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ELEMENTARY PARTICLES AND FIELDS
Theory

Weak and Electromagnetic Mechanisms of Neutrino-Pair Photoproduction in a Strongly Magnetized Electron Gas

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Abstract—Expressions for the power of neutrino radiation from a degenerate electron gas in a strong magnetic field are derived for the case of neutrino-pair photoproduction via the weak and electromagnetic interaction mechanisms (it is assumed that the neutrino possesses electromagnetic form factors). It is shown that the neutrino luminosity of a medium in the electromagnetic reaction channel may exceed substantially the luminosity in the weak channel. Relative upper bounds on the effective neutrino magnetic moment are obtained.

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1. INTRODUCTION

Neutrino emission is a dominant mechanism of the energy loss of stars at a late stage of their evolution [1]. We will consider processes proceeding in outer regions of neutron stars under conditions such that matter is nearly transparent to neutrinos, so that nascent neutrinos carry away their energy entirely, whereby the star being considered is cooled. In surface layers of such stars, magnetic fields of strength H in excess of 10^{12} G may exist; for the class of neutron stars called magnetars, the magnetic-field strength may reach values in the range of 10^{14}–10^{16} G [2].

Dominant processes of neutrino production in outer regions of neutron stars are the annihilation of an electron—positron pair, $e^-e^+ \to \nu\bar{\nu}$; the photoproduction of a neutrino pair on an electron, $\gamma e^\pm \to e^\pm \nu\bar{\nu}$; photon decay, $\gamma \to \nu\bar{\nu}$; and two-photon annihilation, $\gamma\gamma \to \nu\bar{\nu}$. Each of these processes was investigated at various values of the temperature and matter density. The basic results of those investigations were presented in the review article of Yakovlev et al. [3]. However, the respective calculations become more involved in the presence of a magnetic field. The luminosity of a degenerate nonrelativistic electron gas via neutrino emission was found in [4]. The luminosity owing to these processes in a degenerate electron gas was determined in [5] for the case of a superstrong magnetic field (the contribution of electron—positron annihilation was disregarded there because of the smallness of the positron fraction).

Various Standard Model extensions featuring extra particles or introducing new properties for known particles have been considered in recent years. If such assumptions are valid, there are additional reaction channels contributing to the energy loss of astrophysical objects. Hypothetical electromagnetic form factors of the neutrino are one of such possibilities. In the minimal standard model of electroweak interactions, the neutrinos are massless and do not possess electromagnetic dipole moments. After a simple extension of this model, the massive Dirac neutrino develops a magnetic dipole moment (MDM) that is caused by one-loop radiative corrections, $\mu_\nu \simeq 3.2 \times 10^{-19}(m_\nu/1\ \text{eV})\mu_\text{B}$ [6] (where m_ν is the neutrino mass, while μ_B is the Bohr magneton), and which is many orders of magnitude lower than the existing laboratory, astrophysical, and cosmological bounds on μ_ν. The electromagnetic properties of the neutrinos were discussed in the review article of Giunti and Studenikin [7] (see also [1]). Upper bounds on the neutrino electric and magnetic dipole moments (d_ν and μ_ν, respectively) from astrophysical and cosmological considerations are about $(10^{-12}$–$10^{-10})\mu_\text{B}$ and depend substantially on the choice of model (see [8], p. 559, and references therein). In particular, a conservative constraint obtained from an analysis of the spectrum of solar neutrinos [9] was presented in [8]:

$$\mu_\nu < 0.54 \times 10^{-10}\mu_\text{B}. \tag{1}$$

The constraint obtained recently in the GEMMA laboratory experiment devoted to studying antineutrino—

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electron scattering is [10]

$$\mu_\nu < 3.2 \times 10^{-11}\mu_{\rm B}. \quad (2)$$

The interaction of the Dirac neutrino with an electromagnetic field owing to the possible electromagnetic dipole moments (EMDM) μ_ν and d_ν of the neutrino can be described in terms of the vertex operator for $\gamma\bar{\nu}\nu$ coupling. This vertex operator has the form [11–13] (see also [1, 7])[2)]

$$V^\alpha(Q) \quad (3)$$
$$= \mu_{\rm B}\sigma^{\alpha\beta}Q_\beta\left[f_{2\nu}(Q^2) + i\gamma^5 g_{2\nu}(Q^2)\right],$$

where Q is the photon 4-momentum and $\sigma^{\alpha\beta} = (\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha)/2$. Taking into account the relative smallness of Q^2 in the processes being considered, we use below the static values of the neutrino electromagnetic form factors: $f_{2\nu} = f_{2\nu}(0) = \mu_\nu/\mu_{\rm B}$ and $g_{2\nu} = g_{2\nu}(0) = d_\nu/\mu_{\rm B}$.

