Estimation of discount rates for assessing the efficiency of investment projects in the framework of optimal financial asset management.

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Abstract

Singled out for this study is a model of optimal asset management for a firm in a deterministic situation (i.e., given full information) that would allow to refine and modify a traditional definitions of discount rate as the highest rate of return on alternative investment opportunities.

It is shown that, in general, the discount rate (determined as the rate of decline in the value of money in the optimal plan) is time-variable and depends not only on the general situation in the market, but also on the financial position of the firm.

The constructed model provides opportunity for formulating the precise conditions for financial feasibility of a project and justifying the structure of its efficiency criterion. It appears that the criterion of efficiency in question differs from the net present value (NPV) -- in addition to the discounted sum of cash flows from the project, it must take into account the market value of fixed assets created under the project.

We also explore the accounting issues of inflation, uncertainty and risk in evaluating the effectiveness of projects and setting the discount rate.

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Discount rate and investment opportunities

"The use of money is all the advantage there is in having it"
Benjamin Franklin
"Time and money are mostly interchangeable."
Winston Churchill

The usual definition of discount rate in investment analysis -- which holds that this rate represents the maximum rate of return on alternative investments into financial projects that are available to a firm -- is not sufficiently constructive: it doesn't make evident what alternative projects (investment opportunities) should be taken into account. This imprecision (and it is inherent to all other textbook definitions) creates difficulties in practical applications where the choice of discount rate is called into question.

It is often the case that the alternative available investments are the placement of money on a deposit or into long-term sovereign securities. So cases are there where the discount rate is set equal to a deposit rate or an yield on government bonds. However, less certain are the instances where the choice is to be made keeping in mind that these rates or yields vary over time, or where the returns on 1-year deposits are less than those for 2-year deposits, etc.

Furthermore, it is not at all clear why the above definition limits the domain of available investments to only financial investments. Why investments into, say, raising flowers can't be placed into this domain? And, generally, why can't investments into other investment projects permissibly be treated as alternative investments? It is indeed possible that a firm's 'portfolio' may contain other projects with the returns above those of the deposits. Why a project with the highest return can't be selected to provide an indication for the discount rate when evaluating any other project of that firm? It appears [1], this would result in an irrational behavior. This is seen in the example that follows.

Example 1. Idion Inc may invest into a deposit yielding 10% per annum, into Project A that requires investments of $100 and returns annual income of $30, or alternative Project B requiring investments of $250 and annually returning the income of $51. Given the discount rate of 10%, NPVs for Project A and Project B are, respectively, 30/0.1 - 100 = 200 and 51/0.1 - 250= 260. So Project B appears superior to Project A.

However, the Idion firm selects the discount rate to be equal to the return on Project A, i.e. 30%. Consequently, NPV for Project B becomes negative: 51/0.3 - 250 = -80. This would seem to necessitate the rejection of B. This conclusion may also seem to be reinforced by a consideration that taking the discount rate of 20%, i.e. equal to the return on B, would indicate a positive NPV for A amounting to 30/0.2 - 100 = 50. However, it is not beneficial for the firm to reject Project B. To see this, assume that the firm held a free cash balance of $300 at the moment it was selecting between the projects. What would be the outcome of the different investment decisions?

1) If Project A is to be implemented by investing $100 into it, the remaining balance of $200 will have to be placed on the deposit. Overall, this would bring the following annual income to the firm 30+200×0.1=50;
2) If Project B is to be proceeded with by investing $250, the remaining $50 would go into the deposit. It would thus create the following annual income to the firm: 51 + 50×0.1 = 56. Consequently, the firm would earn more by going forward with Project B, rather than rejecting it in favor of A.

As this example makes clear, the return on investments into the alternative project B can't be reflected in the discount rate in view of the fact that this project

is not divisible and replicable – investments into it can only be made in a fixed volume and at the fixed moment of time. On the other hand, when we were analyzing this example we had to place the "narrow" task of comparing the two alternatives into the context of a "broader" task of rationally managing free cash resources of the firm.

In this connection, the "traditional" understanding of the discount rate to which we alluded at the outset, we have seen, has required further specification, and in [24] this rate is already being interpreted as the *maximum return on alternative* ***investment directions*** *available to the investor*. "*Investment directions*" in this context are suggested to mean *divisible and replicable investment projects which lend themselves to having any volume of funds invested into them at any moment of time*.

Such definition would indeed exclude from its scope any specific project that can be realized only once. However, it also excludes "ordinary" investments of funds into deposits or securities. For example, a project "to place funds on a Citigroup deposit" would return one amount if undertaken today, and a totally different amount –if undertaken a year hence. This difficulty can be easily avoided: if the returns on alternative investments are different in each year, we can consider the discount rates to be time-variant as well. However, in this instance we would have to specify the definition further: the discount rate **specific to each step** should be defined as the *maximum return on alternative investment directions available* ***at a given step*** *to the investor,* whereas "investment directions" should be interpreted to mean *divisible and replicable investment projects which lend themselves to having any volume of funds invested into them* ***at a given step***.

Even that definition is not flawless: it is unclear what sense attaches to the notion of *return* on divisible and replicable projects, investments into which are distributed over time and that produce different amounts of income in different years.

Example 2. An investor puts up his funds into the acquisition of residential or office space: a part of the funds is invested at the stage when the building is being constructed, and a part of them goes into finishing and decorating the already constructed building. Subsequent to that, the investor lets out the space such that the rents he would receive tend to be variable over time, as well as the deposit rates and the rates attaching to other investment directions. In this situation it is unclear:

1) How to determine the return on such investment direction?
2) What is the benchmark for comparison to make sure that the given investment direction is the best one?
3) If it happens that the return in the given investment direction is the maximum one and it is accepted for the discount rate, what is the time reference for this rate (beginning of the construction, end of the finishings)?

