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## RELIABILITY, STRENGTH, AND WEAR RESISTANCE OF MACHINES AND STRUCTURES

# On the Theory of Stage-by-Stage Fatigue Failure of Metals upon a Complex Stress State

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**Abstract—**This article presents a phenomenological model of brittle fatigue failure of metals and alloys upon proportional (simple) loading. The model is developed in the frames of physicomechanical approach as a set of hypotheses on the gradual development of defects in metals at various scale and structural levels (such as brittle micro- and macrocracks). On its basis, defining relationships are formulated for probability of failure with regard to defects of each level. Fatigue curves are obtained for various ultimate metal states as a consequence of the evolution of micro- and mesodefects, and the lifetime is determined with regard to a predefined level of accumulated defects. The results for structural steels are given as an example.

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## INTRODUCTION

Stage-by-stage fatigue failure is related with the evolution of a metal structure. Fatigue failure is a multilevel and different-scale kinetic process; it is characterized by obligatory stages of the formation of dislocation substructures, micro-, short, and macrocracks. Herewith, each stage is controlled by its physical mechanisms, and the failure occurs as a consequence of the interaction between objects of different scales, from the atomic level to meso- and macroscopic levels. Random space distributions of structural characteristics result in static fatigue failure, and a probabilistic approach to its description is required.

An overview of physical theories (structural, energy, static) demonstrates that physical models define the laws of defect evolution in solids on the basis of dislocation theories, nonequilibrium thermodynamics, etc.; they describe physical mechanisms of fatigue failure, including the synergetic method. Herewith, most models contain parameters of structure and physical mechanisms of failure not identified in macroexperiments, thus preventing strength macrocharacteristics of resistance against fatigue. The problem is how to interrelate structural regularities and physical mechanisms of failure with the dependences of physical parameters describing failure as a process of mechanics of deformable solid body.

Most phenomenological theories of fatigue strength (including gradient and nonlocal theories, as well as the theory of micromorphous continuums) consider in their terms the evolution of material damage but do not consider the architecture of the internal structure metal, physical mechanisms, and stages of fatigue failure. Thuserewith, According to experimental data, in agreement with the experimental data, certain states of metal structure and inherent physical mechanism of failure correspond to various levels of development of fatigue failure. The mechanics of brittle failure, in turn, explores the development of single, brittle macrocracks at the macrolevel on the basis of deformation, force, and energy approaches.

In the proposed approach, brittle fatigue failure is studied in terms of metal physics as stage-by-stage nucleation and the development and merging of defects of all levels, which made it possible to develop a theory of fatigue strength for the entire time range up to macrofailure.

In the determined theoretical relationships, the amplitude of maximum main stress is selected as a variable, while material functions are determined on the basis of fatigue tests upon symmetrical uniaxial and biaxial stresses according to experimentally substantiated criteria of fatigue strength of metals. The consistency of the obtained results is provided by a comparison of calculations for a wide range of steels, aluminum, magnesium, and titanium alloys upon various plain stresses by the known approaches.

Brittle fatigue failure of metals and alloys are analyzed in the regions of multicycle and gigacycle fatigue at $N_f \in [5\times10^3, 10^{10}]$ cycles and simple cyclic loadings, such as:

$$\begin{gathered}\sigma_{kk}(\tau) = \alpha_k \sigma_a f(\tau), \quad f(\tau) = \sin(\omega\tau + \theta), \quad k = 1, 2, 3, \quad \tau \in [0, t], \\ |\sigma_{11}| \geq |\sigma_{22}| \geq |\sigma_{33}|, \quad \alpha_k = \frac{\sigma_{kk}}{|\sigma_{11}|}, \quad \alpha_0 = \frac{\sigma_0}{\sigma_{11}}, \quad \sigma_0 = \frac{1}{3}\sum_{k=1}^{3}\sigma_{kk},\end{gathered} \tag{1}$$

where $\sigma_a$ is the amplitude of maximum main stress, $N_f$ is the lifetime of complete failure, $\omega$ is the frequency of stress variation, and $\alpha_k$ is the ratio of main stresses.

On the basis of analysis of experimental and theoretical results of fatigue failure in metals at the micro-, meso-, and macrolevels in terms of solid state physics, materials science, and mechanics of a deformable solid body, the following major hypotheses of the proposed model [1–10] were formulated.

**1.** It is known [11–15] that, with an increase in the number of loading cycles, the evolution of defects and dislocations existing in material after solidification leads to the formation of ordered self-organizing dislocation substructures of two types: a banded structure related to single sliding and defect accumulation and a cellular structure related to multiple sliding. Upon the achievement of a critical defect density, there is a transition from a banded substructure to brittle microcracks; the stability of their sizes makes it possible to increase their density significantly. The subsequent merging of microcracks leads to the nucleation of brittle nonpropagating cracks in grain and at the grain interface. When their density reaches the ultimate value, they are merged with the formation of short cracks with the sizes of grain, forming a well-known river pattern on the surface and generated extrusions and intrusions. As a consequence of the merging of short cracks, transcrystallite and grain boundary microcracks reaching the French lines of irreversible damaging are formed. Further evolution leads to occurrence of single macrocracks and final brittle failure of the metal.