In the presence of neutrino EMDM, an additional electromagnetic channel opens up for neutrino-pair emission. In view of this, the electromagnetic mechanism of photoproduction ($\gamma e \to e\nu\bar{\nu}$), neutrino bremsstrahlung on a nucleus [$e^-(Ze) \to e^-(Ze)\nu\bar{\nu}$], electron–positron annihilation ($e^-e^+ \to \nu\bar{\nu}$), and plasmon decay ($\gamma \to \nu\bar{\nu}$) were studied in [14–16]. The contribution of about m_ν from the interference between weak and electromagnetic interactions to these processes was also investigated. The luminosities $L_{\rm em}$ and $L_{\rm w}$ via, respectively, the electromagnetic and weak mechanisms of the production of $\nu\bar{\nu}$ pairs were compared. It was shown that both mechanisms may play a significant role in the energy loss of stars. The neutrino luminosities of the nucleon matter of neutron stars in the bremsstrahlung processes $nn \to nn\nu\bar{\nu}$ and $np \to np\nu\bar{\nu}$ owing to the electromagnetic mechanism were found in [17]. They proved to be several orders of magnitude smaller than the known luminosities in these processes induced by the weak mechanism.

In the present study, we restrict ourselves to the case of a degenerate electron gas in a strong magnetic field H; that is,

$$T \ll \mu - m, \quad H > (\mu^2 - m^2)/(2e), \quad (4)$$

where T is the temperature, $\mu \simeq \mu(T=0) \equiv \varepsilon_{\rm F} = \sqrt{m^2 + p_{\rm F}^2}$ is the chemical potential of the electron gas, and m is the electron mass; also, $\varepsilon_{\rm F}$ and $p_{\rm F}$ are the Fermi energy and momentum, respectively. In this case, medium electrons are in the ground Landau level (the principal quantum number is $n = 0$) and

$$p_{\rm F} = 2\pi^2 n_e/h, \quad (5)$$

where n_e is the electron concentration and $h = eH$ (the electron charge is $-e < 0$). Considering that, under the conditions in (4), typical energy values for the initial (thermal) photon characterized by the dispersion law specified by Eqs. (13) and (14) (see below) satisfy the strong inequality $k_0 \ll \sqrt{eH}$, we conclude that, in the processes under consideration, both the final and the virtual electron also occupy only the zeroth Landau level, where the electron spin can only be antiparallel to the magnetic-field direction.

The wave function for an electron in the ground state in a constant uniform magnetic field $\mathbf{H}||Oz$ specified by the 4-potential $A^\mu = (0, 0, xH, 0)$ has the form [18]

$$\psi(t, \mathbf{r}) = \left(\frac{h}{\pi}\right)^{1/4}(2p_0L_yL_z)^{-1/2} \quad (6)$$
$$\times \exp\left(-ip_0t + ip_yy + ip_zz - \frac{\eta^2}{2}\right)u(p_{||}),$$

where L_y and L_z are the normalization lengths along the y and z axes, respectively; $p_0 = \sqrt{m^2 + p_z^2}$ is the electron energy; $\eta = \sqrt{h}(x + p_y/h)$; and the bispinor $u(p_{||})$ has the form

$$u\left(p_{||}\right) = \frac{1}{\sqrt{2}}\left(0, \delta_+, 0, \delta_-\right)^T, \quad (7)$$
$$\delta_\pm = \sqrt{p_-} \pm \sqrt{p_+}, \quad p_\pm = p_0 \pm p_z,$$
$$p_+p_- = p_{||}^2 = m^2.$$

This bispinor is normalized by the condition

$$\bar{u}(p_{||})u(p_{||}) = 2m, \quad \bar{u} = u^+\gamma^0,$$

while the density matrix is given by

$$u(p_{||})\bar{u}(p_{||}) = (\hat{p}_{||} + m)\Sigma_-,$$

where $p_{||} = (p_0, 0, 0, p_z)$ and $\Sigma_- = (1 - \Sigma_3)/2$ with $\Sigma_3 = i\gamma^1\gamma^2$.

In the expression for the electron Green's function [19], we restrict ourselves to taking into account the contribution of the ground-state level ($n = 0$). We then have

$$G(x, x') \simeq \left(\frac{h}{\pi}\right)^{1/2}\int\limits_{-\infty}^{\infty}\frac{dp_y}{2\pi} \quad (8)$$
$$\times \exp\left[-\frac{1}{2}(\eta^2 + \eta'^2) + ip_y(y - y')\right]$$
$$\times \int\frac{d^2p_{||}}{(2\pi)^2}\exp[-ip_0(t - t')$$

[2)]We use the system of units where $\hbar = c = k_{\rm B} = 1$ and $\alpha = e^2/4\pi \simeq 1/137$ and assume a pseudo-Euclidean metric with signature $(+---)$; also, $\hat{a} = \gamma^\beta a_\beta$ is the contraction of the Dirac matrices γ^β with the 4-vector $a^\beta = (a^0, \mathbf{a})$ and $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$.

$$+ ip_z(z - z')]G(p_{||})\Sigma_-.$$

In this expression, $\eta' = \eta(x \to x')$, while

$$G(p_{||}) = S(p_{||}) + 2\pi i\delta(p_{||}^2 - m^2) \times N_{\mathrm{F}}(p_0)(\hat{p}_{||} + m) \tag{9}$$

is the Fourier transform of the Green's function in the (0, 3) two-dimensional space. The first term in (9) describes the vacuum contribution,

$$S(p_{||}) = \frac{\hat{p}_{||} + m}{p_{||}^2 - m^2 + i0}, \tag{10}$$

while the second term is the contribution of matter consisting of electrons and positrons, which obey the Fermi distribution

$$N_{\mathrm{F}}(p_0) = \theta(p_0)(e^{(p_0-\mu)/T} + 1)^{-1} + \theta(-p_0)(e^{(-p_0+\mu)/T} + 1)^{-1}. \tag{11}$$

Under the conditions in (4), one can neglect the contribution of positrons [the second term on the right-hand side of (11)] because it is suppressed by the factor $\exp[-(m + \mu)/T]$.

