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On the Higgs feature of gravity

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Abstract. The gauge gravitation theory, based on the equivalence principle besides the gauge principle, is formulated in the fibre bundle terms. The correlation between gauge geometry on spinor bundles describing Dirac fermion fields and space-time geometry on a tangent bundle is investigated. We show that field functions of fermion fields in presence of different gravitational fields are always written with respect to different reference frames. Therefore, the conventional quantization procedure is applicable to fermion fields only if gravitational field is fixed. Quantum gravitational fields violate the above mentioned correlation between two geometries.

Keywords. Gauge theory; gravity; Higgs field.

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The physical nature of gravity as a Higgs-Goldstone field is now clarified by means of gauge approach to gravitation theory (Sardanashvily 1980; Ivanenko and Sardanashvily 1983, 1987). Gauge theory of gravity can be built directly by reformulation of gravitation theory in the fibre bundle terms.

The fibre bundle formulation of gauge theory is based on the mathematical definition of matter fields as global sections φ of some differential vector bundle $\lambda = (V, G, X, \Psi_\lambda)$ with a typical fibre V, a structure group G, and a base X which is a smooth orientable paracompact 4-manifold. Hereinafter, $tl\lambda$ denotes a total space of λ, and π is the canonical projection of $tl\lambda$ on X. An atlas $\Psi_\lambda = \{U_i, \psi_{\lambda i}\}$ (where U_i and $\psi_{\lambda i}$ denote patches and trivialization morphisms of λ) defines some reference frame such that a section φ of λ is represented by a collection of V-valued functions $\{\varphi_i(x) = \psi_{\lambda i}(x)\varphi(x), x \in U_i\}$ with respect to Ψ_λ. Atlas transformations

$$\Psi_\lambda = \{U_i, \psi_{\lambda i}\} \to \Psi'_\lambda = \{U_i, \psi'_{\lambda i} = g_i \psi_{\lambda i}\}, \quad g_i \in G(U_i), \tag{1}$$

(where $G(U_i)$ denotes a group of G-valued functions on U_i) do not alter sections φ, but change their representations by field functions. Therefore, the invariance of a matter field Lagrangian under these transformations can be naturally required. This requirement necessitates introduction of gauge potentials, represented by coefficients of a local connection 1-form A_i on λ, and a G-invariant metric g in fibres of λ. Such a metric, however, is not a dynamic variable because it can be always brought into the canonical form by some gauge transformation. Transformations (1) compose the gauge pseudo-group $G_1(X)$ which is the direct limit of groups $G(\{U_i\}) = \prod_i G(U_i)$ with respect to inclusions $\{U_i\} \to \{U'_i\}$.

Another type of gauge transformations is generated by equivariant mappings of the total space $P = tl\Lambda$ of the principal bundle Λ associated with λ (Daniel and Viallet 1980):

$$p \to ps(p), \quad p \in P, \tag{2}$$

where s is a G-valued function on P such that

$$s(pg) = g^{-1}s(p)g, \quad g \in G.$$

Sections φ of λ can be defined by V-valued functions f on P:

$$\varphi(\pi(p)) = [p]f(p), \quad f(pg) = g^{-1}f(p),$$

where $[p]$ denotes restriction of the canonical mapping $\gamma_G: P \times V \to tl\lambda$ to the subspace $\{p\} \times V$ (Kobayashi and Nomizu 1963). Therefore, mappings (2) induce transformations

$$\begin{aligned}\varphi(\pi(p)) = [p]f(p) \to \varphi'(\pi(p)) &= [p]f'(p)\\ &= [p]f(ps(p)) = [p]s^{-1}(p)f(p),\end{aligned} \tag{3}$$

of section φ. Let $\Psi = \{U_i, \psi_i\}$ be an atlas of Λ, and let

$$\Psi_\lambda = \{U_i, \psi_{\lambda i} = [z_i(x)]^{-1}\}, \quad z_i(\pi(p)) = p\psi_i^{-1}(p), \quad p \in P,$$

be the associated atlas of λ. Transformation (3) of φ yields the transformation

$$\varphi_i(x) = [z_i(x)]^{-1}\varphi(x) = f(z_i(x)) \to \varphi_i'(x) = f(z_i(x)s(z_i)) = s^{-1}(z_i)\varphi_i(x)$$