A peculiar feature of cyclic loadings is that the development of failure upon uniaxial loading in the range of $N_f \in [5\times10^6, 10^{10}]$ cycles takes place upon elastic macro-deformation and at an amplitude that equals the fatigue point of metal, leading to brittle macro-failure along short cracks according to mechanisms of intergranular or transcrystallite cleavage. At $N_f \in [5\times10^3, 5\times10^6]$ cycles in brittle materials, fatigue failure leads to the formation of single, brittle macrocracks upon elastic deformation and to brittle macrofailure. In plastic materials the development of a dislocation cellular substructure according to mechanisms of twinning and sliding leads to motion of the grain assemblies. As a result, the nucleation of a viscous microcrack with pit microrelief occurs, and nonelastic deformations equal to the magnitude of elastic deformations develop, preventing the development of brittle cracks. Fatigue failure is comprised of brittle and viscous fibrous constituents.

Therefore, the evolution of defects that existed initially in polycrystalline bodies after solidification develops according to two routes, leading to brittle or viscous macrofailure.

**2.** The analytical data of metal microstructure have been systemized, and a system of six scale and structural levels is introduced on its basis. These levels correspond to various progress stages of brittle failure according to various physical mechanisms, and brittle failure is classified according to defects (failures) of the $i$th scale–structural level, $i = 1, \ldots, 6$. A defect of the $i$th level is characterized by linear size $l_i = l_i(\tau)$ and density $q_i = q_i(\tau)$ in representative volume $V_c$ (where an initial crack of failure mechanics can originate). The ultimate state of a defect of the $i$th level is determined by the ultimate linear size $l_{f,i}$ and density $q_{f,i}$.

Vacancy clusters, discontinuities, pores, grids near grain boundaries, sub-microcracks and so on, with an average length of $l_1 \leq 10^{-3}d$, are physical images of first-level defects that develop upon loading in banded structures. When the amplitude of uniaxial loading exceeds the ultimate sensitivity to cyclic stresses and critical density of first-level defects, a transition occurs from a banded structure to a microcrack, which is defined as a second level microdefect with a length of $l_2 \in (10^{-3}, 10^{-2}]d$. With a further increase in stress amplitude as a consequence of the merging of microcracks, nonpropagating cracks originate: third-level defects, $l_3 \in (10^{-2}, 10^{-1}]d$. Their merging leads to the formation of short (propagating) cracks: fourth-level defects, $l_4 \in (10^{-1}, 1]d$ and a transition from the micro- to mesolevel. With a stress amplitude equal to ultimate fatigue, brittle macrofailure occurs along fourth-level defects upon macroelastic deformation. With a further increase in amplitude, the fourth-level defects grow and expand outside the grain boundaries. Fifth- and sixth-level defects form, and their evolution leads to single, brittle microcracks studied in the failure mechanics [16, 17].

At the microlevel ($i = 1, 2, 3$) mechanical properties vary insignificantly upon microelastic deformation, and the properties are stabilized. At each microlevel, defects of other levels can appear, but their contribution to the chance of failure can be disregarded due to continuous merging. Mechanical properties of metal vary intensively at the mesolevel ($i = 4, 5, 6$).

This defect classification according to six levels for materials of similar internal metal structure is somewhat conventional. In the study of specific materials, it would be necessary to adjust it.

**3.** It is assumed that defect formation at any level occurs as a consequence of the successive nucleation, growth, and merging of defects of previous levels.

**4.** The following continuous increasing averaging function of the $i$th level is introduced: $l_i^* = l_i^*(\tau)$, $\tau \in [0,t]$, with a length of $l_i^*(\tau) = l_i(\tau)(q_i(\tau)V_c)^\gamma$, $\gamma = \text{const}$, $i = 1, \ldots, 6$. The ultimate state of defects of the $(i-1)$th level and formation of the $i$th level defects is characterized by achievement of the function $l_{i-1}^* = l_{i-1}^*(\tau)$ and its ultimate value $l_{f,i-1}^*$ in the time $t_i = t_i(\sigma_a)$, $i = 1, \ldots, 6$.

Damage at the $i$th level (fatigue failure at the $i$th level) $\Omega_i = \Omega_i(\tau)$ is defined as follows: $\Omega_i(\tau) = l_i^*(\tau)/l_{f,i}^*$, $0 \le \Omega_i \le 1$, $i = 1, \ldots, 6$. For each tine $\tau$, $\tau \in [0,t]$ the values $l_i^*$ and damaging $\Omega_i$ are random variables. Thus, it is necessary to introduce a probability function of failure along defects of the $i$th level (probability of failure at the $i$th level), which is the main defining function of the proposed mathematical model of fatigue failure.