In order to calculate correctly processes in magnetized matter, it is necessary to take into account the dispersion of photons. The photon-dispersion law in the presence of a plasma and a strong magnetic field differs substantially from the vacuum dispersion law [20]. Photons of two different polarizations propagate under these conditions. According to the numbering of four eigenvectors of the photon-polarization operator in [20], these are modes 2 and 3. Their polarization vectors are

$$\epsilon_\alpha^{(2)} = \frac{\tilde{F}_{\alpha\beta}k^\beta}{H\sqrt{k_{||}^2}}, \quad \epsilon_\alpha^{(3)} = \frac{F_{\alpha\beta}k^\beta}{H\sqrt{k_\perp^2}}, \tag{12}$$

where $F_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$ are, respectively, the tensor of the external electromagnetic field and the tensor dual to it; k^α is the photon 4-momentum; $k_\perp^2 = k_x^2 + k_y^2$; $k_{||}^2 = k_0^2 - k_z^2$; and $k^2 = k_{||}^2 - k_\perp^2$. It is well known [4, 20] that, in an superstrong magnetic field, it is sufficient to take into account the interaction of electrons only with mode-2 photons. In our case, where $T \ll m$, we can disregard the effect of the thresholds for the production of electron–positrons pairs on the dispersion of photons; in this approximation, the dispersion equation for mode-2 photons has the form

$$k^2 = k_0^2 - k_z^2 - k_\perp^2 = \omega_p^2, \tag{13}$$

where ω_p is the plasma frequency [20]. We can say that the photon acquires a nonzero mass. Under the conditions in (4), it is approximately given by the expression

$$\omega_p = k_0(\mathbf{k} = 0) = \left(\frac{2\alpha}{\pi}\frac{H}{H_0}\frac{p_{\mathrm{F}}}{\varepsilon_{\mathrm{F}}}\right)^{1/2} m, \tag{14}$$

which can be derived from the general formula (3.2) in [20] [p. 96]. Here, $H_0 = m^2/e = 4.41 \times 10^{13}$ G.

Other special features of the photon dispersion in a magnetic field begin playing a significant role in the vicinity of the energy thresholds for the production of electron–positron pairs and for photon absorption by an electron, which, after that, goes over to a different Landau level. In our case, the lowest threshold is the first pair-production threshold $k_{||}^2 = 4m^2$ corresponding to the production of an electron and a positron in the zeroth Landau level. For $T \ll 2m$, however, photons of energy satisfying the condition $k_0 \ll 2m$ are operative, so that one can disregard the behavior of the photon dispersion curve even in the vicinity of the threshold in question.

In the case of the electromagnetic mechanism of neutrino-pair photoproduction, the electron involved emits a virtual photon, and it is this photon that produces a pair. The respective photon propagator, in which, in our case, we can take into account only the mode-2 contribution [see Eq. (12)], has the form

$$D_{\alpha\beta}(k) = \frac{\epsilon_\alpha^{(2)}\epsilon_\beta^{(2)}}{k^2 - \varkappa_2}. \tag{15}$$

Here, $\varkappa_2$ is the corresponding (in general, com- plex-valued) eigenvalue of the photon polarization operator; $\mathrm{Re}\varkappa_2 = \omega_p^2$ [see Eq. (13)] under the conditions in (4), while $\mathrm{Im}\varkappa_2 = -k_0\Gamma$, where Γ is the total probability for virtual-photon absorption (see, for example, [21]). Under the conditions in question, both pair production by an electron and photon absorption by an electron via the inverse magnetobremsstrahlung mechanism are kinematically forbidden. Thus, the Compton scattering $\gamma e^- \to e^-\gamma$ involving medium electrons is a dominant process that contributes to $\mathrm{Im}\varkappa_2$ (see below).

It should also be noted that the renormalization of the photon wave function ($\epsilon_\alpha^{(2)} \to \sqrt{Z_2}\epsilon_\alpha^{(2)}$) in a magnetized medium can be disregarded under the conditions being considered ($\sqrt{Z_2} \simeq 1$) [5].

