Acta Astronautica 150 (2018) 39–43

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

# Thermo-mechanical loads on a spacecraft in low Earth orbits

V.B. Bednova *, M.V. Yumashev **

*Moscow M.V. Lomonosov State University, Leninskie Gory 1, Moscow, 119992, Russia*

A R T I C L E I N F O

*Keywords:*
Space
Debris
Flux
Micro-particles
Thermomechanical
Failure

A B S T R A C T

The present paper develops model for evaluating the results of high energy micro-particle space debris irradiation of space structures. The closed form solution formulas for determining temperature fields, thermomechanical failure are presented. Fracture of brittle materials under local heat exposure is studied.

## 1. Introduction

The issue of space debris may reach the so-called "cascade effect" in mid-term perspective, when the main mechanism of debris self-production will be caused by collisions between pieces of debris [1–6]. The cascade effect, along with orbital explosions, can result in the emergence of space debris fields composed of ultra-small particles, which would not cause significant damage to spacecraft on their own in case of individual collisions. But given a sufficient density of the debris field, conversion of kinetic energy of moving particles on collision with the spacecraft can result in substantial emission of thermal energy, causing major local heat release on the spacecraft surface. Non-uniform heating of the spacecraft body results in thermomechanical stresses generation, which, in turn, can cause local damage or destruction, causing the loss of the spacecraft. As for now, most of protection strategies are aimed at preventing the spacecraft loss due to concentrated momentum application [7–10]. However, one can imagine an impact flux of micro-particles to a spacecraft surface at a speed of 10 km/s, with a high collision frequency. The impact of each particle does not create enough momentum to destroy the shield, but when the particle brakes on the surface, a huge amount of energy could be released (about 1 MW). In this case, most of the energy is released in the form of heat, which leads to a significant local heating of the surface.

The present study describes the mechanisms of thermo-mechanical stressed state generation in structural elements being a result of local heating and suggests a possible way to mitigate the threat. The paper develops model for evaluating the results of high energy micro-particle space debris irradiation of space structures. The closed form solution formulas for determining temperature fields and thermo-mechanical failure criteria are suggested.

## 2. Fracture of brittle materials under local heat exposure

The area of the particle flow impact in this statement of the problem is considered as a boundary region for determining the stresses and temperatures in the entire structural element.

Let's consider an infinite disk with a circular interaction area of radius $a$ (Fig. 1). Let's enter into consideration cylindrical system of coordinates $(r', \varphi', z')$.

It is considered that the thickness of a disk $2h$ is small (relative to radius). On inner surface $(r' = a)$ body condition is

$$\theta(r', \varphi', z')\big|_{r'=a} = \theta_0, \tag{1}$$

where $\theta_0$ — constant. It is considered that temperature $\theta$ changes only in a direction of a basic vector $r'$.

According to the approached method developed in works [11–14], [17,18], it is entered concentric circle of radius $l'(\tau)$, on which conditions of interface of border of heated-up and cold zones are satisfied

$$\left.\frac{\partial\theta}{\partial r'}\right|_{r'=l'} = 0, \tag{2}$$

$$\theta\big|_{r'=l'} = 0. \tag{3}$$

* Corresponding author.
** Corresponding author.
*E-mail addresses:* msu.mmf.vb@gmail.com (V.B. Bednova), yumashevmikhail@gmail.com (M.V. Yumashev).

https://doi.org/10.1016/j.actaastro.2018.02.041
Received 15 February 2018; Accepted 25 February 2018
Available online 6 March 2018

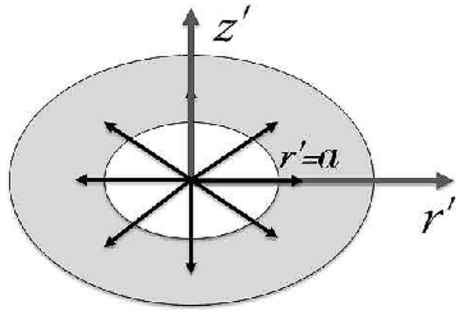


**Fig. 1.** A thin disk. A cylindrical coordinate system is introduced. Axis $r^{'}$ is directed from the geometric center. The temperature condition is given on the surface of a circular interaction area $r^{'} = a$.