**5.** The probability of failure at the $i$th level $Q_i = Q_i(\tau)$, $i = 1, \ldots, 6$, is the probability of an event when the averaged function of the $i$th level $l_i^* = l_i^*(\tau)$ achieves its ultimate value $l_{f,i}^*$ at time $\tau$ (defects of the $i$th level—ultimate state), $\tau \in [0,t]$.

According to the Il'yushin postulate of macroscopic definability, the fatigue failure in the time interval $\tau \in [0,t]$ is determined by metal deformation. The development of fatigue failure is defined by elastic deformation, and the probability of brittle failure can be considered a function of loading. The set of defining relations for the function $Q_i = Q_i(\tau)$, $i = 1, \ldots, 6$ is arranged.

**6.** According to experimental results, the development of fatigue failure depends on the type of stress and strain state. Plotting of the model for arbitrary simple loading (1) is based on the properties of material failure upon three basic loading types under planar stress: uniaxial symmetric loading $\alpha_1 = 1, \alpha_2 = \alpha_3 = 0$, symmetric shift $\alpha_1 = 1$, $\alpha_2 = -1$, $\alpha_3 = 0$, and biaxial uniform loading $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = 0$.

**7.** Failures along the defects of the mesolevel are considered independent events. The probability function of failure along defects of the mesolevel $Q = Q(\tau)$ is introduced in the form of $Q(\tau) = \sum_{i=4}^{6} \frac{Q_i(\tau)}{1 - Q_i(\tau)} \prod_{j=4}^{6} (1 - Q_j(\tau))$, where the functions $Q_i = Q_i(\tau)$ are the probabilities of failure along defects of the $i$th meso-level, $i = 4, 5, 6$.

The fatigue curve of metal along the mesolevel defects is defined as follows: $Q(t_f) = 1$; from here, it is possible to define metal lifetime along the mesolevel defects $t_f = t_f(\sigma_f)$ for arbitrary process (1) ($\sigma_f$ is the amplitude of maximum main stress upon failure along mesolevel defects).

**8.** The defining relations for the probability function of failure $Q_i = Q_i(\sigma_a, n)$ $(0 \le Q_i \le 1)$, $i = 1, \ldots, 6$, along $i$th level defects assuming their uniform distribution in volume $V_c$ as a function of amplitude of main stress $\sigma_a$ and the number of loading cycles $n$ for three main loadings according to the methods of mechanics of deformable solid body and the theory of dimension and similarity are preset as follows:

at the microlevel $i = 1, 2, 3$ at $\sigma_a \ge \sigma_{i-1}$, $\log n \ge \log n_i(\sigma_a)$, $\sigma_0 = 0$, $n_1 = 1$

$$Q_i = F_i\left(\frac{\sigma_a - \sigma_{i-1}}{\sigma_i - \sigma_{i-1}}\right) R_i\left(\frac{\log n - \log n_i(\sigma_a)}{\log N_i - \log n_i(\sigma_i)}\right), \quad i = 1, 2, 3, \tag{2}$$

$$Q_i(n_{i+1}) = 1, \quad i = 1, 2, 3, \tag{3}$$

at the macrolevel at $\sigma_3 \le \sigma_a$, $\log n \ge \log n_4(\sigma_a)$

$$Q_4 = F_4\left(\frac{\sigma_a - \sigma_3}{\sigma_4 - \sigma_3}\right) R_4\left(\frac{\log n - \log n_4(\sigma_a)}{\log N_4 - \log n_4(\sigma_4)}\right), \tag{4}$$

$$Q_4(n_5) = 1, \tag{5}$$

at $\sigma_4 \le \sigma_a$, $\log n \ge \log n_5(\sigma_a)$

$$\begin{gathered} Q_5 = F_5\left(\frac{\sigma_a - \sigma_4}{\sigma_5 - \sigma_4}\right) R_5\left(\frac{\log n - \log n_4(\sigma_a)}{\log N_5 - \log n_4(\sigma_5)}\right), \\ Q_4 = G_4\left(\frac{\sigma_5 - \sigma_a}{\sigma_5 - \sigma_4}\right) R_4\left(\frac{\log n - \log n_4(\sigma_a)}{\log N_4 - \log n_4(\sigma_4)}\right), \quad Q_6 = 0, \end{gathered} \tag{6}$$

