



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



# Electromagnetic neutrino-atom collisions: The role of electron binding

Konstantin A. Kouzakov<sup>a,\*</sup> and Alexander I. Studenikin<sup>b,c†</sup>

<sup>a</sup>Department of Nuclear Physics and Quantum Theory of Collisions, Moscow State University, Leninskie Gory, Moscow 119991, Russia

<sup>b</sup>Department of Theoretical Physics, Moscow State University, Leninskie Gory, Moscow 119991, Russia

<sup>c</sup>Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia

We present a new theoretical approach to neutrino-impact atomic excitation and/or ionization due to neutrino magnetic moments. The differential cross section of the process is given by a sum of the longitudinal and transverse terms, which are induced by the corresponding components of the force that the neutrino magnetic moment imposes on electrons with respect to momentum transfer. In this context, the recent theoretical studies devoted to the magnetic neutrino scattering on atoms are critically examined.

## 1. INTRODUCTION

The Standard Model predicts the value of the neutrino magnetic moment, in units of the Bohr magneton  $\mu_B = e/(2m_e)$ , as [1–3]

$$\mu_\nu \approx 3.2 \times 10^{-19} \frac{m_\nu}{1 \text{ eV}}, \quad (1)$$

where  $m_e$  and  $m_\nu$  are the electron and neutrino masses, respectively. Any experimental evidence for a larger value of  $\mu_\nu$  will unequivocally indicate physics beyond the Standard Model (a recent review of this subject can be found in Ref. [4]). The best upper limit for  $\mu_\nu$  obtained so far in experiments with reactor (anti)neutrinos is  $\mu_\nu \leq 3.2 \times 10^{-11}$  [5] (see also references in the review article [4]). This is by an order of magnitude larger than the most stringent astrophysical constraint  $\mu_\nu \leq 3 \times 10^{-12}$  [6].

At small energy transfer  $T$  the differential cross section (DCS) for the magnetic neutrino scattering on a free electron (FE) behaves as  $d\sigma_{(\mu)}/dT \propto 1/T$  [7,8], while that due to weak interaction,  $d\sigma_{(w)}/dT$ , is practically constant in  $T$  [8]. Therefore, one can enhance the sensitivities of the

reactor experiments by reducing the low-energy threshold of the detectors in the deposited energy  $T$ . However, the FE picture is applicable only when  $T \gg \varepsilon_b$ , where  $\varepsilon_b$  is the binding energy of an atomic electron. If  $T \sim \varepsilon_b$ , the electron binding effects must be taken into account. In this work, we formulate a theoretical framework for the magnetic neutrino scattering on atomic electrons which is similar to that developed for the penetration of relativistic charged projectiles in matter [9]. Within this approach the DCS is given by the sum of two components due to the longitudinal and transverse atomic excitations, respectively, which are induced by the corresponding components of the force imposed by the neutrino magnetic moment on electrons with respect to the direction of the momentum transfer  $\mathbf{q}$ . It also enables us to clearly distinguish between the contributions from excited atomic states taken into account in Refs. [10] and [11], respectively, and to inspect consistently the results of those studies.

The article is organized as follows. Sec. 2 delivers general theory for the magnetic neutrino scattering on atomic electrons as well as a critical account of the theoretical studies published recently [10,11]. The conclusions are drawn in

\*Email address: [kouzakov@srd.sinp.msu.ru](mailto:kouzakov@srd.sinp.msu.ru)

†Email address: [studenik@srd.sinp.msu.ru](mailto:studenik@srd.sinp.msu.ru)

Sec. 3. The units  $\hbar = c = 1$  are used throughout unless otherwise stated.

## 2. THEORY OF MAGNETIC NEUTRINO-IMPACT ATOMIC EXCITATION AND IONIZATION

We specify the incident neutrino energy and momentum by  $E_\nu$  and  $\mathbf{p}_\nu$ , respectively. The atomic recoil is neglected under the assumption  $T \gg 2E_\nu^2/M$ , where  $M$  is the nuclear mass. The atomic target is supposed to be unpolarized and in its ground state  $|0\rangle$  with the corresponding energy  $E_0$ . We treat the initial and final electronic systems nonrelativistically under conditions  $T \ll m_e$  and  $\alpha Z \ll 1$ , where  $Z$  is the nuclear charge and  $\alpha$  is the fine-structure constant. The incident and final neutrino states are described by the Dirac spinors assuming  $m_\nu \approx 0$ . The neutrino electromagnetic vertex associated with the neutrino magnetic moment is employed in the low-energy limit

$$\Lambda_{(\mu)}^i = \frac{\mu_\nu}{2m_e} \sigma^{ik} q_k, \quad (2)$$

where  $q$  is the virtual-photon four-momentum. Note that the  $\mu_\nu$  related contribution to the neutrino-atom scattering couples neutrino states with different helicities and therefore it does not interfere with that due to weak interaction.