Let's enter dimensionless parameters

$$T(r,t)=\frac{\theta-\theta^*}{\theta_m-\theta^*},\ r=\frac{r^{'}}{a},\ l=\frac{l^{'}}{a},\ t=\frac{\tau}{\tau_0},\ \tau_0=\frac{\rho\cdot c\cdot a^2}{k},$$

where $\theta^*$ — initial temperature, $\theta_m$ — melting point temperature, $a$ — inner radius, $\rho$ — density, $c$ — specific thermal capacity, $k$ — heat conductivity factor.

The heat conductivity equation in dimensionless variables in cylindrical system of coordinates for a one-dimensional case becomes:

$$L(T)=\frac{\partial T}{\partial t}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0. \tag{4}$$

The equation decision we will search in a following kind:

$$T(r,t)=\begin{cases} T_0\left(\frac{l(t)-r}{l(t)-1}\right)^2, & 1\le r\le l(t)\\ 0, & r>l(t),\end{cases} \tag{5}$$

where function $l(t)$ is determined from a condition of integrated satisfaction of the equation of heat conductivity:

$$\int_1^{l(t)}L(T)rdr=0. \tag{6}$$

Substituting (5) in (6) and integrating, we receive the equation

$$(l^2-1)\frac{dl}{dt}=12, \tag{7}$$

in which we obtain dependence $l(t)$ on time:

$$\frac{l^3}{3}-l=12t-\frac{2}{3}. \tag{8}$$

So, equations (5) and (8) give us analytical solution of the equation of heat conductivity for temperature field in an infinite thin disk.

## 3. Stress-strain state in a thin disk

Let's consider a problem of definition of stresses and deformations in an infinite thin disk with a circular interaction area of radius $a$. It is considered that the boundary of interaction area has been formed at the expense of instant evaporation of a material, and the border temperature $T_0$ is known. Temperature distribution depends only on radial coordinate $r'$, and the disk surface is free from loadings.

### 3.1. Elastic solution

At the moment $t_0$ the temperature on the interaction area surface has been changed immediately to $T_0$, and further this temperature is supported by an external source at constant level. In this case plane stress conditions will be realized. As shown, for example, in work [16], in such assumptions the stress components have the form

$$\tilde{\sigma}_{r^{'}}=-\frac{\alpha E}{r^{'2}}\int_a^{r^{'}}\theta r^{'}dr^{'}+\frac{EC_1}{1-\nu}-\frac{EC_2}{(1+\nu)r^{'2}}, \tag{9}$$

$$\tilde{\sigma}_{\varphi^{'}}=\frac{\alpha E}{r^{'2}}\int_a^{r^{'}}\theta r^{'}dr^{'}-\alpha E(\theta-\theta_0)+\frac{EC_1}{1-\nu}+\frac{EC_2}{(1+\nu)r^{'2}}, \tag{10}$$

$$\tilde{\sigma}_{r^{'}\varphi^{'}}=0. \tag{11}$$

The boundary conditions are:

$$\tilde{\sigma}_{r^{'}}|_{r^{'}=a}=0, \tag{12}$$

$$\tilde{\sigma}_{r^{'}}|_{r^{'}\to\infty}=0, \tag{13}$$

$$\tilde{\sigma}_{\varphi^{'}}|_{r^{'}\to\infty}=0. \tag{14}$$

Taking into account boundary conditions and introducing dimensionless stresses $\sigma=\frac{\tilde{\sigma}}{E\alpha(\theta_m-\theta_0)}$ we obtain next expressions:

$$\sigma_r=-\frac{1}{r^2}\int_1^r Tr\,dr, \tag{15}$$

$$\sigma_\varphi=\frac{1}{r^2}\int_1^r Tr\,dr-T. \tag{16}$$

### 3.2. An elastic-plastic solution

There are occur plastic deformation due to the strong heating of the region around the area of impulsive action. Consider a perfect plasticity model with the Tresca criterion for the principal dimensionless stresses ordered as $\sigma_1>\sigma_2>\sigma_3$:

$$\frac{|\sigma_1-\sigma_3|}{2}=\sigma_T, \tag{17}$$

where $\sigma_T$ – is the dimensionless yield strength of a strongly heated material.

