.

Then, the law of preservation of angular momentum relative to the earth's rotation axis determines the distribution of wind velocity near the earth's surface (the presence of trade winds). The law of entropy preservation in the adiabatic approximation is also important. In addition, we should take into account physical phenomena such as cyclogenesis, whose correct reproduction requires a good spectral approximation of some linear operators (by proper and singular numbers); 30- to 60-day fluctuations in the tropics; the propagation of quasi-stationary waves; and many other processes responsible for climate characteristics. Of special importance is the solution of the equations of transfer of small admixtures with large spatial gradients, which imposes an essential requirement on the condition of monotonicity of difference schemes.

Note also the problem of displaying computational algorithms on supercomputer architecture. At present, the development of computers and computational algorithms is associated with parallel computing. The current assessments of computational algorithms may differ significantly from the established assessments related to sequential computation. A researcher who often uses massive parallel-computing systems has to choose an algorithm that may be inefficient for sequential computation but easily parallelizable. Since the number of arithmetical operations is huge in the process of solving climate problems, it is advisable to design computer systems directly oriented toward solving these problems.

## MODELING CLIMATE AND ITS CHANGES

In the early 2000s, the Coupled Model Intercomparison Project phase 3 (CMIP3) was announced to reproduce the contemporary climate and predict its changes with the state-of-the-art models of the atmosphere and ocean. In 2004–2005, experiments were conducted according to the IPCC scenarios [1]. All in all, the project involved 23 coupled models created in different countries and with different parameters. The results obtained with these models became the basis of climate change predictions for the 21st century in the high-profile IPCC Fourth Assessment Report [1]. The sole Russian model that participated in this project was the coupled model of the general circulation of the atmosphere and ocean, developed at the RAS Institute of Numerical Mathematics (INM) under the name of the Institute of Numerical Mathematics Climate Model, version 3.0 (INMCM3.0) [1]. The calculations of the contemporary climate and its changes in the 20th century, as well as probable climate changes in the 21st and 22nd centuries, with the INMCM3.0 model were given in [20].

Thus far, the INMCM4 version of the model has been developed [21], which is engaged in the CMIP5 program to form the basis for the IPCC Fifth Assessment Report to be published in 2013. Along with computation blocks for the general circulation of the atmosphere and ocean, the INMCM4 model contains a block of the carbon cycle. In addition to these two versions, a model of the earth system is being designed to include a block describing the atmospheric chemistry together with the above blocks [22].

The INMCM4 model and its reproduction of contemporary climate were described in detail in [21]. In the atmosphere, the resolution is 2 × 1.5° longitudinally and latitudinally and has 21 levels vertically up to an altitude of about 30 km. The time increment in the dynamic block is 5 min. In the ocean model, the resolution is 1 × 0.5° longitudinally and latitudinally and has 40 levels vertically. The time increment is 2 h. At each step of the ocean model, the exchange of boundary conditions occurs between the atmosphere and the ocean. The model also considers processes that occur in the cryosphere, in the dry-land active layer, and on the dry land surface, including the vegetation cover and the carbon cycle with its evolution of plant, soil, oceanic, and atmospheric carbon. The model was implemented on parallel computers with MPI and OPEN MP. It takes the RAS INM cluster of 48 processors 24 hr of real time to calculate 8 simulation years.

The CMIP5 program envisages an all-round comparison of all existing climate models, reproducing the climate of the past, present, and future. Numerical experiments suggested by CMIP5 are based on realistic and methodological calculations. The realistic calculations simulate the climate of the past or present according to a prescribed scenario. The predictions of probable future changes are built according to these calculations. The methodological calculations contain an idealized scenario and allow us to understand better the reasons why the models yield any particular result. Moreover, each intended numerical experiment has a priority; i.e., it is desirable that the model run experiments with the highest priority first and then, if resources are available and scientific interest is present, experiments with lower priorities.

The INMCM4 model ran only those numerical experiments that had the highest priority. The realistic experiments were the following: 1.1, reproducing climate changes between 1850 and 2005 with the concentrations of greenhouse and other gases, volcanic and anthropogenic aerosols, and solar radiation variations observed; 1.2, modeling climate changes between 2006 and 2100 with scenarios RCP8.5 (very warm) and RCP4.5 (most probable) for the concentrations of greenhouse and other gases; 1.3, modeling climate changes between 1850 and 2100 with the anthropogenic emissions of carbon dioxide as a result of fuel combustion and land management instead of the concentrations inferred in experiments 1.1 and 1.2, which presupposed an interactive calculation of the carbon cycle; in other respects, the experimental conditions corresponded to experiments 1.1 and 1.2 (the RCP8.5 scenario); 1.4, modeling the preindustrial climate for 500 yr; and 1.5, reproducing the climate between 1979 and 2008 with the prescribed observable ocean temperatures and sea ice amounts, i.e., without the use of the ocean model.