Using first-order perturbation theory and the photon propagator in the Coulomb gauge, the transition matrix element for the considered process according to Eq. (2) is given by [12]

$$\begin{aligned} M_{fi}^{(\mu)} &= \frac{2\pi\alpha\mu_\nu}{m_e|\mathbf{q}|} (\bar{u}_{\mathbf{p}_\nu - \mathbf{q}\lambda_f} u_{\mathbf{p}_\nu\lambda_i}) \\ &\times \left\{ \frac{2E_\nu - T}{|\mathbf{q}|} \langle n | \rho(-\mathbf{q}) | 0 \rangle \right. \\ &+ \sqrt{\frac{(2E_\nu - T)^2 - \mathbf{q}^2}{\mathbf{q}^2 - T^2}} \\ &\left. \times \langle n | \hat{\mathbf{e}}_\perp \cdot \mathbf{j}(-\mathbf{q}) | 0 \rangle \right\}, \quad (3) \end{aligned}$$

where  $u_{\mathbf{p}\lambda}$  is the spinor amplitude of the neutrino state with momentum  $\mathbf{p}$  and helicity  $\lambda$ ,  $|n\rangle$  the final atomic state, and  $\rho(-\mathbf{q})$  and  $\mathbf{j}(-\mathbf{q})$  the

Fourier transforms of the electron density and current density operators, respectively,

$$\rho(-\mathbf{q}) = \sum_{j=1}^Z e^{i\mathbf{q}\cdot\mathbf{r}_j}, \quad (4)$$

$$\mathbf{j}(-\mathbf{q}) = -\frac{i}{2m_e} \sum_{j=1}^Z (e^{i\mathbf{q}\cdot\mathbf{r}_j} \nabla_j + \nabla_j e^{i\mathbf{q}\cdot\mathbf{r}_j}), \quad (5)$$

and the unit vector  $\hat{\mathbf{e}}_\perp$  is directed along the  $\mathbf{p}_\nu$  component which is perpendicular to  $\mathbf{q}$  ( $\hat{\mathbf{e}}_\perp \cdot \mathbf{q} = 0$ ). Using Eq. (3), the DCS can be presented as

$$\frac{d\sigma_{(\mu)}}{dT} = \left( \frac{d\sigma_{(\mu)}}{dT} \right)_\parallel + \left( \frac{d\sigma_{(\mu)}}{dT} \right)_\perp, \quad (6)$$

$$\begin{aligned} \left( \frac{d\sigma_{(\mu)}}{dT} \right)_\parallel &= \frac{\pi\alpha^2\mu_\nu^2 (2E_\nu - T)^2}{m_e^2 4E_\nu^2} \\ &\times \int_{T^2}^{(2E_\nu - T)^2} \left( 1 - \frac{T^2}{Q^2} \right) \\ &\times S(T, Q) \frac{dQ^2}{Q^2}, \quad (7) \end{aligned}$$

$$\begin{aligned} \left( \frac{d\sigma_{(\mu)}}{dT} \right)_\perp &= \frac{\pi\alpha^2\mu_\nu^2 (2E_\nu - T)^2}{m_e^2 4E_\nu^2} \\ &\times \int_{T^2}^{(2E_\nu - T)^2} \left[ 1 - \frac{Q^2}{(2E_\nu - T)^2} \right] \\ &\times R(T, Q) \frac{dQ^2}{Q^2}, \quad (8) \end{aligned}$$

where  $Q = |\mathbf{q}|$  and

$$\begin{aligned} S(T, Q) &= \sum_n |\langle n | \rho(-\mathbf{q}) | 0 \rangle|^2 \\ &\times \delta(T - E_n + E_0), \quad (9) \end{aligned}$$

$$\begin{aligned} R(T, Q) &= \sum_n |\langle n | \hat{\mathbf{e}}_\perp \cdot \mathbf{j}(-\mathbf{q}) | 0 \rangle|^2 \\ &\times \delta(T - E_n + E_0). \quad (10) \end{aligned}$$

The sums in Eq. (9) run over all atomic states  $|n\rangle$ , with  $E_n$  being their energies, and, since the ground state  $|0\rangle$  is unpolarized, do not depend on the direction of  $\mathbf{q}$ .