Taking into account the results of the elastic solution the Tresca flow law yields:

$$|\sigma_\varphi|=\sigma_T. \tag{18}$$

Let $r=\lambda$ – is coordinate which divides plastic and elastic regions in the sample.

Let consider the equilibrium equation in the plastic region ($1<r<\lambda$):

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\varphi}{r}=0. \tag{19}$$

Solving (19) with regard (18) and with boundary condition $\sigma_r|_{r=1}=0$, we obtain a distribution of radial stress in the plastic region:

$$\sigma_r^p=\sigma_T\left(1-\frac{1}{r}\right),\ 1\le r<\lambda. \tag{20}$$

Thereby in the elastic region $r>\lambda$

$$\sigma_r^e=-\frac{1}{r^2}\int_\lambda^r Tr\,dr+C_1-\frac{C_2}{r^2},\quad \sigma_\varphi^e=\frac{1}{r^2}\int_\lambda^r Tr\,dr-T+C_1+\frac{C_2}{r^2} \tag{21}$$

and in the region of plastic flow adjacent to the area of heating the relations (18) and (20). From the condition $\sigma_r|_{r\to\infty}=0$ we obtain $C_1=0$. Constant $C_2$ and coordinate which divides plastic and elastic regions $\lambda$ obtained from the conditions on the boundary of elastic and plastic regions ($r=\lambda$):

$$\sigma_\varphi^e = \sigma_T,\ \sigma_r^e = \sigma_r^p \ at\ r = \lambda. \tag{22}$$

Then stress state will be described by next expressions:

$$\sigma_r = \begin{cases} \sigma_T\left(1-\frac{1}{r}\right), & 1 \le r \le \lambda \\ -\frac{1}{r^2}\int_1^r Tr\,dr + \frac{\lambda^2}{r^2}\left(\frac{1}{\lambda}\int_1^\lambda Tr\,dr + \sigma_T\left(1-\frac{1}{\lambda}\right)\right), & \lambda \le r \le l \\ -\frac{1}{r^2}\int_1^l Tr\,dr + \frac{\lambda^2}{r^2}\left(\frac{1}{\lambda}\int_1^l Tr\,dr + \sigma_T\left(1-\frac{1}{\lambda}\right)\right), & r > l, \end{cases} \tag{23}$$

$$\sigma_\varphi = \begin{cases} \sigma_T, & 1 \le r \le \lambda \\ -T + \frac{1}{r^2}\int_1^r Tr\,dr + \frac{\lambda^2}{r^2}\left(\frac{1}{\lambda}\int_1^\lambda Tr\,dr + \sigma_T - T(\lambda)\right), & \lambda \le r \le l \\ \frac{1}{r^2}\int_1^l Tr\,dr + \frac{\lambda^2}{r^2}\left(\frac{1}{\lambda}\int_1^l Tr\,dr + \sigma_T\right), & r > l. \end{cases} \tag{24}$$

The comparison of elastic and elastic-plastic solutions for circumferential stresses is shown in Fig. 2.

## 4. Brittle fracture in the sample

Let us consider the problem of heating an infinite disk with a circular interaction area. During fast change in temperature on the surface of interaction area time-depended stress state arises inside the sample. This state is characterized by the fact that the circumferential tensile stresses $\sigma_\varphi$ arise, which can reach values of tensile strength [15]. After this moment the brittle fracture occurs. For analyzing the process of fracture in time we use the idea of a front of the wave of fracture [2,3]. In the fracture zone circumferential stresses equal to zero and redial stresses can be defined from equilibrium equation. On the boundary between damaged and solid zones circumferential stresses equal to tensile strength and for redial stresses the junction conditions are used.