of functions φ_i with respect to the atlas Ψ_λ. This transformation looks like the transformation between atlases Ψ_λ and $\Psi'_\lambda = \{U_i, s^{-1}(z_i)\psi_{\lambda i}\}$. Thus, for any transformation (3) of matter fields $\varphi \to \varphi'$ there exists the atlas change $\Psi_\lambda \to \Psi'_\lambda$ such that $\psi_{\lambda i}\varphi' = \psi'_{\lambda i}\varphi$. Therefore, a matter field Lagrangian, invariant under the gauge pseudo-group $G_{\text{I}}(X)$, is invariant under transformations (3) called the second type gauge transformations. Their group $G_{\text{II}}(X)$ is isomorphic with the group of global sections of the fibre bundle Λ' with the typical fibre G provided by the adjoint representation $G \to gGg^{-1}$ of the structure group G.

Let us consider Dirac fermion matter fields φ. These are described by global sections of a 4-dimensional spinor bundle $\lambda = (V, L, X)$ with the structure Lorentz group $L = \text{SO}(3, 1)$. Herewith, λ is endowed with some Lorentz connection A_i, and the Dirac operator

$$\Delta_D = h_a^\mu \gamma^a D_\mu - m \tag{4}$$

must be defined on φ. Here, quantities $h_a^\mu \gamma^a$ show that any atlas Ψ_λ of the spinor bundle λ must be associated with some atlas Ψ_T of the tangent bundle TX over the manifold X. This fact is the cornerstone of gravitation theory. The atlas Ψ_T is non-holonomic in general, and so is characterized by a tetrad field h which describes a gravitational field and defines some geometry on the tangent bundle TX. Therefore, gravitation theory can be formulated as gauge theory on the tangent bundle associated with spinor bundles. Such a theory is based on the equivalence principle besides the gauge (relativity) principle.

The structure group of TX is $GL^+(4, R)$. An atlas $\Psi_T = \{U_i, \psi_{Ti}\}$ of TX defines a space-time reference frame, namely, a vierbein $\{t_i(x)\} = \psi_{Ti}^{-1}(x)\{t\}$ (where $\{t\}$ is a fixed basis of the typical fibre R^4 of TX) can be erected at every point of X. Functions $t_i(x)$ play the role of local sections $z_i(x)$ of the associated principle bundle $\Lambda = LX$ of linear frames, which are defined by the associated atlas Ψ of Λ. Conversely, any collection of such functions $\{t_i(x)\}$ defines associated atlases Ψ of Λ and Ψ_T of TX. In consequence, reference frame changes compose the pseudo-group $GL^+(4, R)_1(X)$ of space-time gauge transformations. Take notice of the special case of holonomic atlases $\Psi_T = \{U_i, \psi_{Ti} = \partial\chi_i\}$ which correlate with coordinate atlases $\Psi_X = \{U_i, \chi_i\}$ of the manifold X such that the vierbeins $t_\mu(x) = \partial_\mu$ are oriented along coordinate lines. The pseudo-group $GL^+(4, R)_1(X)$ contains the pseudo-sub-group of holonomic gauge transformations

$$\psi'_{Ti}(\psi_{Ti})^{-1} = g_i: \quad t_\mu(x) \to t'_\mu(x') = t_\gamma(x)\frac{\partial x^\nu}{\partial x'^\mu}$$

accompanied by coordinate transformations $\chi'_i\chi_i^{-1}: x^\mu \to x'^\mu$.

Thus, the relativity principle in the fibre bundle terms is identical to the gauge principle, and gravitation theory can be built directly as gauge theory. The relativity principle necessitates introduction of a connection Γ_i on TX and a metric g in fibres of TX. But in contrast to metrics on matter bundles, g is a dynamic variable because it can be brought into the canonical form $g_i = \psi_{Ti}g = \eta$ only with respect to a non-holonomic atlas Ψ_T in general. The equivalence principle confines Γ_i to a Lorentz connection and g to a pseudo-Riemannian metric parallel with respect to Γ_i.

The equivalence principle we modify postulates the existence of a reference frame where Lorentz invariants could be defined everywhere on X, and these would be conserved under parallel transport. This principle possesses the adequate mathematical formulation in the fibre bundle terms. It requirs that both the holonomy group of the connection Γ_i on TX and the structure group of TX must be contracted to the Lorentz group.