Let us consider now that many projects often use loans to make investing into them feasible. Of course, cash inflows and outflows associated with the loans are accounted for in the cash flow for the project. However, loans can be there as well even *in the absence* of the project, and such possibility has to be somehow reflected in the discount rate, which is often ignored in the traditional approaches to the discount rate selection.

Evidently, the reason for such difficulties stems from the fact that the discount rate is not there in the environment in all its purity (thus it proves so challenging to provide any specific recommendations on how to "derive" it there from), but manifests itself with some aberrations in the observed financial indicators (futures quotations, interest rates, rates of return on different projects, etc.)

We shall make an inquiry into the above challenges and try to understand which specific characteristics of the economic environment have influence on the rate. The germ of the idea for our approach is already apparent from the example 1.: a correct understanding of the economic content and objective of the discount rate, which the firm should employ in evaluating the efficiency of its participation in investment projects, is only possible by aligning the rate with the rational financial policy of the firm. The development of such a policy is one of the elements in a much more comprehensive system of **strategic planning** [13] which embraces financial, investment and operational aspects of the company performance.

Such system provides a framework for long-term development objectives of the firm and ensures that the priorities are formulated for the attainment of these objectives subject to established (or voluntarily imposed) constraints on the firm's behavior. The interrelationship between investment decisions and strategic planning was originally emphasized in [3]. [26] provides the following description for this interrelationship: "As investment decisions are linked in a complex way with the long-term planning, they can't be regarded as a separate stand-alone activity. Along with any other strategic decisions, they should bear the imprint of such factors as technology, objectives, and internal environment of the firm… Do the investment decisions really represent the logical implication of much broader organizational objectives? The traditional model of the investment process assumes that this is the case. However, as has been shown, there is evidence that these relationships are not one-way, but imply a much more complex interaction between the strategy and the investment process. Investment decisions form an integral part of the strategic planning, but such that rather runs in parallel with it than flows out as its ramification. It is getting more and more obvious that investments into the business are inseparable from the complex mechanism of strategy formulation. The making of investment decisions cannot be properly carried out or understood outside of the entire organizational context or in isolation from persons designing the corporate strategy."

This also entails the need to analyze for real and financial sectors the issue of interconnected optimization of the investment decisions with the general operational activities of the firm. To such issue we will refer as "the optimization of financial policy of a firm". We shall start off with discussing the simplest case, where the findings can be obtained practically in the absence of any mathematical modelling.

Optimal financial policy in a perfect market. Market discount rate.

> "Assets offering perpetual cash-flow streams are like abominable snowmen: often referred to but seldom seen".

In this subsection we shall discuss the optimal financial policy of a firm operating in a context of some ideal (perfect) equilibrium market.

Suppose that in some planning period, split into steps $0, 1, \ldots, T$, there circulates in the market some quantity of financial titles (FT)[1] of different types, with each of them having a determined price (rate) at which it can be bought or sold (it is supposed that the purchasing price is equal to the selling price (bid and ask rates are equal)). (Bank) deposits opened up at different steps or durating for variable intervals of time we would consider as different types of FT. The use of FT entitles a firm to receive from it certain monetary receipts (for example, in the form of interest payments, dividends, coupon yields, or revenues). We shall call these **gross incomes**. It should be noted that some types of FT (say, bonds) give rise to a periodic gross income, while others (say, bills) generate gross income only at their sale (closing). Further, we will conditionally assume that a gross income arising from an acquired FT starts accruing as from the next step.

Let's assume as well that the quantity of FTs bought or sold by the firm at a step t is not limited and does not influence their prices (rates) and gross incomes. It means that such transactions would represent *divisible* and *replicable* projects – a circumstance which would allow treating them as the directions of investment.

Among the assets yielding income there exist such which are not exactly market assets. Evidently, a firm with the more qualified personnel will be able to function better, other things being equal, therefore "personnel investments" would, too, yield income. Moreover, such investments are duplicable to a certain extent (it is possible to train and employ two persons as well as ten). Why not to consider such for "the directions of investment"? Such question is not to be answered lightly. A number of economists consider it legitimate to include so-called human capital among the assets of a firm. However, it would not be possible to include human capital among FTs in our model, as the possibility of *sale* of any considered asset at any moment plays an essential role in our model. Given that with the abolition of slavery and serfdom assets like that cannot be put on sale, such assets can't be reckoned among "absolutely market assets". On the other hand, a firm does really have a latitude to run different human resource policies and the model presented in Section "Optimization of operating activities" provides for an opportunity to optimize such a policy.

If a firm has free monetary funds to invest, it can give them out as a loan, i.e. purchase the right to income acquisition at the following step or over the next few following steps. Such rights, and we shall call them **deposits**, also are included among FTs. Thus, having purchased at a step t-1 a single-step [single period] deposit for amount S, the firm is entitled to the receipt of gross income $(1+r_t)\,S$ at the next step, with r_t being net return on the deposit (at the step t). Upon such a receipt the deposit is closed, so the monetary flow on it would be zero from then

[1] Under the scope of the term "financial titles", which is borrowed from the works of Kruschwitz, should be implied replicable rights and assets freely traded on the market and bringing in income, or, generally, benefits of some other kind (possibly uncertain). Financial instruments (say, stocks or deposits) and non-unique assets, such as equipment and cars, thus, fall into this category of financial titles.

on. It is convenient to allow that there circulates on the market as well such FT as the **termless** deposit. Purchasing it at a step t-1 for amount 1 would imply that in exchange for that sum the buyer acquires the right to draw an established interest at each following step, i.e. the right to receive the stream of income: r_t, r_{t+1}, …, in perpetuity. Therefore, the termless deposit can be represented as a combination of single-step deposits: investing the amount of 1 on a single-step deposit, closing it at the following step, skimming the interest for oneself, and investing the sum 1 again and again in the same way -- results in the cash flow from a termless deposit being replicated. In the obverse of this situation, a single-step deposit is also a certain operation with the termless one: the construct of purchasing a termless deposit, drawing its interest at the next step and selling that deposit then and there also models the cashflow from a single-step deposit.