From the analysis of relations for the stresses, obtained earlier, it follows that the maximum value of $\max\{\sigma_\varphi\}$ as a function of dimensionless coordinate $r$ is achieved if $r < l(t)$. Note that the absolute maximum of the dimensionless stresses on the dimensionless variable $r$ and $l(t)$ can be determined by solving the following equations:

$$\frac{\partial\sigma_\varphi}{\partial r} = 0, \quad \frac{\partial\sigma_\varphi}{\partial l} = 0. \tag{25}$$

Hence, we obtain two algebraic equations for finding critical values of $r$ and $l$ for which $\sigma_\varphi$ is maximum value.

If at some point in time, i.e., at a certain parameter $l = l_p$, stress component $\sigma_\varphi$ reaches the limit strength of $\sigma_p$, then the formation of the fracture zone is appeared, the boundaries of which $a(t)$ and $b(t)$ determined by the junction conditions for the stresses that are introduced below.

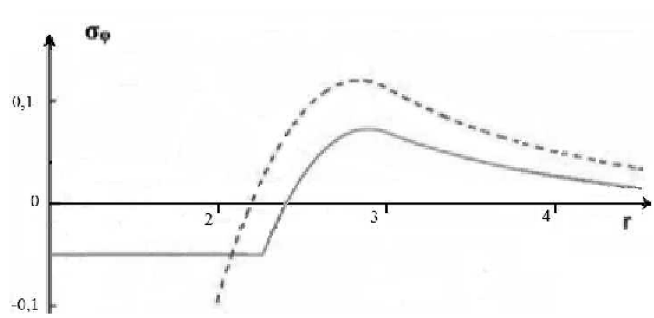


**Fig. 2.** A comparison of the elastic (dotted line) and elastic-plastic (continuous line) solutions for circumferential stresses in a thin disk with a circular interaction area $r = a$ for one moment of time.

We find that the zone of destruction is developing quickly, covering some domain. Therefore, all space is divided into three volume: the first corresponding to $1 < r < a$ – solid body, a second, corresponding to a$< r < b$ – zone with radial cracks and the third, where $b < r$, again a solid body (see Fig. 3).

Thus, the stress state of the sample with a brittle destruction zone is determined by the system of formulas:

$$\sigma_r^1 = -\frac{1}{r^2}\int_1^r Tr\,dr + C_1^1 - \frac{C_2^1}{r^2}, \tag{26}$$

$$\sigma_r^2 = \frac{C_1^2}{r}, \tag{27}$$

$$\sigma_r^3 = -\frac{1}{r^2}\int_1^r Tr\,dr + C_1^3 - \frac{C_2^3}{r^2}, \tag{28}$$

$$\sigma_\varphi^1 = \frac{1}{r^2}\int_1^r Tr\,dr - T + C_1^1 + \frac{C_2^1}{r^2}, \tag{29}$$

$$\sigma_\varphi^2 = 0, \tag{30}$$

$$\sigma_\varphi^3 = \frac{1}{r^2}\int_1^r Tr\,dr - T + C_1^3 + \frac{C_2^3}{r^2}. \tag{31}$$