A connection Γ on the principal bundle Λ, associated with TX, defines a holonomy bundle $\Lambda(p)$ for any point $p \in P = tl\Lambda$, and $\Lambda(p) = \Lambda(p')$ if p and p' belong to the same parallel curve (Kobayashi and Nomizu 1963). A holonomy bundle is reduced subbundle of Λ whose structure group is the holonomy group $K(p) \subset GL^+(4, R)$ of the connection Γ at $p \in P$. All holonomy bundles are isomorphic with each other, and $K(pg) = g^{-1}K(p)g$, $g \in GL^+(4, R)$. In accordance with the equivalence principle, let some holonomy group $K(p)$ be a subgroup of the Lorentz group L. Let us consider the subset $Q = \{qg; q \in tl\Lambda(p), g \in L\}$ of P. Then, the following is true (Kobayashi and Nomizu 1963).

(i) The subset Q is the total space of a reduced subbundle Λ_Γ with the structure group L. In consequence, the structure group of TX is contracted to the Lorentz group, in accordance with the equivalence principle.

(ii) Since $tl\Lambda(p) \subset Q$ for any $p \in Q$, the connection Γ on Λ is reduced to a connection Γ' on Λ_Γ such that the connection form ω' of Γ' takes on values in the Lie algebra of the Lorentz group, and $\omega' = \omega|_Q$ where ω is the connection form of Γ.

(iii) There is one-to-one correspondence between reduced Lorentz group subbundles of Λ and global sections h of the associated bundle λ_Σ in quotient spaces $\Sigma = GL^+(4, R)/L$. Therefore, the reduced subbundle Λ_Γ defines uniquely a global section h of λ_Σ such that $\pi'(Q) = h(\pi(Q))$ where π' is the canonical projection of P on P/L. Herewith, the section h is parallel with respect to the connection Γ. Hereinafter,

$\Lambda_{\Gamma h}$ or Λ_h will denote the reduced Lorentz group subbundle associated with h.

(iv) There exists an atlas Ψ^h of Λ such that its transition functions are elements of gauge Lorentz groups $L(U_i \cap U_j)$. This atlas is defined by local sections $\{z_i^h\}$ of Λ which take on values in $tl\Lambda_{\Gamma h}$. With respect to the atlas Ψ^h, the local connection 1-form $\Gamma_i = (z_i^h)^*\omega = (z_i^h)^*\omega'$ takes on values in the Lie algebra of the Lorentz group, and functions $\psi_{\Sigma i}^h h$ of the field h takes on values in the centre of Σ.

Note that with respect to some atlas Ψ of Λ, a field h can be represented by a collection of tetrad functions $h_i = \psi_i(z_i^h)$, but up to right multiplying them by elements of gauge Lorentz groups $L(U_i)$ because of nonunique choice of sections z_i^h. Functions

$$h_i(x) = \psi_i(z_i^h(x)) = [z_i(x)]^{-1}[z_i^h(x)],$$

are identical with gauge transformations between atlases $\Psi_T^h = \{U_i, \psi_{Ti}^h(x) = [z_i^h(x)]^{-1}\}$ and $\Psi_T = \{U_i, \psi_{Ti}(x) = [z_i(x)]^{-1} = h_i(x)\psi_{Ti}^h(x)\}$ of TX.

The fibre bundle λ_Σ is isomorphic to the fibre bundle of pseudo-Euclidean bilinear forms in tangent spaces T_x over X. Its global section g, isomorphic to h, is a pseudo-Riemannian metric on X such that metric functions g_i are reduced to the Minkowski metric $g_i = \psi_{Ti}^h g = \eta$ with respect to the atlases Ψ_T^h, and g is parallell with respect to the connection Γ_i, i.e. the well-known metricity condition $(d - \Gamma_i)g_i = 0$ holds. These quantities Γ_i and g satisfy both the relativity principle and the equivalence principle, and so can be used as field variables of gauge gravitation theory. But they are not independent from each other. If g is given the connection Γ is fixed with accuracy to torsion. If Γ is given, the set of reduced subbundles Λ_Γ and consequently the set of the corresponding fields h (or g) are subsets of the quotient space Σ.