If a firm purchases a deposit, some other firm (say, a bank) is selling it. Thus, selling a deposit is equivalent to obtaining a loan. In this case, the seller of the deposit obtains a loan amounting S to continue 1 step under the same rate r_t. In a perfect market setting considered by us, any market participant having funds can extend to another participant a loan (moreover, issuance of a debenture by a firm can be considered as a sale of some FT). Thus, as it is easy to see, **there is an equality between the rates for single-step loans or deposits** within every step**.** Indeed, if the deposits were traded at the market for different rates, any market participant would prefer investing into the deposit with a greater rate. All the other deposits with lesser rates would go undemanded, i.e. would have ceased to trade, which is not the scenario in our situation.

It appears even a stronger statement is justified: **purchasing any FT and then selling it at the next step would have provided the same return as investing into the single-step deposit** (and, hence, these transactions are represented by identical cash flows). Indeed, assume that a firm purchases a financial title FT at the step t-1 at some price P; at the following step it receives income D from FT and then sells FT at some price Q. This operation gives raise to the cash inflow of $D+Q$ and the gross return of $(D+Q)/P$. We shall compare it to investing the sum P into the single-step deposit which would provide cash inflow $(1+r_t)\ P$ at the step t. Here three situations would be possible.

1. $D+Q > (1+r_t)\ P$. Here any market participant can: take on a loan of P at the step t-1 with which to fund the purchase of FT, and then sell FT at the next step t repaying the loan and the interest on it. It is easy to see that after the execution of such a strategy he will have the amount of $D+Q - (1+r_t)\ P > 0$ left, i.e. he “will make the money out of nothing”. Naturally, all market participants will make use of such opportunity, and it is impossible that it will ever eventuate on the equilibrium market. Clearly, in such a situation, if it were to happen, either the price of FT at the step t-1 would have had to increase, or the price of FT at the step t to go down, or the loan interest to have been raised.

2. $D+Q < (1+r_t)\ P$. Here it pays for any market participant owning FT at the beginning of step t-1 to sell it in order to put up the received funds into the single-

step deposit, and subsequently to repurchase FT out of the income (from closing the deposit) received at the next step. As a result, he will have a greater amount accumulated at the step t compared to situation if he had continued holding on to the use of FT at the step t-1. Thus, no market participant will have any use for FT at this step, and so it will not start circulating on the market contrary to the initial assumption.

3. Thus, the only feasible and unique situation is when $D+Q = (1+r_t)\ P$. But it carries with it the implication that the return on purchasing FT for one step is the same as the return on purchasing the termless deposit for one step (or purchasing the single-step deposit, for that matter) -- which is what was needed to be demonstrated.

It follows from the statement which was now proved that the prices for FT at two neighboring steps and the gross incomes that arise are connected by a simple ratio:

$$P_{t-1} = \frac{D_t + P_t}{1 + r_t}, \tag{1}$$

where D_t — is a gross income from FT at the step t. From this it demonstrably follows that the price of FT at any step is equal to the discounted amount of pure gross incomes at all subsequent steps (including the receipts from selling FT on the final step), and that the discount rate for each step coincides with the (depositary or loan) interest rate:

$$P_{t-1} = \frac{D_t}{1 + r_t} + \frac{D_{t+1}}{(1 + r_t)(1 + r_{t+1})} + \ldots + \frac{D_T + P_T}{(1 + r_t)\ldots(1 + r_T)}.$$

In particular, if FT ceases to circulate at the step T, i.e. its then price becomes zero, its price on any antecedent step should be equal to the discounted sum of income arising from it on all subsequent steps, including the last:

$$P_t = \frac{D_{t+1}}{1 + r_t} + \frac{D_{t+2}}{(1 + r_t)(1 + r_{t+1})} + \ldots + \frac{D_T}{(1 + r_t)\ldots(1 + r_{T-1})}. \tag{2}$$

A transposition of the term for the price of FT from the left-hand side of the balance to the right results in the formulation for the **general principle of "zero-effect"**: a project to purchase a FT and sell it back at some later step is characterized by zero NPV (provided the discount rates coincide with the interest rates).

We assumed that any FT circulating on the market can be purchased and sold at any moment. One may advance an objection: if you open a term deposit for 2 years at a bank, no one will permit you to close it in a year and draw the interest. It is indeed so, but the value of an FT is an economic concept, rather than a juridical one. Yes, I would not be able to close the deposit ahead of the schedule, but I can conclude an agreement with a third party whereby that party undertakes to transfer me some amount of money today in return for my liability to hand over to him the returns from the deposit on its closing. The maximum amount for which a counterparty can be found that is willing to transact on a like agreement can be construed as constituting the

market value of a “prematurely terminated“ term deposit, and the conclusion of such an agreement be treated as a sale of "prematurely terminated" term deposit.

Example 3. Assume that, with the exception of cash, there are only deposits traded on the market: an annual deposit (with the rate of 10 %) and a two-year deposit (with the rate of 32 %). Interest accrues on the deposits at their closing. An investor has 1 dollar and is intent on maximizing his capital after 2 years.

It is obvious that he should invest in the two-year deposit, so that 1 dollar invested today is equivalent to 1.32 dollars in 2 years. But what 1 dollar in a year is equivalent to? Assume that the investor gains access to one additional dollar after a year. Then the best use of it will be investing it into the annual deposit. Thus, 1 dollar in year 1 will be equivalent to 1,1 dollars in year 2. From this it follows that 1 dollar today is equivalent to 1.32 dollars in 2 years which are, in turn, equivalent to 1.32/1,1 = 1,2 dollars of year 1. Accordingly, the discount rate in the 2nd year is equal to 0.1 (10 %), but makes up 0.2 (20 %) during the 1st year – *these figures do not coincide with the returns on both deposits*.