Six unknown constants $C_1^1,\ C_2^1,\ C_1^2,\ C_2^2,\ C_1^3,\ C_2^3$ and two functions of time $a(t)$, $b(t)$ we can find from the following conditions

$$\sigma_r^1\big|_{r=1} = 0, \tag{32}$$

$$\sigma_r^1\big|_{r\to\infty} = 0, \tag{33}$$

$$\sigma_r^1\big|_{r=a} = \sigma_r^2\big|_{r=a}, \tag{34}$$

$$\sigma_r^2\big|_{r=b} = \sigma_r^3\big|_{r=b}, \tag{35}$$

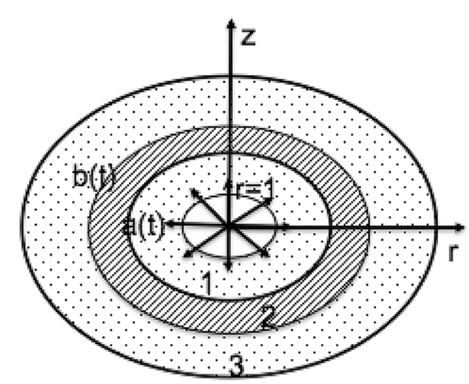


**Fig. 3.** The location of the three calculated zones for the stresses in a disk with a circular interaction area.

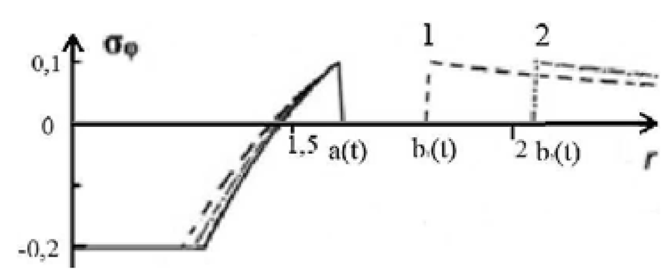


**Fig. 4.** Stresses in the disk for two moments of time (1–0,01, 2–0,02) after the formation a damage zone in the region a.$(t) < r < b_i(t)$, $i = 1, 2$.

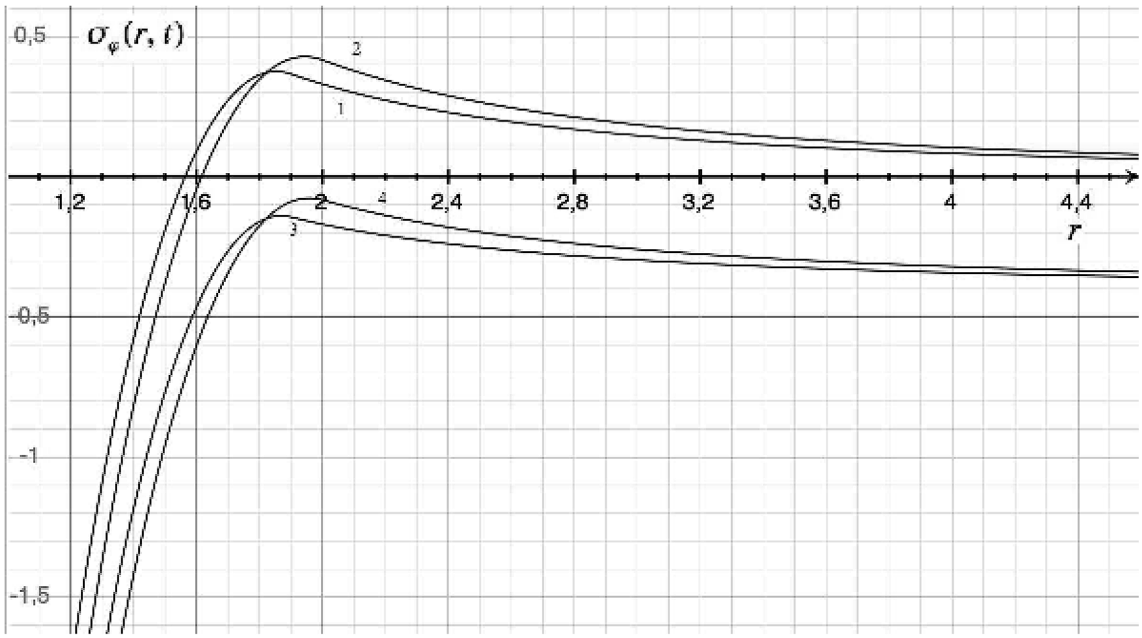


**Fig. 5.** A comparison of circumferential stresses without additional pressure (lines 1, 2) and with additional pressure $\sigma_0$ (lines 3, 4) for two moments of time: $t = 0.01$ (lines 1, 3) and $t = 0.02$ (lines 2, 4).

