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FORMULA FOR THE DEFLECTION OF THE PLANAR HINGED-PIVOT FRAME

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A scheme of a statically definable frame with two hinged fixed supports is proposed. The induction method using the computer mathematic system Maple obtained analytical expressions for the deflection of this structure under the action of concentrated force and load uniformly distributed over the nodes of the crossbar's belts. It is shown that for some values of the number of panels in the crossbar and racks, the truss is kinematically degenerate. The determinant of the system of equilibrium equations vanishes. The corresponding scheme of possible node velocities is given.

Keywords: arch, planar truss, deflection, induction method, Maple, analytical solution

ФОРМУЛА ДЛЯ ПРОГИБА ПЛОСКОЙ ШАРНИРНО-СТЕРЖНЕВОЙ РАМЫ

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Рассматривается статически определимая рама с двумя шарнирными неподвижными опорами. Методом индукции с привлечением системы компьютерной математики Maple получены аналитические выражения для прогиба этой конструкции под действием сосредоточенной силы и нагрузки, равномерно распределенной по узлам поясов ригеля. Показано, что при некоторых значениях числа панелей в ригеле и стойках ферма кинематически вырождается. Определитель системы уравнений равновесия обращается в нуль. Приведена соответствующая схема возможных скоростей узлов.

Ключевые слова: рама, плоская ферма, прогиб, метод индукции, Maple, аналитическое решение

Introduction

Hinged-pivot frames can enter as an integral part in spatial structures, for example, hulls of industrial buildings. As a rule, the calculation of such trusses is carried out numerically, taking into account all the features of the structure. Analytical solutions can serve either for checking and debugging numerical solutions, or for a preliminary simple evaluation of the design being designed. The most effective are those analytical solutions that are represented not by a complicated formula with the greatest number of parameters of the system. The greatest difficulty in obtaining such formulas is the consideration of numerical characteristics, for example, the number of rods or panels.

In [1-5] by induction, calculation formulas were obtained for some statically determinate planar frame-trusses. Polynomial formulas for deflection and forces are found in some critical rods with respect to stability or loss of strength.

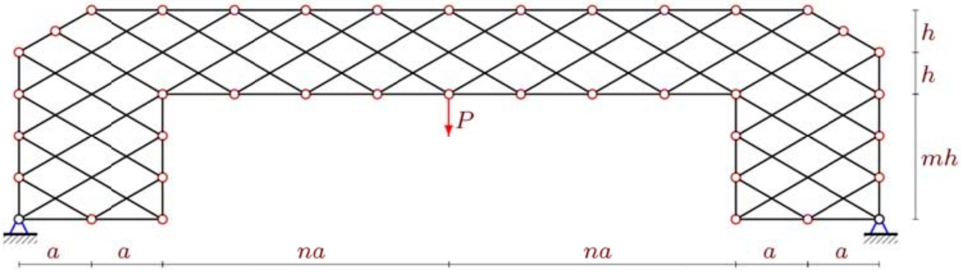

Fig. 1. Truss: n=4, m=3

Calculation of forces in a symbolic form is carried out according to the program [6], in which the coordinates of the nodes and the order of connection of the hinges and rods are entered. The first calculations showed that for odd n and even m there is no solution to the problem, the determinant of the system of equilibrium equations vanishes. The system becomes instantly kinematically variable. This is confirmed by the obtained picture of the possible node velocities (Fig. 2). The majority of nodes remain stationary, the speeds of others are equal to u, v and u'. Obviously the ratio of velocities $u/h = v/a = 2u'/c$, where $c = \sqrt{a^2 + h^2}$. Let us consider the case $m = 2k_1 - 1$, $n = 2k_2$ and $k_1 = k_2 = k$.

