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Micromechanics of discontinuities and high porosity bands formation in the unconsolidated sedimentary rocks

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Abstract. Formation of thin macrofractures and high porosity bands parallel to the compression axis in the unconsolidated sedimentary rocks is treated on the basis of unified approach by considering migration of microdefects (pores) with respect to particles of the medium. The migration of pores is driven by a common cause, namely, a trend of a system to lower its total energy. The mechanism of how discontinuities develop along the maximum compressive stress $T_{\max}$ is discussed and quantitatively investigated. A single pore splits into two separate holes which move away along the $T_{\max}$ axis. The trace left by moving hole is interpreted as a macro-discontinuity. Multiple pores migrate so that they form a system of chains extending along the $T_{\max}$ axis. We associate these chains with observed high porosity bands.

Introduction

Uniaxial compression of rocks leads to the formation of macrofractures (extremely thin discontinuities) and high porosity bands parallel to the compression axis. Usually formation of cracks along compression is treated in the framework of the brittle fracture mechanics by considering micro-defects or inclined micro-cracks as sources of initiation of the Mode I crack parallel to maximum compressive stress $T_{\max}$ (see, [1] and references therein). The standard model of the formation of high porosity bands associates them with the narrow zones of unstable localization of plastic deformations [2]. It is assumed that transverse tensile deformations are localized within the band, which cause the band to dilate. We propose an alternative approach, which explains formation of both discontinuities and high porosity bands from a unified viewpoint. As a unique source of initiation of these discontinuities in unconsolidated sediments (mostly in sands and loose sandstones) we consider the microdefects in sediment packing (pores) which are able to migrate through the geomaterial. The migration of pores is driven by a common mechanism, namely, a trend of a system to lower its total energy (small variations in total energy are equal to the increment of free energy minus the work of external forces).

Pore driving force

Consider an elastic body B bounded in a reference configuration κ by a relatively smooth surface S possessing the outward oriented surface element $d\mathbf{s}$. Let the body B be quasi-statically deformed in such a manner that its actual self-equilibrated configuration undergoes small continuous variation concurrently with variation of κ which is characterized by change of S described by the surface vector field $\delta\mathbf{x}$. In the absence of body forces infinitesimal variation δW_B of the elastic strain energy W_B of the body B can be written in the form

$$\delta W_B = \delta A + \int_S w\delta\mathbf{x} \cdot d\mathbf{s} \tag{1}$$

where δA is the work done on the body B, the second term in the right hand side represents the energy flux due to absorbed or escaped material particles, w stands for the density of W_B per unit volume in the reference configuration.

We are interesting in a special case of the body B containing defect in the form of a pore possessing the traction free surface S in configuration κ. During variation the pore is shifted in the reference configuration κ as a 'rigid whole' by the vector $\delta\mathbf{x}=const$ with respect to material points (Fig. 1). Then, variation δE of the total energy of the system in view of (1) can be expressed as

$$\delta E_B = \delta W_B - \delta A = -\delta\mathbf{x} \bullet \mathbf{f}, \quad \mathbf{f} = -\int_S w d\mathbf{s}. \tag{2}$$

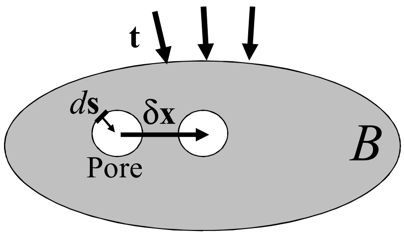

Fig. 1. Infinitesimal variation of the microdefect (pore) position.

According to Eshelby [3], vector $\mathbf{f}$ is the force acting on a defect. From equation $\delta E = -\delta\mathbf{x} \bullet \mathbf{f}$ which is valid both for finite and infinite bodies it follows that the total energy diminishes ($\delta E < 0$) if the displacement vector $\delta\mathbf{x}$ makes an acute angle with the $\mathbf{f}$ direction. In what follows calculations are restricted to the case of linear elasticity and the plane strain state. The pore is represented as a traction free circular hole of radius a. Then for the strain energy density w around the hole one has $w = (1-\nu)T_\theta^2/4\mu$ where μ is the shear modulus, ν is the Poisson's ratio, and T_θ is the circumferential normal stress along the surface of the hole.

Evolution of a single pore

Consider a plane x_1, x_2 subjected to compressive stresses $p_1 >> p_2 > 0$ at infinity. Let two equal circular holes of radius $r=a$ be spaced $2l$ apart along the x_1 axis (Fig. 2). The following cases are possible: (*i*) coinciding holes ($l=0$), (*ii*) overlapping holes ($l<a$), and (*iii*) separated holes ($l>a$). These cases in the framework of our model correspond to different stages of the individual discontinuity evolution, namely, to splitting of an isolated hole into two separate holes followed by their mutual moving apart accompanied by leaving the trace $|x_1| \le l-a,\ x_2 = 0$ behind them.