The same result can be obtained in a different way. The two-year deposit cannot be closed ahead of the schedule; however, it carries a certain value X in a year after its opening. To find that value, it shall be noticed that, having invested 1 dollar into the two-year deposit and having sold it a year later, the investor should receive the same return as upon investing his funds into the annual deposit. But the interest on the annual deposit is 10 %, therefore $X = 1{,}1$. On the other hand, the rate for the second year can be estimated bearing in mind that the return equal to this rate is also afforded by purchasing the open deposit at the price of $X = 1.1$ and cashing in the income on it (1,32) at its closing. But the gross-return on such a transaction would be equal to 1.32/1,1 = 1.2, hence the discount rate associated with the step 2 is equal to 20 %.

Let us now consider that it is possible to purchase and sell FTs not only individually, but in whole “bundles” comprising FTs of different kinds (purchasing FTs in “bundles” entails neither the receipt of additional incomes, nor the incurring of additional expenditures, as against individual transactions). Therefore we can contemplate the fact that there circulate in the market both individual FTs, and the bundles (portfolios) formed of them. Among such bundles the most interesting ones are represented by multi-step deposits. For example, the purchase at the step 0 of a two-step deposit amounting to 1 (dollar) would be equivalent to placing the same amount in the single-step deposit and then re-investing the returns thus received at the next step into the new single-step deposit. It is obvious that such two-step deposit should return the gross income of $(1+r_1)\,(1+r_2)$ at the step 2.

Let's remark now that every FT represents a right to receiving income in the subsequent period. In particular, formula (2) provides the estimation of value for the right to receive a sequence of incomes $(D_{t+1}, \ldots, D_T)$. We shall find out now what, at t = 0, is the value X_t of the right to receive income f_t at the step t. From the discussion above it is apparent that such right is equivalent to the purchase, at the step 0, of a t-step deposit and therefore has a value $X_t = \dfrac{f_t}{(1+r_1)\ldots(1+r_t)}$.

In other words, the firm is indifferent between receiving the income f_t at the step t or receiving the income X_t at the step 0 -- both scenarios are viewed by it as the equivalents. This fact permits to regard the amount X_t as a today's (current) equivalent of the future income f_t.

Operations with FTs are usually regarded as **financial** activities. However, apart from them, it is customary for firms to engage in other activities which are also cost-intensive and bring in other types of income (for example, production,

performance of construction and civil works, participation in real investment projects, including those already under way, and not necessarily associated with its own production). We shall refer to such activities as **operational**.

Here our terminology somewhat differs from the standard one: in assessing investment projects and in property valuation there is maintained a conventional distinction between investment, operational and financial activities, whose classification is organized on a different basis. Such distinction is inconvenient for our purposes, as our object is to separate that activity which we intend to optimize (operations with financial titles and other replicable projects) from that which has been predetermined by earlier implemented decisions (exclusive, non-recurring operations and projects).

We shall assume that the firm settles only in cash with its counterparties in respect of its (operational) activities. Net cash inflows from these activities -- **net operating incomes** – shall be exogenously imputed at this stage (we shall not consider their optimization until we reach section "Optimization of operating activities").

In the considered situation of the ideal market, each firm confronts the following problem: into which FTs should it consider investing its income from financial and operational activities, and the sales of which FTs should be considered to finance its expenses on operational activities, if it so happens that the corresponding net inflows are negative? Such problem has a simple solution: all FTs should be sold at the initial step; if at a given or any subsequent step the net operating income generated appears to be positive, the corresponding funds should be placed on deposit, otherwise, when the income of the firm does not cover its expenses, deficit funds should be brought in by taking on a loan in the corresponding amount. Any other policy of investments will yield the same result. Indeed, if the policy of a firm provides for the purchase of any FT (other than the deposits or loans) at any step, it has been shown that precisely the same cash flow can be generated by disposing of the FT in question at the market, putting the proceeds in the deposit, and repurchasing the FT at the next step out of funds received after closing of the deposit (if it continues to have positive value by then).

Assume, further, that the firm investigates the possibility of participating in a **real** investment project that generates net cash flows f_0, f_1..., f_T at the corresponding steps 0..., T, (as a rule, the amount f_0 is negative and reflects initial investments into the project). The amounts f_t , on the one hand, thus reflect an increment in the net operating income of the firm due to the realization of the project, and, on the other hand, represent a difference between monetary inflows (g_t) and monetary outflows (h_t) arising due to operational activities: $f_t = g_t - h_t$. Therefore, participation in the project entitles the firm to the right to receive (over steps 0,.., T) monetary receipts, g_0..., g_T ,respectively, and simultaneously imposes on it the obligation to incur expenses h_1..., h_T. But, this is equivalent to the firm purchasing, at step 0, a bundle of rights to the reception of income (g_1..., g_T) and at the same time selling the bundle of rights to the reception of income (h_1..., h_T). As we have seen above, such operation for the firm would be equivalent to receiving the following amount at the step 0:

$$\left[\frac{g_1}{1+r_1}+...+\frac{g_T}{(1+r_1)...(1+r_T)}\right]-\left[\frac{h_1}{1+r_1}+...+\frac{h_T}{(1+r_1)...(1+r_T)}\right]=$$

$$=\frac{f_1}{1+r_1}+...+\frac{f_T}{(1+r_1)...(1+r_T)}.$$

On the other hand, having agreed to participation in the project, the firm commits itself at step 0 to incur expenses amounting to ($-f_0$). Thus, participation in the project for the firm would be equivalent to the reception of the following amount at step 0:

$$NPV = f_0+\frac{f_1}{1+r_1}+\frac{f_2}{(1+r_1)(1+r_2)}+...+\frac{f_T}{(1+r_1)...(1+r_T)}.$$

Therefore, if *NPV* appears negative, the firm should refrain from participating in the project, otherwise – it makes sense to agree to such participation. It is important to note that even at $NPV = 0$ the participation in the project will be as efficient for the firm as investing into any other FTs circulating on the market (it once again confirms the zero effect principle discussed above).