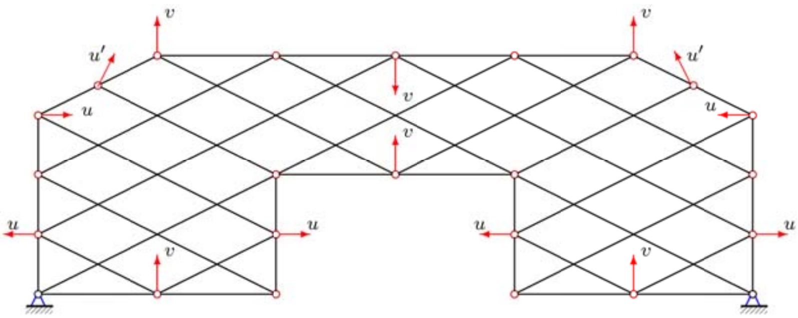

Fig. 2. Scheme of possible rates of the variable truss: n=2, m=2

The deflection is determined by the Maxwell-Mohr's formula [7]:

$$\Delta = \sum_{i=1}^{K-4} S_i s_i l / (EF),$$

where: EF — the stiffness of the rods, S_i — the forces in the rods from the load, s_i – the forces in the rods from the action of a single vertical force applied to the middle node of the lower girdle of the crossbar, l_i – the length of the rods, K=8(n+m)+20 — the number of rods in the design without taking into account four rods simulating the fixed hinge supports, which are adopted to be rigid. The calculation shows that the general form of the formula for deflection does not depend on the number of panels and has the form:

$$\Delta = P(C_1 a^3 + C_1 a^3 + C_1 a^3)/(4h^2 EF). \quad (1)$$

The following non-monotonically increasing sequence is obtained for the coefficient a^3: 39, 99, 1191, 379, 5895, 979, 16583, 2027, 35687, 3651, 65639, 5979, 108871, 9139, 167815, 13259. Identifying the formation of terms of this sequence allows the operator **rgf_findrecur** package **genfunc** system Maple, which returns the next recursion equation of the eighth order

$$C_{1,n} = 4C_{1,n-2} - 6C_{1,n-4} + 4C_{1,n-6} - C_{1,n-8}.$$

The solution of the equation is given by **rsolve**

$$C_1 = (8(10-9(-1)^k)k^3 + 6(5(-1)^k - 1)k^2 + 26(3(-1)^k - 1)k - 36(-1)^k + 69)/3.$$

Similarly, other coefficients are obtained

$$C_2 = 4(8(1-(-1)^k)k^3 + 6((-1)^k - 1)k^2 + ((-1)^k + 4)k + 1),$$

$$C_3 = 2(32(1-(-1)^k)k^3 + 30((-1)^k - 1)k^2 + 2(4(-1)^k + 5)k - 6(-1)^k + 3)/3.$$

In the case of loading by a uniformly distributed load (Fig. 3), the solution becomes more complicated. The sequence of coefficients required for analysis is lengthened, which leads to an increase in the counting time. In the case of symbolic transformations, computer mathematics systems work much more slowly. In some cases (including for the structure under study), the solution can not be obtained simply because of the not really long time of the transformations.

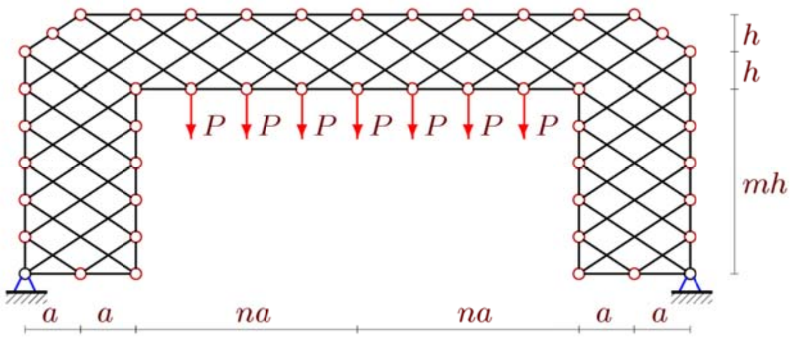

Fig. 3. Truss with load on the lower belt: n=4, m=5

The required length of the sequence is 20, and the recurrent equation is of the tenth order:

$$C_{1,n} = 5C_{1,n-2} - 10C_{1,n-4} + 10C_{1,n-6} - 5C_{1,n-8} + C_{1,n-10}.$$

The coefficients in (1) for this load have the form

$$C_1 = ((132-160(-1)^k)k^4 + 4(9(-1)^k - 37)k^3 + (76(-1)^k - 60)k^2 + (148-114(-1)^k)k - 33)/3,$$
$$C_2 = 64(1-(-1)^k)k^4 + 32((-1)^k - 3)k^3 + (42-8(-1)^k)k^2 - 4(-1)^k(-1)^k k - (-1)^k - 3,$$
$$C_3 = 2(64(1-(-1)^k)k^4 + 12(4(-1)^k - 9)k^3 + (19(-1)^k + 41)k^2 + 3(1-9(-1)^k)k + 6(-1)^k - 3)/3.$$

Simultaneously with calculating the deflection in the same cycle in terms of the number of panels by induction, one can obtain an expression for the expansion (horizontal reaction of the hinged support) $X = 2kPa/h$ and vertical reaction $Y = (4k-1)P/2$. In the case of loading by one force (Figure 1), these reactions have the form $X = (1-(-1)^k)Pa/(2h)$, $Y = P/2$.

When the upper belt is loaded, the coefficients of the solution (1) have the form

$$C_1 = (4(33-40(-1)^k)k^4 - 4(39(-1)^k + 1)k^3 + 4(22(-1)^k - 57)k^2 + 2(15(-1)^k + 8)k - 57(-1)^k + 102)/3,$$
$$C_2 = 64(1-(-1)^k)k^4 - 32((-1)^k + 1)k^3 + 6(4(-1)^k - 9)k^2 + 4((-1)^k + 6)k + (-1)^k + 7,$$
$$C_3 = 2(64(1-(-1)^k)k^4 - 4(4(-1)^k + 11)k^3 + 67((-1)^k - 1)k^2 + (7(-1)^k + 29)k - 15(-1)^k + 12)/3.$$

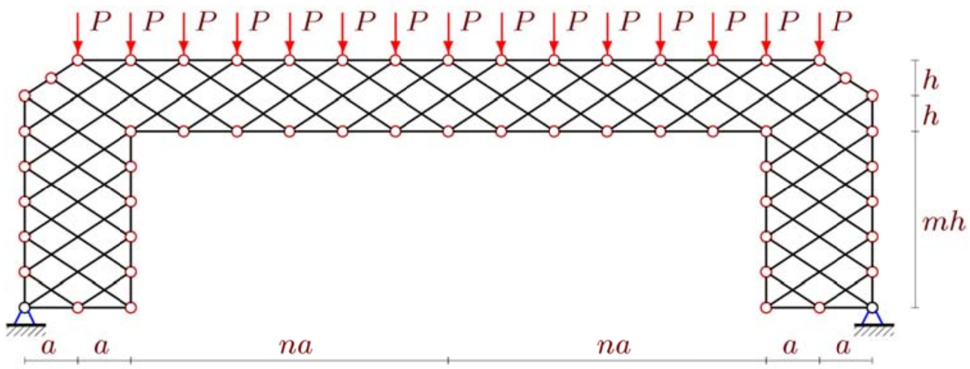

Fig. 4. Truss with load on the upper belt: n = 6, m = 5

A characteristic feature of the solution obtained is the strong jumps in the deflection when the number of panels changes. This is due to the presence in the coefficients of the terms with a factor $(-1)^k$. That is why to evaluate the deformation it is necessary to calculate the deflection at several neighboring nodes.

An overview of some papers containing the derivation of analytical dependences for the arch of arch and frame structures is given in [8].

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