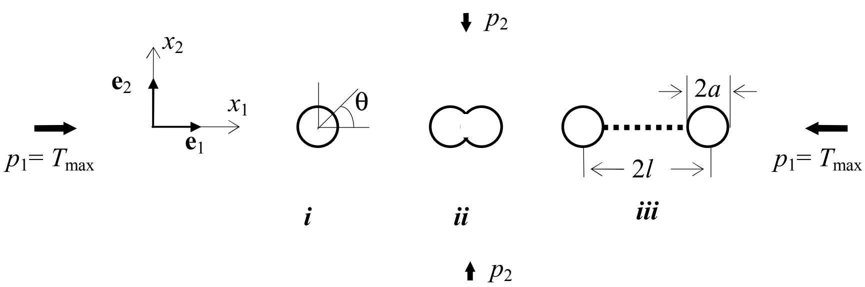

Fig. 2. Various stages of the defects splitting and moving apart along the maximum compressive stress.

Suppose the trace left by the moving holes be completely closed. Then the known solutions of plane elasticity for two equal circular holes can be used for the determination of $\mathbf{f}$. For example, when $l=0$ the force $\mathbf{f}$ acting on the right half-round is equal to $\mathbf{f} = \mathbf{e}_1 c(46p_1^2 - 52p_1p_2 + 126p_2^2)/15$, where $c = a(1-\nu)/4\mu$. For $\varepsilon = a/2l < 0.25$ the force $\mathbf{f}$ varies inversely with cube of the distance between the holes and for the right hole is expressed in the form

$$\mathbf{f} = \mathbf{e}_1 16\pi c\varepsilon^3 (p_1 + p_2)(2p_1 - p_2) + O(\varepsilon^4). \tag{3}$$

Inner product $f = \mathbf{f} \bullet \mathbf{e}_1$ with the $\mathbf{f}$ calculated for the right hole represents the ratio of the released total energy to increase of distance between the holes, i.e. $f = -\delta E / \delta 2l$ ($\delta \mathbf{x} = \mathbf{e}_1 \delta l$). Of interest is the qualitative analogy between defects and electric charges. One can conclude that holes lying along the maximum compressive stress p_1 behave as though they are like charges. They repel each other creating a closed macro-discontinuity. By contrast, holes located along the line perpendicular to the p_1 axis attract each other, at least when the holes are kept well apart.

To take into account more realistic factors let us introduce a force $\mathbf{f}_c$ resisting the defect motion. We assume that under conditions of quasi-static defect motion the released total energy is expended on creating a new surface (i.e., the surface of the closed 'trace'). For definiteness, let the defect have been moved along the p_1 axis. This, with $2l$ taken as the trace length, yields $-\delta E = 4\gamma\delta l$ where γ is the density of the effective surface energy. The resistance force for the right hole is $\mathbf{f}_c = -f_c \mathbf{e}_1$ ($f_c = 2\gamma \geq 0$). The hole keeps still if $f \leq f_c$. If $f > f_c$, defect dynamically propagates. For nearly uniaxial stress state nothing changes in the microstructure if $p_1 \leq (0.65\gamma / c)^{1/2}$. Actual splitting of the defect into two separate ones takes place if p_1 reaches its critical value $p_1{}^* = (1.12\gamma / c)^{1/2}$. The basics of the approach above have been proposed in [3, 4].

Evolution of multiple pores

Let us consider the problem of the co-migration of N pores in an infinite elastic plane under the action of compressive stress p along the vertical axis x_2 [5]. In order to solve this problem, the concept of a driving force on a defect must be expanded on a set of N pores possessing positions $\mathbf{x}_j$, j=1,...,N. In this case the variational equation $\delta E = -\delta \mathbf{x} \bullet \mathbf{f}$ evidently recasts in the form

$$\delta E = -(\delta \mathbf{x}_1 \bullet \mathbf{f}_1 + \delta \mathbf{x}_2 \bullet \mathbf{f}_2 + \ldots + \delta \mathbf{x}_N \bullet \mathbf{f}_N) \tag{4}$$

where $\delta \mathbf{x}_j$ is a potentially possible displacement of the j^{th} pore, $\mathbf{f}_j$ is a force acting on the j^{th} pore from the other $N-1$ pores, i.e. $\mathbf{f}_j = \mathbf{f}_{1,j} + \ldots + \mathbf{f}_{j-1,j} + \mathbf{f}_{j+1,j} + \ldots + \mathbf{f}_{N,j}$. In what follows we analyze the case $\varepsilon_{i,j} = a / |\mathbf{x}_i - \mathbf{x}_j| < 0.25$ in order to make use of Eq. 3 which is used in the generalized form [5]

$$\mathbf{f}_{i,j} = f^* (\varepsilon_{i,j})^3 (-\mathbf{e}_1 \cos\varphi_{i,j} + 2\mathbf{e}_2 \sin\varphi_{i,j}), \quad f^* = \pi a (1-\nu) p^2 / 2\mu, \tag{5}$$

where $\varphi_{i,j}$ is the inclination of the vector $\mathbf{x}_j - \mathbf{x}_i$ to the x_1 axis. As before, the necessary condition for j^{th} pore migration is $|\mathbf{f}_j| > f_c$.