$$\sigma_\varphi^1\Big|_{r=a} = \sigma_p, \tag{36}$$

$$\sigma_\varphi^3\Big|_{r=b} = \sigma_p, \tag{37}$$

$$u^1\big|_{r=a} = u^2\big|_{r=a}, \tag{38}$$

$$u^2\big|_{r=b} = u^3\big|_{r=b}. \tag{39}$$

Numerical analysis of equations (26)-(39) shows that at the time $l_p$ when $\sigma_\varphi$ reaches a value $max\{\sigma_\varphi\}$, instantly damage zone arises of very small diameter.

At $l_p + 0$ when analyzing the development process of the fracture should put $a = const$. Solving the system now with the condition $a = const$, we obtain the dependence for the determination of $b$ as a function of time. Results of numerical estimations of the damage zone over time are shown in Fig. 4.

Consideration both the fact of the destruction and the emergence of plastic flow is essential. The system of equations for this case becomes even more complicated. The equations of system (26)–(39) must be considered together with system (23). Analyzing the results, we conclude that the inclusion of nonlinear material properties significantly affects the description of the fracture process.

## 5. A method of preventing the destruction of the disk

Consider the problem of the disk with an interaction area where the border is set uniformly distributed pressure. The boundary conditions will be

$$\sigma_r\big|_{r\to\infty} = \sigma_0, \tag{40}$$

$$\sigma_r\big|_{r=1} = 0. \tag{41}$$

This is a problem Lame whose solution is:

$$\sigma_r = \sigma_0\left(1 - \frac{1}{r^2}\right), \tag{42}$$

$$\sigma_\varphi = \sigma_0\left(1 + \frac{1}{r^2}\right). \tag{43}$$

To obtain the solution of the original problem of the heating of the disk on a small the central region, now taking into account specified on the outer pressure boundary $\sigma_0$, must be added to the expressions for stress of (15) and (16) expressions obtained from (42) and (43):

$$\sigma_r = \begin{cases} -\frac{1}{r^2}\int_1^r Tr\,dr + \sigma_0\left(1 - \frac{1}{r^2}\right), & 1 \le r \le l \\ -\frac{1}{r^2}\int_1^l Tr\,dr + \sigma_0\left(1 - \frac{1}{r^2}\right), & r > l, \end{cases} \tag{44}$$

$$\sigma_\varphi = \begin{cases} -T + \frac{1}{r^2}\int_1^r Tr\,dr + \sigma_0\left(1 + \frac{1}{r^2}\right), & 1 \le r \le l \\ \frac{1}{r^2}\int_1^r Tr\,dr + \sigma_0\left(1 + \frac{1}{r^2}\right), & r > l. \end{cases} \tag{45}$$

As seen in Fig. 5, due to an additional pressure can reduce the maximum tensile stress, and thus prevent the onset of fracture. Similarly, you can get the optimization problem for the maximum tensile stresses.

## 6. Conclusion

We have determined the stress state in a disk caused by a time-dependent temperature effects on the interaction area in the frame of the elastic-plastic model.

A new analytical model of brittle fracture of the sample, taking into account nonlinear material properties are obtained.

We propose a method for suppressing undesirable thermal stresses caused by local temperature changes due to the impact on the structural element of intense energy flows.

## Acknowledgements

This research is carried out with the assistance of RFFI (The Russian Foundation for Basic Research) Grant 18-01-00151a.

## References

[1] N.N. Smirnov, A.I. Nazarenko, A.B. Kiselev, Modelling of the space debris evolution based on continua mechanics, Space Debris 473 (2001) 391–396.