In addition, the following methodological experiments were conducted: 2.1, modeling climate for conditions under which the $CO_2$ concentration increased starting with the preindustrial level by 1% a year for 140 yr, i.e., until it had quadrupled; 2.2, modeling climate for a situation in which an instantaneous quadrupling of the $CO_2$ concentration occurred at the initial moment and then stayed constant for 150 yr; and 2.3, a situation similar to the previous case but with a prescribed condition of the ocean.

The results of all the conducted experiments, amounting to nearly 8 Tb, were loaded in the CMIP5 database. According to the results of the methodological experiments, the model's equilibrium sensitivity to the quadrupling of the $CO_2$ concentration is 4.1°, which is probably one of the lowest among the existing models. The CMIP5 results have not been published yet, but, according to the CMIP3 data, equilibrium sensitivity to $CO_2$ doubling for all the models stays within 2.1°–4.4° [1]. At the same time, the value of global warming depends logarithmically on the increase in $CO_2$ concentrations. The warming level 150 yr after the instantaneous quadrupling of the $CO_2$ concentration in the model is only 3.1° due to the ocean's thermal inertia.

The *reproduction of the contemporary climate* was described in detail in [21]; this paper also contains a brief analysis of the annual average fields of temperature, pressure, and wind velocity. The comparison of high-latitude distributions of zonally averaged temperatures, calculated using the NCEP reanalysis results [23] for the period 1971–2000 and the results of model experiment 1.3 (the same years), shows that in the troposphere the difference between the model temperature and the observable temperature does not exceed 1°–2°. The exception is the lower troposphere in the Arctic, where the model overstates the temperature by 3°–4°. The lower stratosphere in the model is 4°–6° colder at the high latitudes of both hemispheres

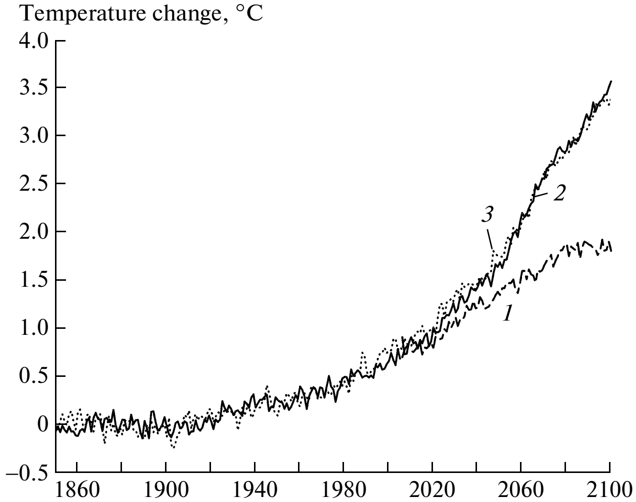


**Fig. 2.** Temperature change compared to 1850–1899 in experiments.
(*1*) The RCP 4.5 scenario, (*2*) the RCP 8.5 scenario, and (*3*) experiment 1.3.

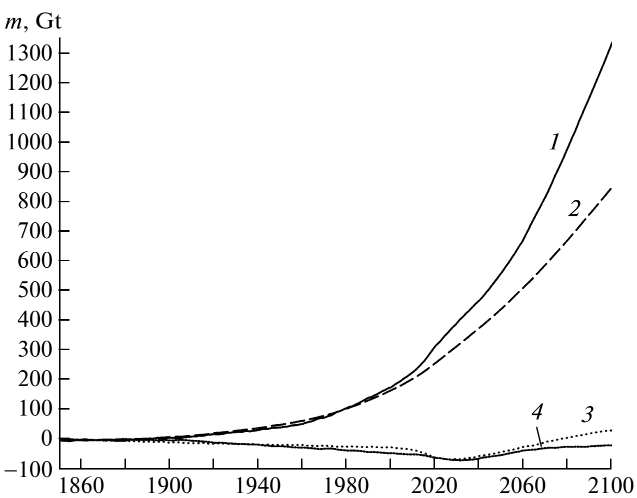


**Fig. 3.** Carbon mass change in the experiment.
(*1*) In the atmosphere, (*2*) in the ocean, (*3*) in plants, and (*4*) in soil in experiment 1.3.

and 2°–4° warmer near the tropical tropopause than in the reanalysis data.