This immediately gives rise to the following question: if a firm has discovered a project with a positive *NPV*, the same discovery can be made by any other market participant. Why won't there be then a "rush" to implement the project thus eliminating the positive NPV opportunity? The answer is very simple: a real project is not like a financial title. Financial titles are bought and sold on financial markets, however, no general markets for projects exist in the world. Projects are not for sale, nor can they be bought, neither can they be divided, nor replicated, each of them is unique and exclusive. Besides, every project is usually tailored to the needs of specific group of its participants. It is theoretically possible to imagine a certain contest in which participants for a project are selected, however, to each possible collectivity of the participants in the project, generally speaking, there will correspond a different organizational-economic mechanism of its realization and different cash flows would be attributable to each participant. For this reason, having learnt about the decision of a given firm to take part in some efficient project, other firms can't but only reflect on whether to commit themselves to designing some other efficient project (possibly, similar to the original project in some respects, for example, providing for the production of similar goods or application of the same technology). However these projects will be distinct from the original one and estimating their efficiency would require carrying out stand-alone independent calculations.

So, given the rational behavior of participants at the ideal market:

1) *NPV (*net present value, i.e. the discounted sum of net cash inflows from the project) *is* the criterion of efficiency for a project (both real, and financial);
2) The discount rate coincides with the deposit rate (equally, with the loan rate) for each step;

3) The *NPV* amount indicates the sum the receipt of which by the firm at step 0 is equivalent to the reception of all incomes from the project and incurrence of all expenses associated with the project;
4) *NPV* amount reflects benefit (or loss) to the firm from participating in the fulfillment of the project -- in comparison with alternative investments of the same funds into FTs trading on the market.

Thus, *NPV* is simultaneously an indicator of absolute efficiency (i.e. it provides an estimation of benefit or loss from the given project) *and* an indicator of comparative efficiency, reflecting a benefit or loss from investments into the project in comparison with other, alternative, investments (since alternative investments into any FT result in *NPV* of zero).

The situation considered by us in the ideal market setting is extremely abstract. In particular, it does not reflect the liabilities associated with tax payments. Let us analyze what effect do certain kinds of tax have on the derived propositions.

1. A number of taxes (VAT, the property tax, etc.) are associated with operational activities of firms. The liability of their settlement will simply alter the amount of net operating income, therefore the obtained conclusions will not be affected, only it will be necessary to have recourse to “after-tax”, rather then “pre-tax”, cash flows during the process of efficiency assessment of the investment projects.
2. Let's assume now that all firms are liable in *profit tax*, levied at the uniform rate *n* on all income received in the form of dividends from stocks, interest received on extended loans and other income arising from FTs. In other words, such *received* income from FTs increases the taxable profit base of firms. At the same time it is necessary to assume that dividend and interest *paid out* by firms reduce the taxable surplus (such a system approximately corresponds to that enacted in the Russian Federation). We shall now consider FTs on which their owner receives income D_t at a step *t*. In this case, having received the income, the owner will deduct from it the liability due in settlement of the profit tax, and his net income will thus constitute a smaller amount $(1-n)D_t$; on the other hand, the cost to the issuer of FT in income payment will reduce his taxable profit, entailing the corresponding reduction in the profits tax for him, consequently the net expense to the issuer will amount to $(1-n)D_t$. Put differently, all cash flows from financial activities (including those due to interest on deposits and loans) become proportioned by the multiple $1-n$ in terms of their final financial impact, and cash flows from operational activities do not change (given the assumptions). In the project efficiency assessment context, it means that now we should consider the same cash flows after tax, additionally, the discount rates should also be transformed to their “after tax” basis, i.e. multiplied by $1-n$. From this it follows that when establishing discount rates the orientation on bank ("pre-tax") rates causes the overestimation in the former.
3. The taxation system usually provides also for the taxation of income from

security sales. In particular, the tax is assessed on the difference between the proceeds on security sale and the costs of its purchase. Thus, the amount of tax is made dependent on the costs of purchase which in our model, in turn, depend on the timing of purchase. In other words, the tax paid appears to be different for FTs of the same kind sold at the same moment of time, but purchased at different times in the past. It appears that such tax substantially alters some results obtained above. The reason for this is clear. In the framework considered before, a situation where an FT is sold and then immediately repurchased would have generated a zero income (it was previously possible to imply that all FTs are put on sale at the initiation of each step for the needed FTs to be repurchased again). Where the proceeds from sale of an FT trigger an increase in the taxable income whereas the costs of purchasing the FTs do not alter the taxable base, the incentive is to trade FTs as infrequently as possible. For this reason the optimum policy is to be formulated at the outset for the entire duration of the planning period, not individually on the step-by-step basis. This problem shall be considered below.

Optimization of financial policy in the imperfect market context

Plan: the best method of accomplishing an accidental result

Ambrose Bierce

Let's now consider the same issue of the optimization of financial policy of a firm but under slightly more realistic assumptions characteristic of an imperfect market. Namely, we will assume that:

- A set of FTs available for purchase can be different for different market participants. This assumption can be associated with administrative restrictions, as well as with the fact that some securities are put on sale only in very large blocks;
- Any market participant can issue his own set of FTs. In particular, loan rates can be different for different market participants;
- There are restrictions on the volume of borrowings from external parties (for example, volumes of loans taken out or emitted debentures are limited);
- FTs are generally characterized by a limited term of circulation (which may also determine the rate at which a given FT is traded: so, the discount bond with the face value of 100 USD redeemable in a year may be trading at $80, whereas a similar bond but redeemable in 2 years - for $60; the analysis and explanation of these dependencies is given in [4, 23]);
- FTs are bought and sold at a market value, however additional costs and taxes are associated with the sale of FTs, and the taxes depend on the step at which the FTs being sold have been purchased.