Then, in gauge theory of gravity, a gravitational field appears due to the equivalence principle which singles out the Lorentz group as the exact symmetry subgroup of space-time symmetries. The physical ground of the equivalence principle is the fact of existence of Dirac fermion fields and the correlation between gauge geometry on spinor bundles λ and space-time geometry on TX.

We shall say that a spinor bundle $\lambda = (V, L, X)$, endowed with a connection A_i, describes Dirac fermion fields in presence of a gravitational field h if λ is associated with some reduced Lorentz group subbundle $\Lambda_{\Gamma h}$ of the principal bundle Λ associated with TX and endowed with a connection Γ. In general, connections A_i and Γ_i differs from each other in torsion. Since any atlas of Λ_h can be expanded up to an atlas Ψ^h of the bundle Λ (Kobayashi and Nomizu 1963), there is an atlas Ψ_T^h of TX for any atlas Ψ_λ of λ such that Ψ_λ and Ψ_T^h are associated with each other, i.e. they are defined by the same local sections z_i^h, and we see tetrad functions $h_i = \psi_i(z_i^h)$ in (4) for the Dirac operator. In consequence, fermion fields φ and φ' in presence of different gravitational fields h and h' are represented by sections of spinor bundles λ and λ' which are associated with different subbundles Λ_h and Λ'_h of the principal bundle Λ. For instance, their field functions $\varphi_i = \psi_{\lambda i}^h \varphi$ and $\varphi'_i = \psi_{\lambda i}^{h'} \varphi'$, and the Dirac operator Δ_D on them are always written with respect to different reference frames because there are no Lorentz gauge transformations between atlases Ψ_T^h and $\Psi_T^{h'}$.

Note that any spinor bundle λ, endowed with some connection A_i, describes locally fermion fields as fields in presence of gravity. If $U \subset X$ is a paracompact set, homotopic to a point, a trivial bundle $\lambda|_U$ is associated with some reduced Lorentz group subbundle Λ_{hU} of the trivial principal bundle Λ_U associated with TU, and a connection A_i on $\lambda|_U$ and Λ_{hU} can be expanded up to some connection on Λ_U and TU.

Thus, fermion fields must be described only in complex with a certain gravitational field h. The total space P of the principal $GL^+(4,R)$ bundle Λ can be represented as a total space of the principal Lorentz group bundle Λ_L with the base P/L. Let $\lambda_L=(V,L,P/L)$ be the fibre bundle associated with Λ_L. Since

$$tl\lambda=\gamma_L(tl\Lambda_h\times V)\subset\gamma_L(P\times V)=tl\lambda_L,$$

for any h, each spinor bundle λ is a subbundle of λ_L over the subspace $tl\Lambda_h/L\subset P/L$. Therefore, any section φ_L of λ_L defines a section $\varphi=\varphi_L|_{tl\Lambda_h/L}$ of λ. Conversely, a section φ of λ can be expanded up to some section φ_L of λ_L because P/L is a paracompact space and $tl\Lambda_h/L$ is a closed subset of P/L. Thus, sections φ_L of λ_L describe a complex of fermion fields and gravitational fields.

Let us consider gauge transformations in gravitation theory with a glance to this complex. There are two kinds of the first type gauge space-time transformations, namely, above mentioned atlas changes of TX and atlas transformations of spinor bundles λ which compose the pseudo-group $L_{\rm I}(X)$. The former transformations do not alter spinor fields functions, whereas the latter transformations do not change a gravitational field. Transformations of Ψ_λ act on tetrad functions

$$h_i\to h_i g,\quad g\in L(U_i), \tag{5}$$

but a tetrad gravitational field h is defined by h_i up to these changes. There is injection of $L_{\rm I}(X)$ into $GL^+(4,R)_{\rm I}(X)$, and its image includes gauge transformations of atlases Ψ^h_T. The second type partner of $L_{\rm I}(X)$ is the Lorentz gauge group $L_{\rm II}(X)$ of (3). These transformations do not act on a gravitational field. They transform tetrad functions by rule (5) and alter a torsion part of the Lorentz connection A_i on λ.