2. PHOTOPRODUCTION OF A NEUTRINO PAIR ON AN ELECTRON (WEAK MECHANISM)

Taking into account Eqs. (6) and (8) and performing integration with respect to spacetime coordinates, we can represent the S-matrix element for the process of neutrino-pair photoproduction on an electron in the form

$$S_{fi} = \frac{(2\pi)^3\delta^{(023)}\,(p+k-p'-q-q')}{L_yL_z(2^5V^3k_0p_0p_0'q_0q_0')^{1/2}}M_w, \quad (16)$$

$$M_w = \frac{eG_{\rm F}}{\sqrt{2}}j_\alpha J^\alpha,$$

where the three-dimensional delta function expresses the conservation laws for the components of the total 4-momentum that are indicated parenthetically, $V = L_xL_yL_z$ is the normalization volume, $G_{\rm F}$ is the Fermi constant, and the neutrino and electron currents are given by

$$j^\alpha = \bar{u}_\nu(q')\gamma^\alpha(1+\gamma^5)u_\nu(-q), \quad (17)$$

$$J^\alpha = \bar{u}(p'_{||})[\Gamma^\alpha S(p_{||}+k_{||})\hat{\epsilon} + \hat{\epsilon}S(p'_{||}-k_{||})\Gamma^\alpha]u(p_{||}).$$

In the latter, we have taken into account only the vacuum contribution (10) to the electron Green's function (9)—the medium contribution is absent because, for the process under consideration, virtual electrons cannot become on-shell particles (see also [4]). Here, q' and q are the 4-momenta of, respectively, the neutrino and the antineutrino {we assume that they are massless—$q'^2 = q^2 = 0$—taking into account the cosmological constraint on the sum of the masses of light (active) neutrinos [8]: $\sum m_\nu \lesssim 1$ eV}; $k_{||}$, $p_{||}$, and $p'_{||}$ are the two-dimensional [in the $(0,3)$ space] momenta of the photon, initial electron, and final electron, respectively; and $\hat{\epsilon} = \gamma^\alpha\epsilon_\alpha^{(2)}$ [see Eq. (12)]; and $\Gamma^\alpha = \gamma^\alpha(g_V + g_A\gamma^5)$, the vector and axial-vector electron–neutrino coupling constants being dependent on the flavor $\ell = e, \mu, \tau$ of the neutrino ν_ℓ [8]. Specifically, we have

$$g_V = \pm\frac{1}{2} + 2w, \quad g_A = \pm\frac{1}{2}, \quad (18)$$

where the upper (lower) signs refer to ν_e (ν_μ and ν_τ) and $w = \sin^2\theta_{\rm W} \simeq 0.231$ is the weak-mixing parameter.

The general expression for the medium luminosity (the rate of the energy loss of a unit volume by neutrino-pair emission in the process being considered) has the form

$$L_{\rm w} = \int \frac{d^3kd^3qd^3q'dp_ydp'_ydp_zdp'_z}{(2\pi)^{13}2^5q_0q'_0k_0p_0p'_0L_x} \quad (19)$$
$$\times (2\pi)^3\delta^{(023)}\left(p+k-p'-q-q'\right)|M_{\rm w}|^2$$
$$\times (q_0+q'_0)n_B(k_0)n_F(p_0)\left(1-n_{\rm F}(p'_0)\right),$$

where

$$n_{\rm B}(k_0) = (e^{k_0/T}-1)^{-1},$$

$$n_{\rm F}(p_0) = (e^{(p_0-\mu)/T}+1)^{-1}$$

are the Bose and Fermi distributions for photons and electrons, respectively.

For the square of the reduced matrix element [see Eq. (16)] in (19), we obtain the expression we note that, in the case being considered, the polarizations of initial and final particles have fixed directions: the polarization of the electrons is antiparallel to the magnetic-field direction, the photons have a type-2 polarization [see Eq. (12)], and the polarization of the massless neutrino (antineutrino) is antiparallel (parallel) to its momentum}

$$|M_{\rm w}|^2 = 2\pi\alpha G_{\rm F}^2R, \quad R = N^{\alpha\beta}J_\alpha J^*_\beta, \quad (20)$$

where the neutrino tensor has the form

$$N^{\alpha\beta} = j^\alpha j^{\beta*} \quad (21)$$
$$= {\rm tr}\left[\hat{q}'\gamma^\alpha(1+\gamma^5)\hat{q}\gamma^\beta(1+\gamma^5)\right]$$
$$= 8(q'^\alpha q^\beta - g^{\alpha\beta}q'\cdot q - i\varepsilon^{\alpha\beta\lambda\rho}q'_\lambda q_\rho)$$

and the components of the electron current [see Eq. (17)] are obtained by means of a direct calculation with allowance for Eqs. (7), (10), and (12):

$$J^\alpha = (J_0, 0, 0, J_z); \quad (22)$$
$$J_0 = \frac{1}{2\sqrt{a}}\left[\frac{1}{\Delta}L(a,b) - \frac{1}{\Delta'}L(a,-b')\right],$$
$$L(a,b) = (ag_A + bg_V)(\delta'_+\delta_+ + \delta'_-\delta_-) - (ag_V + bg_A)(\delta'_+\delta_- + \delta'_-\delta_+),$$
$$a = k_{||}^2, \quad b = 2k_{||}\cdot\tilde{p}_{||} = 2(k_0p_z - k_zp_0),$$
$$b' = 2k_{||}\cdot\tilde{p}'_{||} = 2(k_0p'_z - k_zp'_0),$$
$$\Delta = a + 2k_{||}\cdot p_{||}, \quad \Delta' = a - 2k_{||}\cdot p'_{||};$$
$$J_z = J_0(g_A \leftrightarrow g_V).$$