The comparative analysis of the geographical distribution of air temperatures near the earth's surface according to the NCEP reanalysis data [23] and the model results shows that generally the model reproduces correctly the observable temperature. Errors do not exceed 2°. The only exception is Northern Africa and Southwest Asia, where the temperature is understated by 3°–5°, and some regions around the South Pole, where the temperature is overstated by 3°–6°.

The comparison of the observable and model velocities of zonal winds reveals the model's good reproduction of the regions of trade winds and western winds in temperate latitudes, as well as of subtropical velocity maximums near the tropopause. In the troposphere, the wind velocity error in the model does not exceed 2 m/s, and, in the stratosphere, it reaches 4–8 m/s at the temperate latitudes of the Southern Hemisphere.

The analysis of the annual average pressure distribution shows that the model reproduces well the observed subtropical anticyclones of the Northern and Southern hemispheres and the Aleutian and Icelandic pressure minimums, as well as the pressure minimum around the South Pole and the maximums over the Antarctic and Greenland. Generally, errors do not exceed 1–2 GPa, the exception being a 4- to 5-GPa pressure understatement over Northeast Asia, Alaska, and the adjacent regions of the Pacific and Arctic oceans.

*Modeling climate changes.* The analysis of the results of the experiment allows us to identify the main modes of natural climate variability in the model. To this end, the empirical orthogonal functions (EOFs) of the surface temperature averaged for each five-year period of preindustrial experiment 1.4 were calculated in various geographical regions. These functions represent the eigenvectors of the covariance matrix of the time series of the field of the corresponding climate variable and characterize their contribution to this variable's temporal variability.

Of greatest interest is the North Atlantic–Arctic–Eurasia region where, according to observations, the Atlantic Decennial Oscillation manifests itself (see, for example, [24]). The first temperature EOF over the North Atlantic, Arctic, and Eurasia characterizes a temperature increase at high latitudes of the Northern Hemisphere with its maximum in the Atlantic sector of the Arctic Ocean. The characteristic period of variability in the model is 35–50 yr. The difference between the near-earth temperature averaged over the past 20 yr (1991–2010) and the previous 20 yr (1971–1990), according to the NCEP reanalysis data [23], is characterized by the fact that, against the general warming, we can see a stronger warming in the Atlantic sector of the Arctic. Thus, the observable warming of the Arctic in recent decades may be noticeably enhanced by natural variability that projects on the first EOF.

The globally averaged temperature change between 1850 and 2100, according to expert data, as well as in the first half of the preindustrial experiment, can be seen in Fig. 2. Natural temperature fluctuations in the model climate system do not exceed 0.2°. Warming during the 20th century, according to the model data, was about 0.7°. By the end of the 21st century, warming reaches almost 1.9° under the RCP4.5 scenario and 3.5° under the RCP8.5 scenario. The global warming indicators in experiment 1.3, where the $CO_2$ concentration is calculated according to preset anthropogenic emissions with the carbon cycle model, are very close to the data of experiments 1.1 and 1.2, where the $CO_2$ concentration is prescribed.

The carbon cycle's role is reflected in Fig. 3, which shows an increase in carbon mass in the plants, soil, ocean, and atmosphere compared to 1850. The carbon mass growth in the ocean is much smaller than in the atmosphere. In the plants and soil, a small mass decrease happens between 1850 and 2030 owing to intensive land use and small growth should occur between 2040 and 2100, since the warming and fertilization effect will become stronger.

The *earth system model* includes an atmospheric block with a resolution of $5 \times 4°$ longitudinally and latitudinally and 39 levels vertically up to a height of 90 km [22]. The blocks of the ocean and the carbon cycle are fully identical, as utilized in model INMCM4 and described briefly above. The model also includes a chemical block, which accounts for the variability of 74 small gaseous components of the atmosphere that directly or indirectly affect the photochemical change in the ozone concentration. The model also accounts for the responses of the oxygen, hydrogen, nitrogen, chlorine, bromine, and sulfur cycles. The atmospheric transfer of chemically active admixtures uses wind velocities calculated in the dynamic block, and the velocities of chemical reactions are evaluated with the help of the temperature obtained in the circulation block. The calculated concentrations of ozone are used to compute the velocities of the radiant heating of the atmosphere, and those of methane and water vapor, for atmospheric cooling, accounted for in the circulation block. To account for heterogeneous processes, the formation and evolution of polar stratospheric clouds are considered. The interaction of the chemical and dynamic blocks was described in detail in [25]. This model is necessary primarily for model situations in which climate and environmental changes have a complex nature, for example, when evaluating the consequences of hypothetical geoengineering impacts in order to mitigate global warming [26].

