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Separation of variables for integrable case of the Kowalevski top in a non-Euclidean space

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The integrable Kowalevski case in a non-Euclidean space is a pseudospherical Euler-Poisson equation [1]

$$\dot{\boldsymbol{m}} = (g\boldsymbol{m}) \times \frac{\partial H}{\partial \boldsymbol{m}} + (g\boldsymbol{\gamma}) \times \frac{\partial H}{\partial \boldsymbol{\gamma}}, \quad \dot{\boldsymbol{\gamma}} = (g\boldsymbol{\gamma}) \times \frac{\partial H}{\partial \boldsymbol{m}}$$

with Hamiltonian

$$H = \frac{1}{2}(m_1^2 + m_2^2 - 2k^2 m_3^2) - b_1\gamma_1$$

and Poisson structure

$$\Pi_r = \begin{pmatrix} 0 & k^2 m_3 & m_2 & 0 & k^2\gamma_3 & \gamma_2 \\ -k^2 m_3 & 0 & -m_1 & -k^2\gamma_3 & 0 & -\gamma_1 \\ -m_2 & m_1 & 0 & -\gamma_2 & \gamma_1 & 0 \\ 0 & k^2\gamma_3 & \gamma_2 & 0 & 0 & 0 \\ -k^2\gamma_3 & 0 & -\gamma_1 & 0 & 0 & 0 \\ -\gamma_2 & \gamma_1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Where $\boldsymbol{m} = (m_1, m_2, m_3)$ is an angular momentum, $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$ is unit vector of the axis of symmetry, k^2 is a non-Euclidean parameter, $g = \mathrm{diag}(1, 1, -k^2)$ is a diagonal matrix.

Poisson structure has a Casimir functions

$$C = \gamma_1^2 + \gamma_2^2 - k^2\gamma_3^2, \qquad L = m_1\gamma_1 + m_2\gamma_2 - k^2 m_3\gamma_3.$$

An additional integral F coincides with Kowalevski classic integral [2]

$$F = \left(\frac{m_1^2 - m_2^2}{2} + b_1\gamma_1\right)^2 + (m_1 m_2 + b_1\gamma_2)^2.$$

With Kowalevski [2] we introduce variables

$$x_1 = m_1 + \mathrm{i}\, m_2, \qquad x_2 = m_1 - \mathrm{i}\, m_2$$

and polynomials

$$R_i = k^2[-\frac{1}{4}x_i^4 + hx_i^2 + 2\ell b_1 x_i + cb_1^2 - f], \qquad R = -\frac{1}{2}[R_1 + R_2 + \frac{1}{4}k^2(x_1^2 - x_2^2)^2].$$

Here c, ℓ, h, f are constants of Casimir functions C, L, Hamiltonian H and additional integral F.

The variables of separation define by formulas

$$s_1 = \frac{R - \sqrt{R_1 R_2}}{(x_1 - x_2)^2}, \qquad s_2 = \frac{R + \sqrt{R_1 R_2}}{(x_1 - x_2)^2}.$$

Then $\{s_1, s_2\} = 0$ and the system of pseudospherical Euler-Poisson differential equations reduces to the system of separation equations

$$(s_1 - s_2)^2 \dot{s}_1^2 = 2\varphi(s_1)P(s_1), \qquad (s_1 - s_2)^2 \dot{s}_2^2 = 2\varphi(s_2)P(s_2).$$

Here $\varphi(s)$ and $P(s)$ are the polynomials

$$\varphi(s) = s^3 + k^2 h s^2 + \frac{1}{8}k^4(2cb_1^2 + 2h^2 - 2f)s + \frac{1}{8}k^6 \ell^2 b_1^2,$$
$$P(s) = (2s + k^2 h)^2 - k^4 f.$$

Thus integration of pseudospherical Euler-Poisson equations in a Kowalevski case reduce to the hyperelliptic quadrature.

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