$$Q_5(n_6) = 1; \tag{7}$$

at $\sigma_5 \le \sigma_a$, $\log n \ge \log n_6(\sigma_a)$

$$\begin{gathered} Q_6 = F_6\left(\frac{\sigma_a - \sigma_5}{\sigma_6 - \sigma_5}\right) R_6\left(\frac{\log n - \log n_4(\sigma_a)}{\log N_6 - \log n_4(\sigma_6)}\right), \\ Q_5 = G_5\left(\frac{\sigma_6 - \sigma_a}{\sigma_6 - \sigma_5}\right) R_5\left(\frac{\log n - \log n_4(\sigma_a)}{\log N_5 - \log n_4(\sigma_5)}\right), \quad Q_4 = 0, \end{gathered} \tag{8}$$

$$Q_6(n_7) = 1. \tag{9}$$

In Eqs. (2)–(9), the $n_i = n_i(\sigma_a)$, $i = 1, \ldots, 6$, are the number of cycles of initiation of defects of the $i$th level at the amplitude of maximum main stress $\sigma_a$; in particular, $n_7 = n_7(\sigma_a)$ is the number of cycles of achieving the ultimate state by sixth-level defects. The set of material functions $(\sigma_i = \sigma_i(\alpha_2, \omega), N_i)$, $\alpha_2 = 0, 1, -1$, is introduced when the $i$th level defect achieves the ultimate state and the function $l_i^* = l_i^*(n)$ reaches its ultimate value $l_{f,i}^*$; the numbers of cycles $N_i$ are the basic numbers of cycles for the $i$th level, $i = 1, \ldots, 6$.

The proposed model makes it possible to select function types $F_i = F_i(\sigma_a)$ and $R_i = R_i(n)$, $i = 1, \ldots, 6$ in Eqs. (2)–(9) for specific materials. In this case the consideration is restricted by functions of the following type:

$$F_i = \left(\frac{\sigma_a - \sigma_{i-1}}{\sigma_i - \sigma_{i-1}}\right)^{\beta_i}, \quad R_i = \left(\frac{\log n - \log n_i(\sigma_a)}{\log N_i - \log n_i(\sigma_i)}\right)^{\phi_i}, \quad i = 1, \ldots, 4;$$

$$F_i = \left(\frac{\sigma_a - \sigma_{i-1}}{\sigma_i - \sigma_{i-1}}\right)^{\chi_i}, \quad R_i = \left(\frac{\log n - \log n_4(\sigma_a)}{\log N_i - \log n_4(\sigma_i)}\right)^{\phi_i}, \quad i = 5, 6; \quad G_i = \left(\frac{\sigma_{i+1} - \sigma_a}{\sigma_{i+1} - \sigma_i}\right)^{\chi_i}, \quad i = 4, 5;$$

$\beta_i = \beta_i(\alpha_2, \omega)$, $\phi_i = \phi_i(\alpha_2, \omega)$, $\chi_i = \chi_i(\alpha_2, \omega)$, $\alpha_2 = 0, 1, -1$, $i = 1, \ldots, 6$ are the material functions.

In the recurrent set of Eqs. (2), (4), (6), and (8), a subsequent equation is related with previous one via the number of cycles $n_{i+1} = n_{i+1}(\sigma_a)$ defined according to Eqs. (3), (5), and (7), at which the function $l_i^* = l_i^*(n)$ reaches the ultimate value $l_{f,i}^*$, the $i$th level defect reaches its ultimate state, and the $(i+1)$th level defects start to generate.

**9.** The metal properties $(\sigma_i = \sigma_i(\alpha_2, \omega), N_i, l_{f,i}^*)$, $\alpha_2 = 0, 1, -1$, $i = 1, \ldots, 6$ can be determined on the basis of numerous experiments with processing of polished cross sections by dedicated analyses. Due to labor intensity and an insufficient amount of available data, it is not possible in most cases. Therefore, on the basis of analysis of physical regularities of development of brittle cracks, a set of assumptions is introduced as follows.

It is assumed that ultimate metal fatigues upon basic loading types are material constants for fourth-level defects $(\sigma_4, N_4)$ when the function $l_4^* = l_4^*(n)$ reaches ultimate state $l_{f,4}^*$, and macrofailure along fourth-level defects takes place.

Noticeable modification of the micro- and macrostructure of metal and the average sizes and density of micro- and macrocracks occurs upon variation of the number of loading cycles by decimal exponents. On this basis $N_i$ is assumed for basic numbers of cycles:

$$\log N_i = \log N_4 + 4 - i, \quad i = 1, \ldots, 4; \quad \log N_i = \log N_4 + 3 - i, \quad i = 5, 6.$$

At the microlevel, it is possible in Eqs. (2) and (3) to assume $\beta_i = \chi_i = 1$, $\phi_i = 1/2$, $i = 1, 2, 3$. Herewith, material constants can be presented in the form of amplitudes $\sigma_i$, $i = 1, 2, 3$, at which the appropriate ultimate value $l_{f,i}^*$ is reached at the number of cycles $N_4$. From here, according to the model at $N_4 = 10^6$ cycles, the basic amplitudes are as follows: $\sigma_1 \approx 0.5\sigma_4$, $\sigma_2 \approx 0.604\sigma_4$, $\sigma_3 \approx 0.75\sigma_4$. If the ultimate sensitivities of material to cyclic stresses $\sigma_T^c$ are known, below which structural modifications are not observed, the material constants are then selected as follows: $(\sigma_1(\alpha_2) = \sigma_T^c, N_1)$, $\alpha_2 = 0, 1, -1$.