The longitudinal term (7) is associated with atomic excitations induced by the force that the neutrino magnetic moment exerts on electrons

in the direction parallel to  $\mathbf{q}$ . The transverse term (8) corresponds to the exchange of a virtual photon which is polarized as a real one, that is, perpendicular to  $\mathbf{q}$ . It resembles a photoabsorption process when  $Q \rightarrow T$  and the virtual-photon four-momentum thus approaches a physical value,  $q^2 \rightarrow 0$ . Due to selection rules, the longitudinal and transverse excitations do not interfere (see Ref. [9] for detail).

The properties of Eq. (7) were studied in the work [11], where the transverse component was unaccounted. It is determined by the dynamical structure factor

$$S(T, Q) = \frac{1}{\pi} \text{Im} F(T + E_0, Q), \quad (11)$$

where the density-density Green's function  $F$  is

$$\begin{aligned} F(E, Q) &= \langle 0 | \rho(\mathbf{q}) \frac{1}{E - H - i0} \rho(-\mathbf{q}) | 0 \rangle \\ &= \sum_n \frac{|\langle n | \rho(-\mathbf{q}) | 0 \rangle|^2}{E - E_n - i0}, \end{aligned} \quad (12)$$

with  $H$  being the atomic Hamiltonian. In Ref. [11] the dispersion relation for the function  $F$  was formulated as

$$F(E, Q) = \frac{1}{\pi} \int_0^\infty \frac{\text{Im} F(E, Q')}{Q'^2 - Q^2 - i0} dQ'^2. \quad (13)$$

Consider the limit  $Q \rightarrow 0$ . The electron density operator (4) at  $\mathbf{q} = 0$  is by definition  $\rho(0) = Z$  and hence

$$\begin{aligned} F(E, 0) &= \langle 0 | \rho(0) \frac{1}{E - H - i0} \rho(0) | 0 \rangle \\ &= \frac{Z^2}{E - E_0}. \end{aligned} \quad (14)$$

Using it in Eq. (13) when  $Q = 0$ , we arrive at the sum rule [12]

$$\int_0^\infty S(T, Q) \frac{dQ^2}{Q^2} = \frac{Z^2}{T}. \quad (15)$$

Note that the value of  $F(T + E_0, 0)$  is calculated in Ref. [11] erroneously, namely as

$$F(E, 0) = \frac{Z}{E - E_0}. \quad (16)$$

Following the procedure of Ref. [11], which implies the use of Eq. (15) for evaluating the integral in Eq. (7) under assumptions of small  $T$  and

large  $E_\nu$ , we arrive at the result where the factor of  $Z^2$  occurs. This means that the atomic effects result in a coherent enhancement of the DCS as compared to the case of  $Z$  free electrons, where a typical incoherent-scattering factor of  $Z$  is encountered (the same as, for instance, in the Compton scattering). This conclusion is not consistent with the incoherent character of the considered inelastic scattering process.

Taking into account that  $\mathbf{j} = (\rho\mathbf{v} + \mathbf{v}\rho)/2$ , with  $\mathbf{v}$  being the electron velocity operator, we can estimate the ratio of the functions (10) and (9) as  $\sim v_a^2$ , where  $v_a \ll 1$  is a characteristic velocity of atomic electrons. Therefore, one might expect the transverse component to play a minor role in Eq. (6). However, the authors of Ref. [10] came to the contrary conclusion that this component strongly enhances due to atomic ionization when  $T \sim \varepsilon_b$ . The enhancement mechanism proposed in Ref. [10] is based on an analogy with the photoionization process. As mentioned above, when  $Q \rightarrow T$  the virtual-photon momentum approaches the physical regime  $q^2 = 0$ . In this case, we have