Here, primes refer to the final electron. The quantities $\delta_\pm$ were defined in Eq. (7). It is convenient to perform integration with respect to the neutrino and antineutrino momenta in (19) by inserting the "decomposition of unity" (see [5]),

$$1 = \int d^4Q\delta^{(4)}(q+q'-Q),$$

and by using the well-known relation (see [22], p. 23)

$$\int \frac{d^3q}{q_0}\frac{d^3q'}{q'_0}\delta^{(4)}(q+q'-Q)q^\alpha q'^\beta \quad (23)$$
$$= \frac{\pi}{6}(2Q^\alpha Q^\beta + Q^2g^{\alpha\beta}).$$

This yields [see Eqs. (20) and (21)]

$$\bar{R} \equiv \int d^2Q_\perp\frac{d^3q}{q_0}\frac{d^3q'}{q'_0}\delta^{(4)}(q+q'-Q)R \quad (24)$$

$$= \frac{4}{3}\pi^2 Q_{||}^2 \theta(Q_{||}^2)(Q_-^2 J_+^2 + Q_+^2 J_-^2),$$

where $J_\pm = J_0 \pm J_z$, $Q_\pm = Q_0 \pm Q_z$, and the Heaviside step function takes into account the kinematical constraint

$$Q_{||}^2 = Q_0^2 - Q_z^2 \geq 0. \tag{25}$$

Taking into account Eq. (22), we recast expression (24) into the form

$$\bar{R} = \frac{2^8}{3}\pi^2(g_V^2 + g_A^2)k_{||}^2 Q_{||}^2 \theta(Q_{||}^2) \times \left(\frac{mQ_{||}^2}{\Delta\Delta'}\right)^2 (p_{||}\cdot p'_{||} + m^2). \tag{26}$$

As a result, we obtain the following expression for the luminosity in (19):

$$L_{\rm w} = \frac{\alpha G_{\rm F}^2 g_+^2 m^6}{3\cdot 2^5 \pi^7}\frac{H}{H_0} \times \int \frac{d^3k dp_z dp'_z}{k_0 p_0 p'_0}\theta(Q_{||}^2)\frac{k_{||}^2\left(Q_{||}^2\right)^3}{\Delta^2\Delta'^2} \times Q_0 n_{\rm B}(k_0) n_{\rm F}(p_0)\left(1 - n_{\rm F}(p'_0)\right). \tag{27}$$

Here, $Q_{||} = k_{||} + p_{||} - p'_{||}$ by virtue of the conservation law and we have used the integral relation (see [18], p. 227)

$$\int dp_y = hL_x \tag{28}$$

and the approximate equality [see (26) and (7)]

$$p_{||}\cdot p'_{||} \simeq p_{||}^2 = m^2,$$

which is valid under the condition $k_0 \ll m$. We have also performed summation over three neutrino flavors [see (18)]

$$g_+^2 = \sum_{\ell=e,\mu,\tau}(g_V^2 + g_A^2) = 12w^2 - 2w + \frac{3}{2} \simeq 1.679. \tag{29}$$

In the case of a strongly degenerate electron gas, a dominant contribution to the luminosity in (27) comes from electrons whose energies are close to the Fermi energy [see Eq. (4)]:

$$|p_0 - \varepsilon_{\rm F}| \lesssim T, \quad |p'_0 - \varepsilon_{\rm F}| \lesssim T. \tag{30}$$

To $O(\exp(-(\varepsilon_{\rm F} - m)/T))$ terms, we can single out in (27), an integral of the product of two Fermi factors (see, for example, [3]),

$$\int_{-\infty}^{\infty} dx n_{\rm F}(x)(1 - n_{\rm F}(x-y)) = y(e^y - 1)^{-1}, \tag{31}$$

where $x = (p_0 - \varepsilon_{\rm F})/T$ and $y = (Q_0 - k_0)/T$, and set $p_0 = \varepsilon_{\rm F}$ in the remaining factors.

Further integration in (27) can be performed analytically in two limiting cases—the nonrelativistic case of $p_{\rm F} \ll m$ and the ultrarelativistic case of $p_{\rm F} \gg m$.

In the nonrelativistic case, we have $\varepsilon_{\rm F} \simeq m$ and additionally assume that $\omega_p \ll T$ [this makes it possible to use the vacuum dispersion equation $k^2 = 0$ instead of Eq. (13)]. We then have [see Eq. (22)] $\Delta \simeq 2mk_0$ and $\Delta' \simeq -2mk_0$ (since $k_0 \lesssim T \ll m$), and the kinematical constraint in (25) determines the region of the variable y [see Eq. (31)]: $-v_{\rm F}k_+ \leq Ty \leq v_{\rm F}k_-$, where $k_\pm = k_0 \pm k_z$ and $v_{\rm F} = p_{\rm F}/\varepsilon_{\rm F} \ll 1$. In the significant region, $|y| \ll 1$, which permits replacing the integral in (31) by unity. Further integration with respect to y and with respect to the azimuthal and polar angles of the vector $\mathbf{k}$ becomes trivial, while the integral with respect to the photon energy reduces to a well-known integral from standard reference books:

$$\int_0^\infty x^7(e^x - 1)^{-1}dx = (2\pi)^8/480.$$

As a result, we obtain an expression for the luminosity in the form

$$L_{\rm w} = \frac{2(2\pi)^2}{4725}\alpha G_{\rm F}^2 g_+^2 \frac{H}{H_0}\frac{m}{p_{\rm F}}T^9. \tag{32}$$

This result differs from the respective result in [5] by the factor of $\pi/2$ and coincides with result presented in [4].