Experiments to verify basic characteristics $(\sigma_i(\alpha_2), N_i, l_{f,i}^*(\alpha_2))$, $i = 1, 2, 3, 4$, $\alpha_2 = 0, 1, -1$, can be arranged as follows. At an amplitude equal to ultimate fatigue, a batch of samples is conditioned to macrofailure, the number of cycles $N_4$ is determined, and the sizes of the fourth-level defects are measured in brittle fractures. Their density, as well as the value of $l_{f,4}^*$, is determined. Loading with the amplitude $\sigma_i$ is carried out to the number of cycles $N_i$, $i = 1, 2, 3$, on the basis of microstructure images; the $i$th level defects, including their density, are determined. The value of $l_i^*(N_i)$ is determined and, upon additional loading, on the basis of data of polished cross sections, it is determined whether it is a maximum; otherwise, the number of cycles $N_i$ is adjusted.

At the mesolevel it is assumed for basic loadings $\sigma_6 = \sigma_s$, $\sigma_6 = \tau_s$, $\hat{\sigma}_6 = \hat{\sigma}_s$ ($\sigma_s$, $\tau_s$, $\hat{\sigma}_s$ are the yield points upon uniaxial loading, shift, and biaxial uniform loading, respectively) at a cycle number of $N_6$. On the basis of experimental data with various stress amplitudes in the range of $\sigma_a \in (\sigma_{-1}, \sigma_s)$ and analysis of the microstructure, the stress amplitude $\sigma_5$ at which the function $l_5^* = l_5^*(n)$ reaches ultimate value $l_{f,5}^*$ with $N_5$ cycles is experimentally determined. At $(\sigma_i, N_i)$, $i = 5,6$, both brittle macrofailure of the sample and the formation of single cracks with average length of $L = l_{f,i}^*$, $i = 5,6$ are possible.

In order to determine the functions $\beta_i$, $\phi_i$, $\chi_i$ it is necessary to preset additional basic characteristics of the model.

**10.** The presented approach makes it possible to apply Eqs. (2)–(9) for an arbitrary process (1). Herewith, the amplitude of maximum main stress is selected as a variable, and the material functions $(\sigma_i = \sigma_i(\alpha, \alpha_2, \alpha_3, \omega), N_i)$, $i = 1, \ldots, 6$ (without limitation of generality, it is assumed that $\alpha_1 = 1$) are presented as follows:

$$\sigma_i = \sigma_i(N_i, \omega)\tilde{\sigma}_i(\alpha_2, \alpha_3, \tilde{\eta}_i, \hat{\eta}_i), \quad i = 1, \ldots, 6, \tag{10}$$

where $\tilde{\eta}_i = \dfrac{\sigma_i(\alpha_2 = 0, N_i, \omega)}{\sigma_i(\alpha_2 = -1, N_i, \omega)}$, $\hat{\eta}_i = \dfrac{\sigma_i(\alpha_2 = 0, N_i, \omega)}{\sigma_i(\alpha_2 = 1, N_i, \omega)}$.

For defects of the first three levels, it is possible to apply the following equations:

$$\tilde{\eta}_i = \frac{1}{3}(\tilde{\eta}_4(i-1) + 4 - i), \quad \hat{\eta}_i = \frac{1}{3}(\hat{\eta}_4(i-1) + 4 - i), \quad i = 1,2,3. \tag{11}$$

The functions $\tilde{\sigma}_i = \tilde{\sigma}_i(\alpha_2, \alpha_3, \tilde{\eta}_i, \hat{\eta}_i)$, $i = 1, \ldots, 6$, in Eq. (14) are selected as follows:
for brittle materials

$$\text{at} \quad -1 \le \alpha_2 \le 0 \quad \tilde{\sigma}_i = [6 - \tilde{\eta}_i - \alpha_2(2\tilde{\eta}_i - 6) + \alpha_0(3\tilde{\eta}_i - 15)]^{-1},$$

$$\text{at} \quad 0 \le \alpha_2 \le 1, \quad \alpha_3 \ge 0 \quad \tilde{\sigma}_i = [1 + \alpha_2(\hat{\eta}_i - 1) - \alpha_3(\hat{\eta}_i - 1)]^{-1}, \tag{12}$$