Let us now formulate an economic-mathematical model for optimizing the financial policy of a specific firm that functions in the context of such imperfect market (some simplified models of this kind have been presented in [31, 33]). Such policy will provide for the purchase and sale of various FTs and the issuance of

own borrower's notes over the course of some planning period beginning at step 1 and terminating at step *T*.

As before, it is supposed that the purchase of FTs is possible in any volume so that they can be regarded as divisible and replicable investment projects. However, as it was already noted above, FTs of any one type purchased simultaneously but sold at different times are now associated with different cash flows. Therefore it will now be found convenient to subdivide FTs a little differently: into *groups* and *series*. Each group comprises FTs of one type acquired at the same time. That way, negative numbers *i* will be assigned to groups in which FTs have been acquired prior to the beginning of the planning period. FTs in each group will then be broken down into series *s* (*s* = 1..., *T*), each comprising those FTs which will be put for sale at a step *s*.

FTs belonging in series *s* of group *i* shall be characterized (at step *t*) by a market value c_{ist} and net cash inflow to the owner of a_{ist} (if FTs in series *s* do not circulate on the market at step *t*, we shall assume these amounts equal to 0). That way, net cash inflow a_{ist} accounts for:

- Acquisition costs - if FTs are acquired at step *t*;
- Proceeds from selling FTs - if FTs are to be sold at step *t*;
- Dividends, interest, coupon and other incomes from using FTs received by the firm at step *t*;
- Profit taxes resulting from sale of FTs, taxes on dividends, interest, etc.

We distinguish between FTs and borrower's “own liabilities” emitted by the firm (we shall mark those with the superscript, instead of sub-indexes), the difference being that FTs are acquired first and only then put on sale, whereas the liabilities are issued (emitted) first and are repaid (redeemed) subsequently. Thus, with FTs the corresponding cash flows begin with expenses and end up with cash receipts (inflows), whereas for liabilities – they begin with cash receipts (inflows) and terminate with expenditures.

The firm may issue its own stocks, however the income on them (dividends) is received by shareholders - proprietors of the firm. Therefore, we do not regard the issue of own stocks as emission of liabilities by the firm.

Let's group liabilities by *types*, assigning to different *types* those liabilities that are issued at different times or are issued simultaneously but are redeemable at a different time. We shall assume that among the range of FTs available to a firm at each step are single-step and multi-step deposits (including, if necessary, “hoarding” – i.e. holding of funds at zero or negative interest), and among the range of liabilities that a firm is authorized to issue are single-step and multi-step “credits” (in a limited volume, see below).

Cash flows for liabilities shall be described as follows. The basic feature of any liability of *m*-th type at step *t* is *indebtedness* l^{mt}. At the moment a liability is emitted, the amount of related indebtedness equals to the cash receipts from it, and

zero – prior to this moment or after the retirement of the liability; in the intervening period it reflects the outstanding debt under the liability or the face value of a bond payable at its retirement. We shall stipulate, moreover, that for liabilities issued in the current period, $l^{m0} = 0$. Accordingly, the difference $l^{mt-1} - l^{mt}$ expresses payments in repayment of a liability at step t (or the receipt of funds – at the moment of its issue). Additionally, we shall account for the fact that an interest (coupon yield) is paid on liabilities. Denoting by r^{mt} (at the step t) the interest rate (on indebtedness from the preceding step), the amount of these payments constitutes $r^{mt}l^{mt-1}$. Thus, net cash outflow under a liability at step t is explicitly determined by the dynamics of the indebtedness and equals $l^{mt-1} - l^{mt} + r^{mt}l^{mt-1}$. The assumption that all payments under a liability are proportional to its magnitude is justified in this case: extending loans of a different size and at different rates, creditors factor in the risk of default, which is not present in our deterministic model. Besides, this assumption ensures that liabilities become divisible and replicable projects whose cash flows are arranged "back to front": they begin with the cash inflow, and terminate with outflows (if you please, they can be regarded as investment projects from the point of view of the creditor).

In the course of its operational activities, the firm also deals with the corresponding cash flows discussed above. We shall include in these cash flows also payments under those liabilities of the firm which were in place at step 0 (so index m relates only to those liabilities which will have been issued at step 1 or later). We shall also account for the fact that in the course of its operational activities a firm may create or acquire **fixed assets** - buildings, equipment, plots of land and other assets which are expected to be used in the operations of the firm on a constant basis [9, Glossary]. Unlike FTs, these assets:

1) Have a "complex" character. Indeed, "physically" a plant, for instance, is comprised of "simple" assets - buildings, constructions, machine tools, etc. However these "simple" assets viewed separately are capable of generating only a substantially diminished income than when installed to function in a joint complex;
2) Being created, they change their value over time. Responding to a change in the general mood of stock market, this value may diminish or increase over certain horizons of time, however due to the impact of physical depreciation or functional (technical) obsolescence it necessarily sustains a decrease in due course. In professional valuation [9] such diminution of value is also referred to as depreciation;
3) Are indivisible. So, “half of the plant” (but not of its stocks!) exists neither as a physical entity, nor as an object of commercial operations;
4) Are associated with the realization of specific projects, owing to which transactions with them can’t but be of a limited scope. Whereas stocks can be bought or sold at any time, objects created during the fulfillment of an investment project can be sold only when the project provides for it (to be

specific, we shall assume that these objects are put on sale at the end of a period);

5) Bring in income only indirectly connected with the value of these assets. Moreover, when stock dividends in any given year are constrained by non-negativity, industrial plant operations may generate negative net income during some periods (for example, the property tax is payable even when the item is "idle").

Fixed assets are not partial to operations with FTs and debt liabilities, so no index number is assigned to them.

Considering the above, the operational activity of a firm is reflected in the model by two exogenously and step-by-step determined parameters:

- F_t - net operating income at a step t (cash available to the firm at the beginning of a planning period is reflected in the amount of F_1);
- G_t - value of fixed assets at the beginning of step t (i.e. at the end of step t-1).