In the ultrarelativistic case, we have $\varepsilon_{\rm F} \gg m$ and [see Eq. (14)] $\omega_p \simeq \sqrt{(2/\pi)\alpha H/H_0}\,m \gg T$ at comparatively low temperatures. For the photon momentum, we then have $|\mathbf{k}| \lesssim \sqrt{\omega_p T} \ll \omega_p$ and $k_{||}^2 \simeq k_0^2 \simeq \omega_p^2$; further, $\Delta \simeq -\Delta' \simeq 2k\cdot p_{||} \simeq 2\varepsilon_{\rm F}\omega_p$, and we obtain $Q_{||}^2 \simeq \omega_p^2 + 2\omega_p(Q_0 - k_0)$ upon taking into account the relevant conservation laws. Therefore, the respective integrand is independent of angles in this approximation. After standard integration with respect to $dp_z dp'_z$ with allowance for Eq. (31), the luminosity (27) assumes the form

$$L_{\rm w} = 2\frac{\alpha G_{\rm F}^2 g_+^2 m^6}{3\cdot 2^5\pi^7}\frac{H}{H_0}\frac{4\pi T^2}{\omega_p \varepsilon_{\rm F}^2} \times \int_0^\infty dk k^2 n_{\rm B}(k_0)\int_{-y_0}^\infty dy \frac{y}{e^y - 1}(\omega_p + Ty) \times \frac{\omega_p^2[\omega_p(\omega_p + 2Ty)]^3}{(2\omega_p\varepsilon_{\rm F})^4}, \tag{33}$$

where $k = |\mathbf{k}|$; $y = (Q_0 - k_0)/T$ [see Eq. (31)]; and $y_0 = \omega_p/(2T) \gg 1$, the first factor of 2 in expression (33) taking into account the identical contributions to the integral with respect to p_z from small vicinities of the points $p_z = \pm p_{\rm F}$ [see Eq. (30)]: $||p_z| - p_{\rm F}| \lesssim T \ll p_{\rm F}$. With allowance for the approximation $n_{\rm B}(k_0) \simeq e^{-\omega_p/T} \exp[-k^2/(2\omega_p T)]$, the integral with respect to k reduces to a standard Gaussian integral. The integral with respect to y can be represented in the form

$$\int\limits_{-\infty}^{1} dt(1 - e^{-y_0 t})^{-1} t(t-2)(t-1)^3.$$

The leading term of its asymptotic expansion for $y_0 \gg 1$ (a dominant contribution comes from the region of $0 < t < 1$) is 1/12. As a result, we obtain

$$L_{\rm w} = \frac{\alpha G_{\rm F}^2 g_+^2 \omega_p^9}{576(2\pi)^{11/2}} \times \frac{H}{H_0}\left(\frac{m}{\varepsilon_{\rm F}}\right)^6 \left(\frac{T}{\omega_p}\right)^{3/2} e^{-\omega_p/T}. \quad (34)$$

Expression (34) differs substantially from the respective result obtained in [5] in the form of a single integral and includes the suppressing factor of $(m/\varepsilon_{\rm F})^6 e^{-\omega_p/T}$. This means that, under the conditions $\varepsilon_{\rm F} \gg m$ and $T \ll \omega_p$, the photoproduction of neutrino pairs does not make a significant contribution to the energy loss of the medium—the photon decay $\gamma \to \nu\bar{\nu}$ is a dominant process here [5].

3. NEUTRINO-PAIR PHOTOPRODUCTION ON ELECTRONS (ELECTROMAGNETIC MECHANISM)

The matrix element for electromagnetic mechanism of neutrino-pair photoproduction on an electron is given by [see Eq. (15)]

$$M_{\rm em} = e^2 \frac{\left(j^\alpha_{\rm em}\epsilon^{(2)}_\alpha\right)\left(J^\beta_{\rm em}\epsilon^{(2)}_\beta\right)}{Q^2 - \varkappa_2}, \quad (35)$$

where the neutrino and electron electromagnetic currents ($j^\alpha_{\rm em}$ and $J^\alpha_{\rm em}$, respectively) are obtained from the respective expressions for the weak currents in (17) via the substitutions $\gamma^\alpha(1+\gamma^5) \to V^\alpha(Q)$ [see Eq. (3)] and $\Gamma^\alpha \to \gamma^\alpha$ [that is, the substitutions $g_V = 1$ and $g_A = 0$ must be made in Eq (22)]. After integration with respect to the neutrino and antineutrino momenta with the aid of relation (23), the modulus squared of the matrix element assumes a characteristic resonance form associated with the possibility for a virtual photon to become an on-shell particle; that is,

$$\int \frac{d^3q}{q_0}\frac{d^3q'}{q'_0}\delta^{(4)}(q + q' - Q)\,|M_{\rm em}|^2 = \frac{256\pi}{3} e^4 \bar{\mu}_\nu^2 k_{||}^2 \left(\frac{m^2}{\Delta\Delta'}\right)^2 \times \frac{Q_{||}^2 (Q^2)^2}{(Q^2 - \omega_p^2)^2 + (Q_0\Gamma)^2}, \quad (36)$$

where Γ is the total probability for the Compton scattering ($\gamma e \to e\gamma$) of a massive photon (plasmon) whose 4-momentum is Q ($Q^2 = \omega_p^2$) and where we have introduced the notation

$$\bar{\mu}_\nu^2 = \mu_{\rm B}^2(f_{2\nu}^2 + g_{2\nu}^2) = \mu_\nu^2 + d_\nu^2. \quad (37)$$