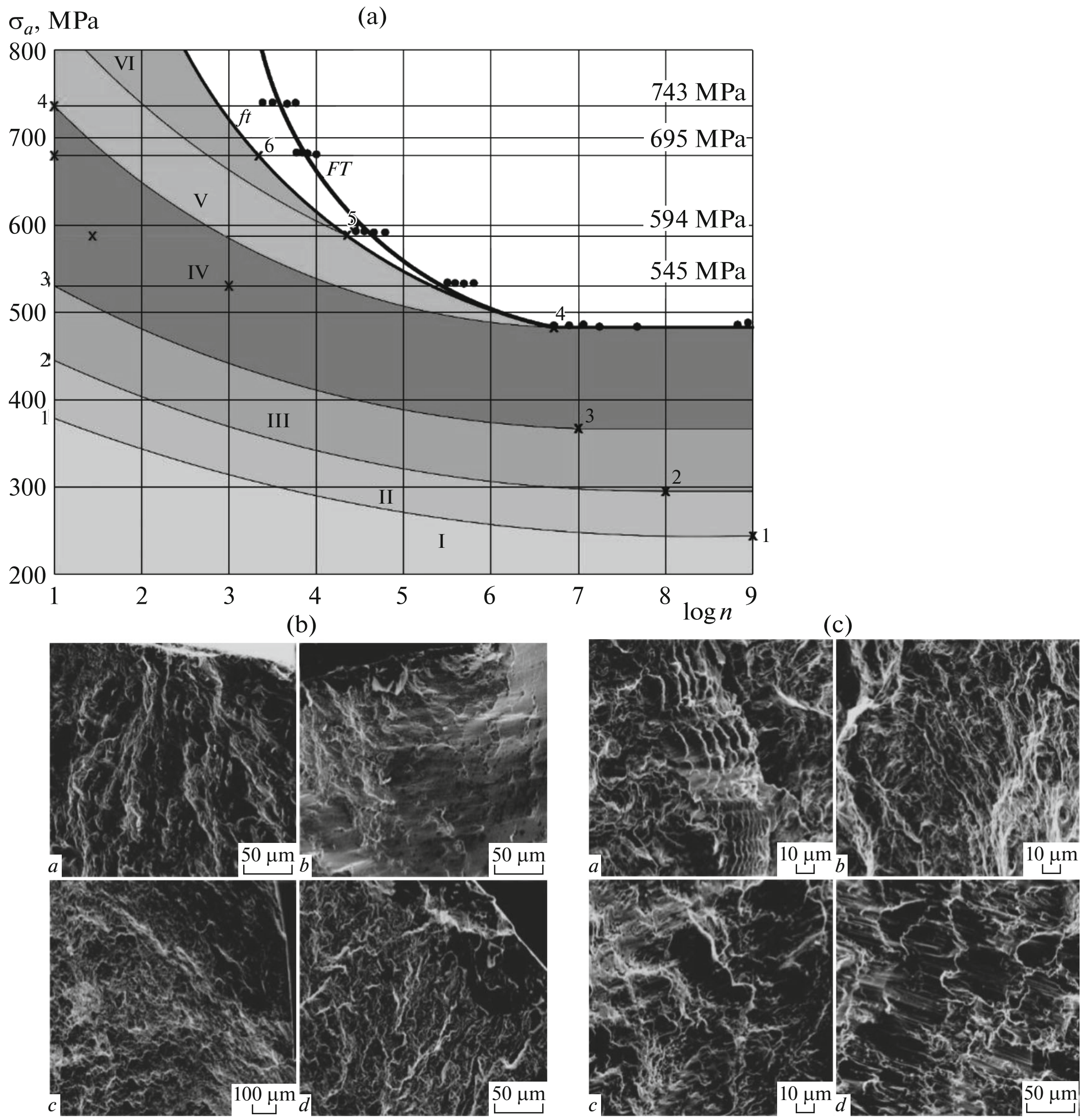


**Fig. 1.**

$$\text{at} \quad 0 \le \alpha_2 \le 1, \quad \alpha_3 < 0 \quad \tilde{\sigma}_i = [6 - \tilde{\eta}_i - \alpha_3(2\tilde{\eta}_i - 6) + \alpha_0(3\tilde{\eta}_i - 15)]^{-1};$$

for plastic materials

$$\text{at} \quad -1 \le \alpha_2 \le 0 \quad \tilde{\sigma}_i = \left[3\alpha_0(1+\alpha_2) + \frac{1}{2}\tilde{\eta}_i^2(1 - \alpha_2 - 3\alpha_0)\right]^{-1/2},$$

$$\text{at} \quad 0 \le \alpha_2 \le 1, \quad \alpha_3 \ge 0 \quad \tilde{\sigma}_i = \left[3\alpha_0(1+\alpha_3) + \frac{1}{2}\tilde{\eta}_i^2(1 - \alpha_3 - 3\alpha_0)\right]^{-1/2}, \tag{13}$$

$$\text{at} \quad 0 \le \alpha_2 \le 1, \quad \alpha_3 < 0 \quad \tilde{\sigma}_i = \left[3\alpha_0(1+\alpha_3) + \frac{1}{2}\tilde{\eta}_i^2(1 - \alpha_3 - 3\alpha_0)\right]^{-1/2}.$$