In the actual practice no firm can emit liabilities beyond a certain limit in its bid to take on a desired volume of loans. It is essential that such limitations are caused not only by uncertainty and risk factors, but also arise out of necessity to stabilize the market. For example, from time to time there arise in the market situations in which the purchase of some FTs for their subsequent sale would yield net returns in excess of the rate on loans of the same duration. If restrictions on credit volumes were not in place, market participants would have aspired to take on as great amount of credit as possible to acquire these FTs so as to re-sell them at a price exceeding the costs of loan repayment. Therefore, credit rationing situations always take place in the actual practice and should be considered in our model. Usually, creditors demand that borrowers comply with a certain ratio between their equity and borrowed funds (capital structure), for example, a 60:40 proportion. Given this, we shall assume that credit indebtedness of a firm should not exceed a certain fraction h of the market value of its assets (in this example, $h = 2/3$). In such situations, assets of the firm, including its FTs and fixed assets, perform (explicitly or by default) the role of collateral under the loan. This allows for interpreting the specified fraction of the market value of assets as their “mortgage value”.

Let's now pass on to formalizing the model to optimize the financial policy of a firm. In this model, a formulated financial policy of a firm determines the acquisition volumes for various FTs and their distribution into series, and also stipulates volumes for emitting various kinds of liabilities. Therefore two groups of non-negative variables shall be unknowns in the model:

v_{is} – quantity of FTs in group i series s, in possession of the firm;

y^m - quantity (volume) of liabilities *of m-th* type issued by the firm.

The model includes three groups of restrictions.

1. At the beginning of the planning period only those FTs which are already in possession of the firm can be allocated into series (with negative concomitant

indexes). By denoting through N_i the number of FTs in group i in possession of the firm at the beginning of planning period, this restriction can be written down as:

$$\sum_s v_{is} = N_i, \qquad (i < 0). \tag{3}$$

2. The balance of cash inflows and outflows: net cash inflows from FTs, operational activities and from the assumption and repayment of liabilities should be non-negative at each step:

$$\sum_{i,s} a_{ist} v_{is} + F_t - \sum_m y^m \left(l^{mt-1} - l^{mt} + r^{mt} l^{mt-1} \right) \geq 0, \ (0 < t < T). \tag{4}$$

3. Restrictions on the issuance of liabilities: For any step, the overall debt under firm liabilities should not exceed the mortgage value of its assets estimated at prices prevailing at this step:

$$\sum_m y^m l^{mt} \leq h \left[\sum_{i,s} c_{ist} v_{is} + G_t \right], \ (0 < t < T). \tag{5}$$

Developing an optimality criterion for financial policy of the firm is a non-trivial matter. As noted in [27], “in the market economy context, the major objective of strategy for enterprise development is considered to be the increase of its market value”. Such objective is also set by a substantial number of companies in the developed countries. The rationale for use of such criterion was analyzed by Michael Jensen. In [10] he writes that 200 years of research in the field of economics and finance indicate that social welfare in the economy is maximized by all firms striving to maximize their own market value. This conclusion is not absolute in its character. Thus, given the presence of monopolies and “externalities”, value maximization doesn’t necessarily result in social welfare being maximized.

Nevertheless, as it is noted in [10], though there exist many objective functions that can guide managers of companies in their decision-making, value maximization remains important as it is most closely aligned with public welfare maximizations. On this basis, we shall proceed to use the criterion based on value maximization of a firm.

In our model the optimal policy is developed at once for the entire extent of the planning period, the criterial indicator relating to the end of this period, i.e. to step T. The value of a firm here can only be estimated by the value of its equity V_T, i.e. the value of firm’s assets minus value of its liabilities (this indicator is also referred to as net asset value). Therefore we will seek for such financial policy which maximizes the equity of the firm by the end of the planning period, i.e. at step T.

Let's notice now that in breaking FTs down into series, we have accepted that each FT will have been sold over the planning period. It implies that, at the final step, all assets of the firm shall be comprised of fixed assets (G_T), or only of cash funds received from operational activities and from the use and disposal of FTs.

The obtained amount should, then, be reduced by the amount of payments in settlement of liabilities at step T and by the value of residual liabilities. But such a reduction, as it relates to liability m, amounts to:

$$y^m(l^{mT-1} - l^{mT} + r^{mT}l^{mT-1}) + y^m l^{mT} = y^m(l^{mT-1} + r^{mT}l^{mT-1}).$$

It follows from here that, when calculating the value of equity V_T, we can assume that all debt liabilities of the firm terminate at the last step T, i.e. that $l^{mT} = 0$. In this situation, the optimality criterion assumes the following form:

$$V_T = \sum_{i,s} a_{isT} v_{is} + F_T + G_T - \sum_m y^m \left(l^{mT-1} - l^{mT} + r^{mT} l^{mT-1} \right) \Rightarrow \max . \qquad (6)$$

We have reached the linear programming problem and its solution would indeed specify a development strategy for a firm -- as it works out where to invest or not invest the capital, what liabilities have to be issued and when. Certainly, a firm management may undertake some other strategy but then the model would allow comparing it with the optimal one and finding out the extent to which following it would reduce the value of firm's equity at the end of the planning period.

It shall be noticed as well that a solution to (3) - (6) problem, i.e. the financial strategy of firm, depends on the *initial structure of investment portfolio* of the firm. Therefore, firms with different capital structures or differing operational activities may estimate the efficiency of purchasing and selling the same FTs differently, and their optimal FT bundles may also vary. Same conclusion can be reached, when the firm policy is optimized under conditions of probabilistic uncertainty [32, ch. 6].