A numerical experiment whose conditions were similar to those of experiment 1.4 was conducted with the earth system model, but, in addition, scenarios of flows from the surface of several tens of small gaseous components were specified. Since the changes of climate proper and the carbon cycle were considered above, here it is advisable to limit ourselves to analysis of the change in the total ozone content (TOC) in the 20th and 21st centuries (Fig. 4). The minimal TOC at the end of the 20th century and at the beginning of the 21st century is predetermined by the maximal emission of chlorine- and fluorine-containing substances. After a decrease in their emissions, reduction takes place, and, at the end of the 21st century, the TOC exceeds that of the 1970s, which is related to the cool-

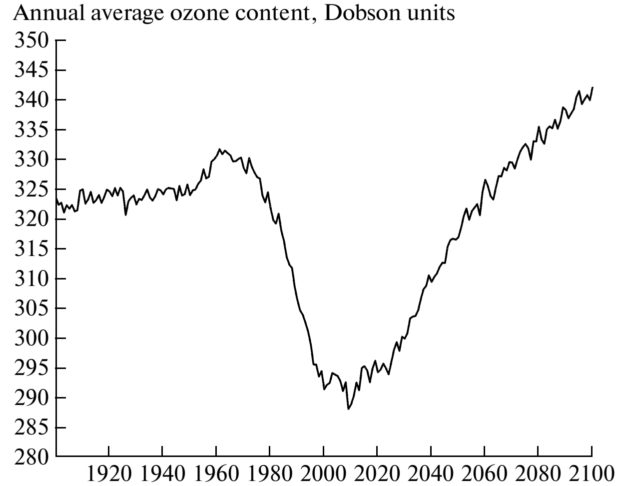


**Fig. 4.** Total ozone content change (Dobson units) in the 20th–21st centuries.

ing of the stratosphere due to the greenhouse effect. However, the exceedance of the TOC level of the 1970s is high only in temperate and at high latitudes, especially at the end of winter in each hemisphere. In the tropics, the TOC at the end of the 21st century does not exceed the 1971–1980 level. This heterogeneity is predetermined by the strengthening of the Brewer–Dobson circulation, i.e., a more intense air rise in the tropical stratosphere and air sinking at high latitudes. Such features of the TOC change in the 20th and 21st centuries were also obtained in other chemical–climate simulations (see, for example, [27]).

This paper pays attention only to certain aspects of modeling climate and its changes, which includes many problems that need individual investigation. In particular, this paper does not consider a new direction, mathematical climate theory. One of its main problems is the justification of the applicability of contemporary climate models for analysis of the sensitivity of the real climate system to external impacts [2].

Computers realize a finite-dimensional approximation of the initial differential model, and, at first sight, it seems that analytical studies of the asymptotic properties of models are not that important because the properties of closeness to reality can be studied directly by comparative analysis of the results of numerical experiments with observation data. However, some scientists have a serious objection to this statement related to external impacts. If it is impossible to conduct target experiments with the climate system, we need a convincing justification that the sensitivity of models under development to small external impacts is close to the sensitivity of the real climate system.

The question arises: is it possible, knowing the path of the system, to calculate its response to small external impacts? What conditions should this system meet to give a positive answer to this question? Moreover, the consequence of the studies should be the formulation of the main mathematical result, a method of constructing the response operator.

The further development of climate models and methods of weather prediction is related to increasing spatial resolution and improving the physical parameterizations of subgrid scales. The level of spatial resolution and largely the complexity of physical parameterizations are limited by the capacity of the most powerful computer systems (supercomputers). Thus, the peak capacity of computer complexes available to the developers of climate models (several hundred or at best thousand arithmetical operations per second) enables long (hundreds of years) calculations on a grid with a resolution of about 100 km. This resolution does not allow us to assess climate change effects at the regional level, while regional differences are of special interest in the future climate.

To obtain regional forecasts, mesoscale models are used with a resolution of 1–10 km and the size of the calculated area varying from several hundred to several thousand kilometers. Mesoscale models of the above resolution are, in turn, unable to reproduce explicitly the structure of atmospheric currents with a spatial scale smaller than several kilometers. Such currents can be reproduced by the large-eddy simulation method, which enables the explicit description of the nonstationary dynamics of large 3-D eddies as the main contributors to the energy of turbulent flows in the boundary layer of the atmosphere. The spatial resolution of large-eddy simulation models of the boundary layer of the atmosphere, depending on the type of turbulent flows, ranges from several meters to several tens of meters, and the size of the calculated area, to several tens of kilometers.