The selection of Eqs. (12) and (13) is provided by a comparison of forecasted cyclic strength according the proposed model with known current criteria and representative amount of experimental data. Thus, for symmetric uniaxial loading with torsion ($-1 \le \alpha_2 \le 0$, $\alpha_3 = 0$), the expression in Eq. (16) is trans-

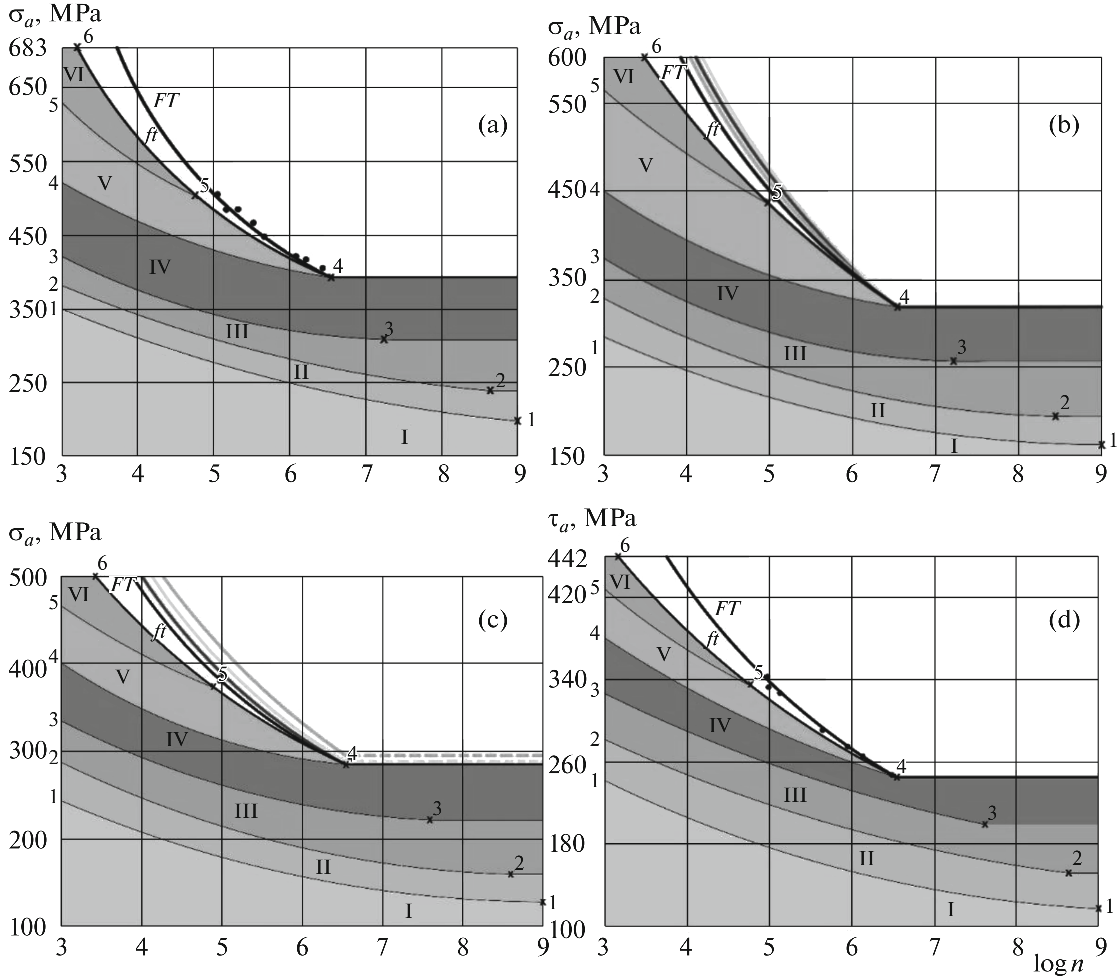


**Fig. 2.**

formed as follows $\tilde{\sigma}_i = [1 - \alpha_2(\eta_i - 1)]^{-1}$, and it agrees with the known criteria by McDiarmid, Matake, Findley, Dang-Van, Zenner, Lee, Papadopoulos, Kenman, and Zavoichinsky [18]. It also satisfactorily corresponds to consequences of the approaches by Sines, Crossland, Kakuno, Kawada, Depero for numerous materials with expanded experimental substantiation. For biaxial loadings ($0 \le \alpha_2 \le 1$, $\alpha_3 = 0$) the expression in Eq. (16) in the form of $\tilde{\sigma}_{1,i} = [1 + \alpha_2(\hat{\eta}_i - 1)]^{-1}$ corresponds to consequences of the approaches by McDiarmid, Dang-Van and others.