Only holonomic transformations from the gauge group $GL^+(4,R)_{\rm I}(X)$ possess the second type partners (Ivanenko and Sardanashvily 1987). These are fibre-to-fibre morphisms of the tangent bundle TX, which are induced by diffeomorphisms γ of the manifold X. They yield transformations of tensor fields τ (as global sections of tensor bundles):

$$L_\gamma:\tau(x)\to\tau'(x)=(\partial\gamma)\tau(\gamma^{-1}(x)) \tag{6}$$

which alter these fields $\tau(x)\to\tau'(x)$ in a point. If diffeomorphism γ is represented as a flow, the generator of morphisms (6) takes the familiar form of the Lie derivative. An action functional $\int_U L\,\mathrm{d}x$, $U\subset X$, is invariant under (6) if $\gamma(U)=U$.

Note that any change of a gravitational field h also effect matter spinor fields φ. Let φ_L be a section of λ_L such that $\varphi=\varphi_L|_{tl\Lambda_h/L}$, and let f be the corresponding V-valued function on P. We choose an atlas $\Psi_L=\{U_{Li},\psi_{\lambda_L i}(q)=[z_{Li}(q)]^{-1},q\in U_{Li}\}$ of the bundle λ_L and some atlas $\Psi=\{U_i,\psi_i\}$ of Λ. Then, the field functions of φ_L read

$$\varphi_{Li}(q)=f(z_{Li}(q))=f(\psi_i^{-1}g_i(q))=f'(g_i(q))$$

where $q\in U_{Li}$, $\pi(q)\in U_i$, and $g_i(q)$ is an element of the coset $\psi_{\Sigma i}(q)\in GL^+(4,R)/L$. The property

$$f'(g_i(q)k)=f(z_{Li}(q)k)=k^{-1}f'(g_i(q)),\quad k\in L,$$

holds. Functions $\varphi_{Li}(q)$ realize the so-called induced representation $L\uparrow GL^+(4,R)$ of the group $GL^+(4, R)$ by the rule

$$g_0: \varphi_{Li}(q) = f'(g_i(q)) \to f'(g_0^{-1} g_i(q)) = f'(g_i(q')k) = k\varphi_{Li}(q'),$$

$$k = g_i^{-1}(q') g_0^{-1} g_i(q) \in L, \quad \psi_{\Sigma i} q' = g_0^{-1} \psi_{\Sigma i} q, \quad q, q' \in U_{Li}.$$

Therefore, any change of a gravitational field $h \to h' = g_0^{-1} h$ induces the transformation

$$\varphi_i(x) = \varphi_{Li}(h) \to k\varphi_i'(x) = k\varphi_{Li}(h'). \tag{7}$$

We can neglect the factor k in the right side of this expression because of gauge equivalence of fields $k\varphi_i'$ and φ_i'. Moreover, we can replace transformation law (7) by the low

$$\varphi_i(x) \to \varphi_i'(x) = \varphi_i(x). \tag{8}$$

But this does not mean that $\varphi'(x) = \varphi(x)$ because

$$\varphi'(x) = (\psi_{\lambda i}^{h'})^{-1} \varphi_i'(x) = (\psi_{\lambda i}^{h'})^{-1} \varphi_i(x) = (\psi_{\lambda i}^{h'})^{-1} \psi_{\lambda i}^{h} \varphi(x) \neq \varphi(x).$$

Functions $\varphi_i(x)$ and $\varphi_i'(x)$ in (8) are always written with respect to different reference frames because atlases $\Psi^h = \Psi_L|_{t l \Lambda_h}$ and $\Psi^{h'} = \Psi_L|_{t l \Lambda_{h'}}$ of the reduced bundles Λ_h and $\Lambda_{h'}$ cannot be expanded up to the same atlas of Λ.

The fact that changes of a gravitational field is accompanied by nonequivalent transformations between spinor reference frames makes impossible for us to quantize a geometric gravitational field in presence of the Dirac fermion matter.