Replacing the matrix element $M_{\rm w}$ by $M_{\rm em}$ (35) in expression (19) and taking into account Eqs. (36) and (28), we perform integration with respect to the transverse component $Q_\perp$ of the virtual-photon momentum with the aid of the (weak) asymptotic expression for the resonance term of the form (a vicinity of the resonance $Q^2 \simeq \omega_p^2$ makes a dominant contribution):

$$\left[(x-a)^2 + \varepsilon^2\right]^{-1} \simeq \frac{\pi}{|\varepsilon|}\delta(x-a), \quad |\varepsilon| \ll 1.$$

As a result, we obtain the following expression for the electromagnetic-mechanism-induced neutrino luminosity of the medium:

$$L_{\rm em} = \frac{\alpha^2\bar{\mu}_\nu^2}{24\pi^5} m^6 \omega_p^4 \frac{H}{H_0} \int \frac{d^3k\, dp_z\, dp'_z}{k_0 p_0 p'_0} \times \theta(Q_{||}^2 - \omega_p^2)\Gamma^{-1}\frac{k_{||}^2 Q_{||}^2}{(\Delta\Delta')^2} \times n_{\rm B}(k_0) n_{\rm F}(p_0)(1 - n_{\rm F}(p'_0)). \quad (38)$$

Here, $\theta(Q_{||}^2 - \omega_p^2)$ is a step function that takes into account the kinematical constraint $Q_{||}^2 \geq \omega_p^2$, which differs from that in Eq. (25).

We note that the contribution of the interference between the weak and electromagnetic mechanisms to the neutrino luminosity is proportional to the (small) neutrino mass, and we therefore disregard it.

In just the same way as in Section 2, we now consider the nonrelativistic and ultrarelativistic limiting cases, in which one can obtain comparatively simple asymptotic expressions for the luminosity in (38).

In the nonrelativistic case ($p_{\rm F} \ll m$ and $\omega_p \ll T$), the total probability for Compton scattering is given by the asymptotic expression

$$\Gamma = \frac{4\alpha^2}{3\pi} T \frac{H}{H_0} \frac{m}{p_{\rm F}} \frac{Q_\parallel^2}{Q_0^2} (1 + n_{\rm B}(Q_0)). \qquad (39)$$

Substituting it into expression (38) and performing calculations similar to those in the case of the weak mechanism, we obtain an asymptotic expression for the neutrino luminosity in the form

$$L_{\rm em} = \frac{\bar{\mu}_\nu^2}{24\pi^3} \omega_p^4 T^3. \qquad (40)$$

Un upper (relative) bound on the effective magnetic moment $\bar{\mu}_\nu$ of the neutrino [see Eq. (37)] can be found by requiring that the neutrino luminosity in the electromagnetic channel not exceed its counterpart in the weak channel—that is, $L_{\rm em} < L_{\rm w}$. Comparing Eqs. (40) and (32) and taking into account Eqs. (14) and (5), we then obtain

$$\frac{\bar{\mu}_\nu}{\mu_{\rm B}} < \frac{1}{15} \left(\frac{2}{7}\right)^{1/2} \frac{g_+}{\alpha} G_{\rm F} m^2 \times \frac{H}{H_0} \left(\frac{n_e}{m^3}\right)^{-3/2} \left(\frac{T}{m}\right)^3. \qquad (41)$$

We now recast this result into a form that is convenient for astrophysical applications; that is,

$$\bar{\mu}_\nu / \mu_{\rm B} < 9.3 \times 10^{-15} H_{13} \rho_6^{-3/2} T_8^3, \qquad (42)$$

where $H_{13} = H/(10^{13}\ {\rm G})$, $\rho_6 = \rho/\left(10^6\ {\rm g/cm^3}\right)$, and $T_8 = T/(10^8\ {\rm K})$. Here, we have used the relation (see, for example, [23]) $n_e \simeq 0.5\rho/m_p$, where ρ is the stellar-matter density and m_p is the proton mass.

By way of example, we indicate that, at [here and in Eq. (51) below, we choose the numerical values of the parameters involved in such a way as to meet the conditions of applicability of the asymptotic formulas for the luminosity]

$$T = 1.8 \times 10^8\ {\rm K}, \qquad (43)$$
$$H = 2.5 \times 10^{12}\ {\rm G}, \quad \rho = 5.4 \times 10^4\ {\rm g/cm^3},$$

the resulting constraint is

$$\bar{\mu}_\nu / \mu_{\rm B} < 1.1 \times 10^{-12}, \qquad (44)$$

where the upper bound is close to the well-known astrophysical bounds [8].