At present, petaflop ($10^{15}$ arithmetical operations per second) computer systems are coming into wide use, and during this decade it is expected that the capacity of supercomputers will reach an exaflop ($10^{18}$ operations per second) level. This means that global atmospheric models will have a resolution typical of the current mesoscale models (1–10 km), and the grid pitch of models used to predict atmospheric circulation at the regional level will be ~100 m. The same resolution (from 1 km to 100 m) will also be available for discretizing hydrothermodynamic equations along the vertical coordinate. The experience obtained by Japanese researchers in modeling global climate processes with a horizontal resolution of 3.5–10 km [28] has laid the foundation for active experimentation with models of very high resolution and has led to the necessity to develop a strategy of further development of climate models taking into account the prospects for high-capacity computing [29].

In this respect, we will have to revise many parameterizations of subgrid processes used in the current models of the general circulation of the atmosphere and ocean. First of all, this affects convection parameterization, since it is beginning to be reproduced explicitly at resolutions of several kilometers and smaller. Increasing attention is being paid to the creation of "seamless" (with an improved local resolution and physical description) simulation systems that allow the creation of a broader spectrum of atmospheric flows within a single computing technology (see, for example, [30]). The transfer to very detailed resolutions cannot be carried out "mechanically" (only by rejecting hydrostatic approximation) or without a large-scale redesign of the existing computing technologies and, in some cases, the reformulation of equation parameterizations and systems currently used for an approximate description of the hydrothermodynamics of the climate system and the earth system in the future.

In conclusion, we should stress that the current climate models and future models can positively be referred to the high-tech class. Bearing in mind that climate change prediction is a problem of national importance, such technologies should be viewed as necessary components of national security. In this respect, it is hard to explain the fact that, despite the presence of the Climate Doctrine, Russia has no national climate program.

## ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project nos. 11-05-12018-ofi-m-2011, 11-05-00435, 11-05-01190, 09-05-00681, and 09-05-01231), a grant of the President of the Russian Federation for Support of Leading Scientific Schools (no. NSh-3521.2010.5), a State Contract (no. 07.514.11.4044), and Program 16 of the RAS Basic Research.

## REFERENCES

1. "IPCC Fourth Assessment Report," in *Intergovernmental Panel on Climate Change*, Ed. by S. D. Solomon, D. Qin, M. Manning, Z. Chen, et al. (Cambridge Univ. Press, New York, 2007).
2. V. P. Dymnikov and A. N. Filatov, *Basics of the Mathematical Theory of Climate* (VINITI, Moscow, 1994) [in Russian].
3. V. Bjerknes, "Das Problem von der Wettervorhersage, betrachtet vom Standpunkt der Mechanik und der Physik," Meteorol. Z. **21**, 1 (1904).
4. L. F. Richardson, *Weather Prediction by Numerical Process* (Cambridge Univ. Press, 1922).
5. R. Courant, K. Friedrichs, and H. Lewy, "Uber die partiellen Differentialgleichungen der matematischen Physik," Math. Ann. **100** (1928).
6. I. A. Kibel', "Application to Meteorology of the Equations of the Dynamics of a Baroclinic Fluid," Izv. Akad. Nauk SSSR. Ser. Geogr. Geofiz., No. 5, 627 (1940).