[2] N.N. Smirnov, A.B. Kiselev, M.N. Smirnova, V.F. Nikitin, Space traffic hazards from orbital debris mitigation strategies, Acta Astronaut. 109 (2015) 144–152. https://doi.org/10.1016/j.actaastro.2014.09.014i.

[3] Vitaly Adushkin, Stanislav Veniaminov, Stanislav Kozlov, Alexandr Doronin, Mikhail Silnikov, Natural and technogeneous contamination of near-Earth space, Acta Astronaut. 135 (2017) 6–9.

[4] N.N. Smirnov, A.B. Kiselev, A.I. Nazarenko, Mathematical Modeling of Space Debris Evolution in the Near Earth Orbits Moscow University Mechanics Bulletin, Allerton Press, 2002, pp. 33–41 (Vestnik Moskovskogo Universiteta. Ser. 1 Matematika Mekhanika) (4).

[5] Vitaly Adushkin, Stanislav Veniaminov, Stanislav Kozlov, Mikhail Silnikov, Orbital missions safety – a survey of kinetic hazards, Acta Astronaut. 126 (2016) 510–516, https://doi.org/10.1016/j.actaastro.2015.12.053.

[6] M. Yu, V.A. Sotskiy, V.V. Veldanov, Selivanov, Growth in the quantity of debris in Space as an effect of mutual mechanical collisions of various types, Acta Astronaut. 135 (2017) 10–14.

[7] D.V. Panov, M.V. Silnikov, A.I. Mikhaylin, I.S. Rubzov, V.B. Nosikov, E.Yu. Minenko, D.A. Murtazin, Large-scale shielding structures in low earth orbits, Acta Astronaut. 109 (2015) 153–161. https://doi.org/10.1016/j.actaastro.2014.12.009.

[8] N.N. Smirnov, A.B. Kiselev, K.A. Kondratyev, S.N. Zolkin, Impact of debris particles on space structures modeling, Acta Astronaut. 67 (3) (2010) 333–343. https://doi.org/10.1016/j.actaastro.2010.03.003.

[9] N.N. Smirnov, K.A. Kondratyev, Evaluation of craters formation in hypervelocity impact of debris particles on solid structures, Acta Astronaut. 65 (2009) 1796–1803, https://doi.org/10.1016/j.actaastro.2009.04.003.

[10] M.N. Smirnova, K.A. Kondratev, Space debris fragments impact on multi-phase fluid filled containments, Acta Astronaut. 79 (2012) 12–19.

[11] G.I. Barenblatt, On some approximate methods in the theory of one-dimensional unsteady filtration in the elastic drive regime, A Journal of Russian Academy of Sciences, Division of Technical Sciences 9 (1954) 108–112.

[12] M.S. Baharev, L.I. Mirkin, S.A. Shesterikov, M.A. Yumasheva, Structure and Strength of Materials with the Laser Action, Moscow University Press, Moscow, 1988.

[13] S.S. Grigorian, Some problems of the mathematical theory of deformation and fracture of hard rocks, PMM 31 (4) (1967) 643–669.

[14] A.M. Lokoshchenko, The Creep and the Long-term Strength of Metals in the Aggressive Medium, Moscow University Press, Moscow, 2000, 178 pp.

[15] L.I. Mirkin, S.A. Shesterikov, M.V. Yumashev, M.A. Yumasheva, Instability of thermal fracture under the conditions of constrained deformation, Physico-Chemical Mech. Mater. (6) (2006) 55–60.

[16] S.P. Timoshenko, J.N. Goodier, Theory of Elasticity, Nauka, Moscow, 1975.

[17] S.A. Shesterikov, M.A. Yumasheva, To the problem of the temperature fracture with the fast heating, A Journal of Russian Academy of Sciences, Mech. Solid. (1) (1983) 128–135.

[18] M.A. Biot, New method in heat flow analysis with application to flight structures, J. Aeronaut. Sci. 24 (1957) 857–873.