In quantum field theory, chronological vacuum expectations F of quasi-free fermion fields (e.g. fermions in presence of a Higgs vacuum) can be constructed as follows. Let A_Φ be a commutative Z_2-graded tensor-algebra of a vector topological space Φ of fermion fields. Expectations F are defined by the form

$$F(\varphi^1 \cdots \varphi^n) = \frac{1}{i^n} \frac{\partial}{\partial \alpha_1} \cdots \frac{\partial}{\partial \alpha_n} \exp(-i\alpha_i \alpha_j M(\varphi^i, \varphi^j))$$

on A_Φ where M is some continuous Hermitian bilinear form on Φ, and α_i are elements of a Grassmann algebra. In algebraic quantum field theory, $\Phi = V \times S(R^4)$ where $S(R^4)$ is the Scwartz space of test functions of Wightman's theory, and the covariance form $M(x, y)$, $x, y \in R^4$, is a Green function of some linear differential operator Δ on Φ, i.e.

$$\Delta M(x, y) = -i\delta(x - y).$$

For instance, if Δ is Dirac operator (4) in the Minkowski space, the function $-iM(x, y)$ is the propagator of free fermion fields.

Fermion fields φ and φ' in presence of different fields h and h' fail to compose a vector space. Field functions of such fields as like as the Dirac operator Δ_D on them are always described with respect to different reference frames. Therefore, the above mentioned quantization scheme is applicable to fermions only if the field h is fixed. In general forms F' and F on A_Φ, which correspond to different gravitational fields, are nonequivalent, and so describe different Higgs vacua (Sardanashvily and Zakharov 1989). Thus, description of fermions in presence of a gravitational field is analogous to their description in presence of some Higgs classical field (Sardanashvily and Ikhlov 1988). Note that any finite system of fermions in presence of a classical Higgs field can be effectively represented as free fermions interacting with some quantum field. Such representation, however, turns out to be impossible for fermions in presence of a gravitational field. Firstly, Dirac operator (4) is defined only if $h_i \neq 0$, i.e. some classical background tetrad field h must always exist. Secondly, one faces difficulty in description of fermion fields φ in presence of a tetrad field h as fermion fields φ' in presence of h', but which additionally interact with the deviation field σ ($h = h'\sigma$) because fields φ and φ' are always written with respect to different reference frames.

To overcome this difficulty, one can assume that, in the quantum case, geometry on spinor bundles does not correlate with space-time geometry on the tangent bundle. In this case, quantities $h^\mu_a = \sigma^b_a h'^\mu_b$ in (4) for the Dirac operator are not tetrad functions of a pseudo-Riemannian metric on X. For instance,

$$\rho^{\mu\alpha}\rho_{\alpha\nu} \neq \delta^\mu_\nu, \quad \rho^{\mu\nu} = h^\mu_a h^\nu_b \eta^{ab}, \quad \rho_{\mu\nu} = g'_{\mu\alpha} g'_{\gamma\beta} \rho^{\alpha\beta}, \tag{9}$$

where g' is the fixed metric tensor corresponding to the tetrad field h'. Therefore, the quantities σ do not describe fluctuations of a conventional gravitational field. For example, let $g'_{\mu\nu} = \eta_{\mu\nu}$, and let σ^b_a be a small deviation from δ^b_a, i.e. $\sigma^b_a = \delta^b_a + \varepsilon^b_a$. Then, one can compare the expression

$$\rho^{\mu\nu} = \eta^{\mu\nu} + \varepsilon^{(\mu\nu)}, \quad \rho_{\mu\nu} = \eta_{\mu\nu} + \varepsilon_{(\mu\nu)},$$

obtained from (9), with the expression

$$g^{\mu\nu} = \eta^{\mu\nu} + \varepsilon'^{(\mu\nu)}, \quad g_{\mu\nu} = \eta_{\mu\nu} - \varepsilon'_{\mu\nu},$$

for small metric fluctuation. In the geometric terms, the deviations σ coincides with coefficients of the well-known soldering form θ many authors tried unsuccessfully to identify with a tetrad gravitational field in the framework of gauge Poincaré models (Cho 1976; Tseitlin 1981). Therefore, to quantize deviations σ, one can use Lagrangians of the Poincaré gauge theory, if all indices are paired by means of a background metric g'. Extension of deviations σ, in our opinion, can result in destruction of a space-time geometry.

Note that, if a gravitational field h is fixed, variations of a connection A_i are reduced only to variations of the torsion, and so do not violate the correlation between gauge geometry on spinor bundles and space-time geometry. Therefore, the standard scheme of gauge field quantization is applicable to A_i.

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