7. I. A. Kibel', *Introduction to Hydrodynamic Methods of Short-Term Weather Forecasting* (Gostekhizdat, Moscow, 1957) [in Russian].
8. J. G. Charney, R. Fjörtoft, and J. von Neumann, "Numerical Integrations of the Barotropic Vorticity Equations," Tellus **2** (1950).
9. A. A. Dorodnitsyn, B. I. Izvekov, and M. E. Shvets, "Mathematical Theory of General Circulation," Meteorol. Gidrol., No. 4 (1939).
10. N. Phillips, "The General Circulation of the Atmosphere: a Numerical Experiment," Quart. J. R. Meteorol. Soc. **82** (1956).
11. J. Smagorinsky, "General Circulation Experiment with the Primitive Equations. I. The Basic Experiment," Mon. Wea. Rev. **91** (1963).
12. A. S. Sarkisyan, *Theoretical Basics and Computation of Oceanic Currents* (Gidrometeoizdat, Leningrad, 1966) [in Russian].
13. K. A. Bryan, "Numerical Method for the Study of the Circulation of the World Ocean," J. Comput. Phys. **4** (1969).
14. S. Manabe and K. Bryan, "Climate and the Ocean Circulation," Mon. Wea. Rev. **97** (1969).
15. G. I. Marchuk, V. P. Dymnikov, V. B. Zalesnyi, et al., *Mathematical Modeling of General Circulation of the Atmosphere and Ocean* (Gidrometeoizdat, Leningrad, 1984) [in Russian].
16. V. P. Meleshko, V. M. Kattsov, P. V. Sporyshev, et al., "The Study of Possible Climate Changes with Models of the General Circulation of the Atmosphere and Ocean," in *Climate Changes and Their Consequences: Proceedings of the Special Session of the Academic Council of the Center of International Cooperation in Environmental Problems Dedicated to the 80th Anniversary of Academician M.I. Budyko (May 19–20, 1999)* (St. Petersburg, 2002) [in Russian].
17. I. I. Mokhov, P. F. Demchenko, A. V. Eliseev, et al., "Estimation of Global and Regional Climate Changes during the 19th–21st Centuries on the Basis of the IAP RAS Model with Consideration for Anthropogenic Forcing," Izv. Atmospheric Oceanic Phys. **38** (5), 555 (2002).
18. V. P. Dymnikov, V. N. Lykosov, E. M. Volodin, et al., "Modeling Climate and Its Changes," in *Current Problems of Computational Mathematics and Mathematical Modeling*, Vol. 2: *Mathematical Modeling* (Nauka, Moscow, 2005) [in Russian].
19. K. S. Gage and G. D. Nastrom, "On the Spectrum of Atmospheric Velocity Fluctuations Seen by MST/ST Radar and Their Interpretation," Radio Sci. **20** (1990).
20. E. M. Volodin and N. A. Dianskii, "Simulation of Climate Changes in the 20th–22nd Centuries with a Coupled Atmosphere–Ocean General Circulation Model," Izv. Atmospheric Oceanic Phys. **42** (3), 267 (2006).
21. E. M. Volodin, N. A. Dianskii, and A. V. Gusev, "Simulating Present-Day Climate with the INMCM4.0 Coupled Model of the Atmospheric and Oceanic General Circulations," Izv. Atmospheric Oceanic Phys. **46** (4), 414 (2010).
22. E. M. Volodin, V. Ya. Galin, A. V. Gusev, et al., "Earth System Model of INM RAS," Russ. J. Numer. Anal. Math. Modelling **25** (5) (2010).
23. E. Kalnay, M. Kanamitsu, R. Kistler, et al., "The NCEP/NCAR 40-Year Reanalysis Project," Bull. Am. Meteorol. Soc. **77** (1996).
24. V. A. Semenov, M. Latif, D. Dommenget, et al., "The Impact of North Atlantic-Arctic Multidecadal Variability on Northern Hemisphere Surface Air Temperature," J. Climate **23** (2010).
25. V. Ya. Galin, S. P. Smyshlyaev, and E. M. Volodin, "Combined Chemistry–Climate Model of the Atmosphere," Izv. Atmospheric Oceanic Phys. **43** (4), 399 (2007).
26. E. M. Volodin, S. V. Kostrykin, and A. G. Ryaboshapko, "Simulation of Climate Change Induced by Injection of Sulfur Compounds into the Stratosphere," Izv. Atmospheric Oceanic Phys. **47** (4), 430 (2011).
27. V. A. Zubov, E. V. Rozanov, I. V. Rozanova, et al., "Simulation of Changes in Global Ozone and Atmospheric Dynamics in the 21st Century with the Chemistry–Climate Model SOCOL," Izv. Atmospheric Oceanic Phys. **47** (3), 301 (2011).
28. M. Satoh, H. Tomita, H. Miura, et al., "Development of a Global Cloud Resolving Model. A Multi-Scale Structure of Tropical Convections," J. Earth Simulator **3** (2005).
29. J. Shukla, R. Hagedorn, B. Hoskins, et al., "Revolution in Climate Prediction Is Both Necessary and Possible. A Declaration at the World Modeling Summit for Climate Prediction," Bull. Am. Meteorol. Soc. **90** (2009).
30. W.-K. Tao, J.-D. Chern, R. Atlas, et al., "A Multiscale Modeling System," Bull. Am. Meteorol. Soc. **90** (2009).