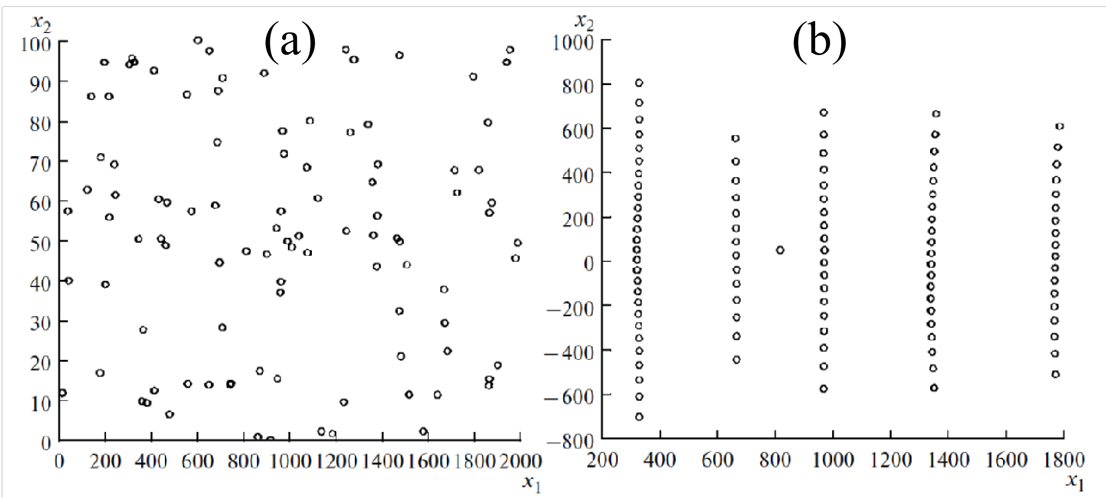

Fig. 3. Migration of pores under vertical compression: (a) the initial pore distribution in the domain Ω; (b) the resulting pore distribution after the completion of migration of all pores.

At the initial state, a random spatial distribution of N pores having a diameter $2a$ was specified in some bounded domain Ω (Fig. 3 a). At each iteration for each j^{th} pore the driving force on a defect, $\mathbf{f}_j$, was calculated, which is caused by the influence of all other pores. The position of the pore was varied along the direction of the acting force $\mathbf{f}_j$ if $|\mathbf{f}_j|>f_c$. The iterative process for the given initial conditions was terminated when the criterion $|\mathbf{f}_j|\leq f_c$ had been met for all pores. Our calculations showed that the migration of pores results in the formation of a relatively regular structure composed of quasi-parallel linear chains extended along the axis of compression (Fig. 3 b). We associate these formations with the systems of high porosity bands. Several series of calculations were carried out. Each series was characterized by its own values of N, f_c, and spatial average pore density, ρ. With fixed N, the pore density ρ was varied by changing the size of the initial domain Ω within which the pores are distributed. Each series of calculations included 960 case computations of the formation of bands. The cases within the series differ by the initial random distribution of pores. For each series, the frequency histograms were compiled for distances h between the high porosity bands. Our results show that the initial pore density ρ (excluding its critically small values) does not have any effect on the shape of the histogram. In particular, this means that the typical distance h between the bands does not depend on ρ. Quite the opposite, variation in the resisting force, f_c, with the other conditions being the same, drastically changes the frequency distribution of distances h. As f_c decreases, h increases; the histograms become more diffused, and their maxima become lower.

Conclusion

The proposed model of discontinuities formation has been verified by discovering systems of extremely thin (0.01-0.1 mm wide) straight lines on the surfaces of some sedimentary layers (e.g., at outcrops of Upper Cretaceous rocks located along the Black Sea coast near Novorossiysk, Russia [5]). Slightly ajar near the surface, these lines in many cases penetrate deep into the rock forming thin planes of closed discontinuities orthogonal to bedding. Basing on various studies, including electron microscopy, we conclude that the observed discontinuities are the remnants of discontinuities originated back in not yet lithified sediments.

According to our model of formation of bands possessing increased porosity, the pores rather than deformations are localized in the narrow zones. An important feature of the model is that the formation of a high porosity band does not have a sense of a loss of stability. Quite the contrary, their formation is treated as a gradual process spread over time. The formation of the systems of high porosity bands is a direct sequence of the model used, without any assumptions on the existence of such systems and any special tuning of the model parameters. Moreover, based on the suggested model, we can predict such bands to always occur in the form of regular systems.

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