In the ultrarelativistic case ($p_{\rm F} \gg m$ and $\omega_p \gg T$), the asymptotic expression for the probability of Compton scattering has the form

$$\Gamma = \frac{\alpha^2}{\pi} \frac{H}{H_0} \left(\frac{m}{\varepsilon_{\rm F}}\right)^6 \frac{T^2}{\omega_p} f\left(\frac{Q_\parallel^2 - \omega_p^2}{2\omega_p T}\right), \qquad (45)$$

where we have introduced the function

$$f(z) = \int_{-z}^{\infty} dt \frac{t}{e^t - 1} = \frac{\pi^2}{3} + \frac{z^2}{2} + z\ln(1 - e^{-z}) - {\rm Li}_2(e^{-z}), \qquad (46)$$

which is expressed in terms of the dilogarithm

$${\rm Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1 - t) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

Substituting Eq. (45) into Eq. (38), we obtain an asymptotic expression for the luminosity in the ultrarelativistic case [its derivation is analogous to the derivation of expression (34)]. We have

$$L_{\rm em} = \frac{K\bar{\mu}_\nu^2}{12(2\pi)^3} \omega_p^7 \left(\frac{T}{\omega_p}\right)^{3/2} e^{-\omega_p/T}, \qquad (47)$$

where the numerical factor K has an integral form [see Eq. (46)]; that is,

$$K = \int_0^\infty dx\, x^2 e^{-x^2/2} \int_0^\pi d\theta \sin\theta \times \int_{-\frac{1}{2}x^2\sin^2\theta}^{\infty} \frac{dy\, y}{(e^y - 1)\, f\left(y + \frac{1}{2}x^2\sin^2\theta\right)} \simeq 2.507. \qquad (48)$$

Using Eqs. (47) and (34) and the condition $L_{\rm em} < L_{\rm w}$, we find a bound on the effective magnetic moment; that is,

$$\frac{\bar{\mu}_\nu}{\mu_{\rm B}} < 4(2\pi)^{-33/4} \left(\frac{2\alpha}{3K}\right)^{1/2} \times g_+ G_{\rm F} m^2 \left(\frac{H}{H_0}\right)^4 \left(\frac{n_e}{m^3}\right)^{-3}. \qquad (49)$$

In the "astrophysical" form, we have

$$\bar{\mu}_\nu / \mu_{\rm B} < 9.3 \times 10^{-17} H_{13}^4 \rho_6^{-3}. \qquad (50)$$

Setting

$$H = 3 \times 10^{15}\ {\rm G}, \quad \rho = 10^9\ {\rm g/cm^3}, \qquad (51)$$

we obtain the following constraint from (50):

$$\bar{\mu}_\nu / \mu_{\rm B} < 7.6 \times 10^{-16}. \qquad (52)$$

Its stringency is very relative because of a strong suppression of neutrino-pair photoproduction via the weak mechanism in the relativistic case [see Eq. (34)].

It was shown in [5] that, within the matter-density interval of $10^8 \lesssim \rho \lesssim 10^{10}\ {\rm g/cm^3}$, a dominant contribution to the neutrino luminosity comes from the plasmon-decay process $\gamma \to \nu\bar{\nu}$. Comparing the corresponding expression for the luminosity from [5] with expression (47), we derive a more lenient bound on the effective magnetic moment; that is,

$$\bar{\mu}_\nu/\mu_{\rm B} < 4(2\pi)^{-7/4}(K\alpha)^{-1/2} \quad (53)$$
$$\times\, g_{V+}G_{\rm F}m^2(H/H_0)^{1/2} = 1.7\times 10^{-12}H_{13}^{1/2},$$

where we have used the effective coupling constant [compare with that in (29)]

$$g_{V+}^2 = \sum_{\ell=e,\mu,\tau} g_V^2 = 12w^2 - 2w + \frac{3}{4} \simeq 0.929.$$

Under the conditions in (51), we obtain the constraint

$$\bar{\mu}_\nu/\mu_{\rm B} < 2.9\times 10^{-11}, \quad (54)$$

where the upper bound is close to the bounds in (1) and (2).

4. DISCUSSION

We have calculated the luminosity of a strongly magnetized degenerate electron gas (under conditions of neutron stars) in the process of neutrino-pair photoproduction on electrons via the weak and electromagnetic mechanisms in the nonrelativistic ($\varepsilon_{\rm F}\simeq m$) and ultrarelativistic ($\varepsilon_{\rm F}\gg m$) cases. We have derived upper bounds on the neutrino effective magnetic moment $\bar{\mu}_\nu$, which are close to the well-known astrophysical and laboratory bounds. The electromagnetic mechanism of neutrino-pair photoproduction proves to be more efficient than the weak mechanism for $\bar{\mu}_\nu \gtrsim 10^{-12}\mu_{\rm B}$ in the nonrelativistic case [see Eq. (44)] and for $\bar{\mu}_\nu \gtrsim 10^{-15}\mu_{\rm B}$ in the ultrarelativistic case [see Eq. (52)], but, in the latter case, plasmon decay is a much more efficient mechanism of the energy loss up to $\bar{\mu}_\nu \lesssim 10^{-11}\mu_{\rm B}$ [see Eq. (54)].

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