For symmetric uniaxial loading with torsion ($-1 \le \alpha_2 \le 0$, $\alpha_3 = 0$), the expression in Eq. (17) is as follows: $\tilde{\sigma}_i = [(1+\alpha_2)^2 - \alpha_2\eta_i^2]^{-1/2}$, which corresponds to the criteria by Gough, Pollard, Lee and satisfactorily corresponds to consequences of the approaches by Carpinteri, Spagnoli and others. For biaxial loadings ($0 \le \alpha_2 \le 1$, $\alpha_3 = 0$), the consequences of the expression in Eq. (17) in the form of $\tilde{\sigma}_i = [(1-\alpha_2)^2 + \alpha_2\hat{\eta}_i^2]^{-1/2}$ satisfactorily corresponds to consequences of the approaches by Gough, Zavoichinsky and others [24].

**11.** On the basis of the proposed model, experimental data on crack development in a representative set of structural steels were analyzed: carbonaceous, austenite-martensite, corrosion resistant, stainless, alloyed, pipe steels; cast irons; the metals molybdenum, nickel, lead, titanium and others; nickels, magnesium, aluminum, titanium alloys under various proportional loadings: uniaxial loading, shift, biaxial and uniaxial loadings with torsion with various amplitude ratios.

As an example Fig. 1 illustrates analyses of experimental data [19] and the study of microstructure of pipe steel grade S135 (0.32% C) (the steel properties are an ultimate strength of $\sigma_u = 1197$ MPa, a yield point of $\sigma_s = 1112$ MPa, and a characteristic mean grain size of $d \approx 0.02$ mm) upon symmetric axial loading with torsion, $\alpha_2 = -0.72$. Figure 1a illustrates basic constants (crosses), the regions of development I—VII, and nucleation boundaries 1—1, ..., 6—6 of defects of the six levels, as well as the fatigue curve along mesodefects $ft$ calculated by the model. On the basis of experimental data, the fatigue curve along complete failure $FT$ is plotted. At four amplitudes of maximum main stress $\sigma_a$ ((a—d) in Fig. 1) upon different loading stages, the microstructure was studied; the respective photos are illustrated in Fig. 1b and, before failure, in Fig. 1c. It follows from the analysis that, according to the model, fourth-level defects in steels are developed at ($\sigma_a = 545$ MPa, $n = 10^3$ cycles), ($\sigma_a = 594$ MPa, $n = 40$ cycles) and ($\sigma_a = 695$ MPa, $n = 10$ cycles) (the points are indicated by crosses in the region IV in Fig. 1 (a)); at ($\sigma_a = 743$ MPa, $n = 10$ cycles), the fourth-level defects reach the ultimate state and fifth-level defects form. The state before failure at the amplitudes $\sigma_a = 545$ MPa and $\sigma_a = 594$ MPa is characterized by reaching of the ultimate state by fifth-level defects; at the amplitudes $\sigma_a = 695$ MPa and $\sigma_a = 743$ MPa, it is characterized by reaching of the ultimate state by sixth-level defects. This is confirmed by experimental data. The calculated fatigue curve $ft$ along brittle defects of the mesolevel is located to the left of the complete failure curve $FT$, and viscous failure develops in material, preventing the development of brittle cracks and increasing the steel lifetime.

The development regions I—VII and nucleation boundaries 1—1, ..., 6—6 of defects of six levels calculated by the model as a function of ratio of amplitudes $\alpha_2$ of symmetric axial loading with torsion are illustrated in Fig. 2 ((a) at $\alpha_2 = 0$, (b) at $\alpha_2 = -0.4$, (c) at $\alpha_2 = -0.8$, and (d) at $\alpha_2 = -1$) for steel grade 12KhN2A with ultimate fatigues $\sigma_{-1} = 390$ MPa, and $\tau_{-1} = 252$ MPa upon uniaxial loading and shift, respectively, ($\sigma_u = 900$ MPa, $\sigma_s = 683$ MPa, $\tau_u = 581$ MPa, $\tau_s = 442$ MPa, $\hat{\eta}_4 = 0.9$ [20]). The following fatigue curves are presented: McDiarmid—Matake, Gough—Pollard, Sines—Crossland; experimental data (black dots) and fatigue curve $FT$ calculated by the model according to complete failure (with consideration for extra growth of brittle cracks according to the Paris equation), which is close to the McDiarmid—Matake fatigue curve. The calculated fatigue curve $ft$ defines the minimum lifetime on the basis of the formation of brittle macrocracks in the region $N_f \in [10^5, 5\times10^6]$ cycles agrees well with the experimental data for steel.

The obtained theoretical results are applied to estimate structure safety in the calculation of the lifetime of structural elements with the use of generated defects on the basis of physical regularities of fatigue failure of material.

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