$$\left. \frac{R(T, Q)}{Q^2} \right|_{Q \rightarrow T} = \frac{\sigma_\gamma(T)}{4\pi^2 \alpha T}, \quad (17)$$

where  $\sigma_\gamma(T)$  is the photoionization cross section at the photon energy  $T$  [13]. The limiting form (17) was used in Ref. [10] in the whole integration interval. Such a procedure is incorrect, for the integrand rapidly falls down as  $Q^2$  ranges from  $T^2$  up to almost  $4E_\nu^2$ , especially when  $Q \gtrsim r_a^{-1}$ , where  $r_a$  is a characteristic atomic size (within the Thomas-Fermi model  $r_a^{-1} = Z^{1/3} \alpha m_e$  [14]). This fact reflects a strong departure from the real-photon regime. Thus, we can classify the enhancement of the DCS claimed in Ref. [10] as spurious. It should be noted in this connection that when the present work (as well as [12]) had already been completed and submitted for publication the authors of Ref. [10] had disproved their claim (see Ref. [15] for detail).

### 3. CONCLUSIONS

To summarize, we have performed a theoretical analysis of the magnetic neutrino scattering

on atomic electrons. For this purpose we have divided the DCS into two components corresponding to the longitudinal and transverse atomic excitations. This allowed us to demonstrate in a physically transparent fashion the deficiencies of the recent theoretical predictions concerning the role of the atomic effects in the magnetic neutrino scattering [10]. No enhancement mechanism due to electron binding effects has been determined [12] (see also Ref. [11]), in contrast to Ref. [10]. At the same time, the attempt to argue the insignificance of the atomic effects by means of analytical calculations [11] needs further elaboration [12,16].

Finally, it is unreasonable to expect the effects of atomic excitation and/or ionization to introduce enhancement of the sensitivities of the experiments searching for neutrino magnetic moments. In this respect, it will be interesting to explore the role of coherent magnetic neutrino scattering on atoms in detectors, which case, however, requires much lower energy thresholds in the deposited energy  $T$  ( $\sim 100$  eV) than presently attainable in the detectors ( $\sim 1$  keV).

## ACKNOWLEDGEMENTS

We are grateful to Mikhail B. Voloshin for useful discussions and valuable comments. One of the authors (A.I.S.) is thankful to Gianluigi Fogli and Eligio Lisi for invitation to attend NOW2010.

## REFERENCES

1. W.J. Marciano and A.I. Sanda, Phys. Lett. B 67 (1977) 303.
2. B.W. Lee and R.E. Shrock, Phys. Rev. D 16 (1977) 1444.
3. K. Fujikawa and R.E. Shrock, Phys. Rev. Lett. 45 (1980) 963.
4. C. Giunti and A.I. Studenikin, Phys. At. Nucl. 73 (2009) 2089, arXiv:08123646 [hep-ph].
5. A.G. Beda et al. (GEMMA collaboration), arXiv:0906.1926 [hep-ex]; arXiv:1005.2736 [hep-ex].
6. G.G. Raffelt, Phys. Rev. Lett. 64 (1990) 2856.
7. G.V. Domogatskii and D.K. Nadezhin, Sov. J. Nucl. Phys. 12 (1971) 678.
8. P. Vogel and J. Engel, Phys. Rev. D 39 (1989) 3378.
9. U. Fano, Annu. Rev. Nucl. Sci. 13 (1963) 1.
10. H.T. Wong, H.-B. Li, and S.-T. Lin, Phys. Rev. Lett. 105 (2010) 0161801.
11. M.B. Voloshin, Phys. Rev. Lett. 105 (2010) 201801, arXiv:1008.2171 [hep-ph].
12. K.A. Kouzakov and A.I. Studenikin, Phys. Lett. B 696 (2011) 252, arXiv:1011.5847 [hep-ph].
13. A.I. Akhiezer and V.B. Berestetskii, Quantum Electrodynamics, second ed., Wiley, New York, 1965.
14. L.D. Landau and E.M. Lifshitz, Quantum Mechanics, Non-Relativistic Theory, third ed., Pergamon, New York, 1977.
15. H.T. Wong, H.-B. Li, and S.-T. Lin, arXiv:1001.2074v3 [hep-ph].
16. K.A. Kouzakov, A.I. Studenikin, and M.B. Voloshin, arXiv:1101.4878 [hep-ph].