Let's analyze the constructed model in greater detail. We shall proceed from a simple case where at the initial moment (at step 0) the firm has only cash V_0, does not conduct any operational activities, has no liabilities nor will have them at the end of the planning period. Then, the initial capital will have increased by V_T/V_0 over the planning period, given the optimal financial policy. This should also specify the time value of cash factor for bringing step 0 estimates up to their step T values, which is, apparently, represented by the following discount rate

$$E = \sqrt[T]{V_T / V_0} - 1. \qquad (7)$$

Generally, this rate depends on the duration of the planning period T (the sooner the firm wishes "to convert its money into money", the less will be the return on its operations). Source [28] describes the behavior of v_{0T} for extended T given the invariable composition of FTs circulating on the market, each of which may yield both positive and negative net income (with loan facilities not taken into account). This behavior is as follows.

Let's assume that investments into each i-th FT are efficient at low discount rates and are inefficient at the high rates. Let $f_i(\lambda)$ be NPV, given the discount rate λ, from unit investments into i-th FT, such that $f_i(0) > 0 > f_i(\infty)$ as per our assumption, $f(\lambda)$ — is a maximum of $f_i(\lambda)$, μ — is the least positive root in the

equation $f(\lambda) = 0$ and $h+1$ — is the multiplicity of the root. Then, the value of v_{0T} increases with the increase in T as $(1+\mu)^T/T^h$, so the discount rate (7) approximates μ. Somewhat different approach to the analysis of this problem is elaborated in [2].

However, it is important to bear in mind that even when interpreting (7) as the discount rate, such rate would have the meaning of **an average** rate over the planning period and would most likely be too crude an estimate to make itself amenable to application in the project efficiency assessment contexts. More correct approach to establishing the discount rate is presented in section "Estimating the efficiency of investment projects and property valuation in the framework of strategic planning".

Let us introduce an assumption that the firm pursues operational activities but that those are reduced only to participation in some real investment project. If such project requires heavy expenditures, the system (3) - (6) may happen to be unsolvable. On the contrary, its resolution would attest that incomes received at each step would be sufficient for financing the project. It would mean that the project will be *financially feasible* (implementable) if and only if the system (3) - (6) has a solution. Emphasis should be placed on the *non-local* character of such criterion for financial feasibility. Put differently, to be convinced of financial feasibility of a project, it is not enough merely to have information on cash flows from the project -- it is necessary to get to know as well what the financial position of the firm will be at each step of the planning period[2].

From this point of view, the propositions appearing in [24, 33] stating that to ensure financial feasibility of the project it is necessary to register at each step a non-negative sum of accumulated compounded (at a deposit rate) net cash inflows from the project (including investments of firm's own equity into the project) seem flawed. They assume that equity of the firm and net inflows from the project are partially accumulated on "single-step" deposits for financing forthcoming expenditures. Certainly, a project will be financially feasible if such a non-negative sum obtains on the deposits through and through. However the inverse statement would be incorrect: sometimes it is possible to ensure financial feasibility through the joint use of some financial instruments, including multi-step deposits, and then the structure of financial feasibility conditions would appear to be different. In other situations accumulated compounded cash inflows from the project may be positive, however the project may require investments of the firm's equity (own funds) where the firm has no such funds, and credit restrictions on the volume of debt preclude it to take on more debt.

Estimating the efficiency of investment projects and property valuation in the framework of strategic planning

[2] This has the meaning that it is not sufficient merely to know that a project requires expenses during a particular year, in addition we need to know that the firm will have funds available to meet these expenses. Under the "local approach", we merely pay attention to the project ignoring the position of the firm in the same period. So, the term "local approach" here means a consideration of the project in isolation from the financial position of the firm.

> The first rule of the transitional economy: For every economist there exists his counterpart of the same magnitude with a contrary opinion.
> The second rule of the transitional economy: And both are wrong.

Let's notice at the outset that the optimal financial policy, i.e. solution to (3) - (6) problem, will not sustain change, if the equality sign in (3) is replaced with the "$\leq$" sign (i.e. allowing for possibility of "free destruction" of some FTs in the possession of the firm). Important information on this solution can be gleaned from analyzing the corresponding dual problem. Its unknowns to be denoted as λ_i, π_t and μ_t, are non-negative and reflect "prices" of the restrictions (3) - (5).

The value of λ_i ($i < 0$) represents a "shadow price" of i-th group FT at step 0. It reflects an increment (gain) of objective function (6), i.e. contribution to equity of the firm at the end of the planning period, from the firm acquiring one additional marginal unit of such FT. Additionally we agree to assume that $\lambda_i = 0$ at $i \geq 0$.

π_t ($0 < t < T$) is the "shadow price" of money (cash) at a step t. It reflects an increment in the objective function (6) due to receipt by the firm of additional 1 dollar at step t. It is natural to regard the "price" of 1 dollar for the firm at the last step as equal to 1 dollar. Therefore, we impute $\pi_T = 1$.

μ_t ($0 < t < T$) is interpreted as the "price" of rights to the assumption of debt or, more succinctly, "value of loan rights". It shows a potential gain in the objective function (6) if the firm were to have been permitted at the step t to issue an extra liability for 1 dollar over and above the established restriction limit (naturally, if the indebtedness is below the limit at any particular step, $\mu_t = 0$). We shall further assume that $\mu_T = 0$ (this is logical since receiving a loan at the final step doesn't change the value of equity).

Using this notation, the dual problem assumes the following form:

$$\sum_{t=1}^{T}\left(\pi_t F_t + h\mu_t G_t\right) + G_T + \sum_i \lambda_i N_i \Rightarrow \min; \tag{8}$$

$$\sum_{t=1}^{T}\left[\pi_t a_{ist} + h\mu_t c_{ist}\right] \leq \lambda_i; \tag{9}$$

$$\sum_{t=1}^{T}\left\{\pi_t\left[l^{mt-1}\left(1+r^{mt}\right) - l^{mt}\right] + \mu_t l^{mt}\right\} \geq 0. \tag{10}$$

From out of complementary slackness conditions in linear programming it follows that the inequality sign in (9) is observed strictly only if the optimal policy does not provide for the purchase of i-th group s-series FTs, and inequality in (10) is observed strictly only if this policy does not envisage issuing the liabilities *of m-th* kind.

Let's return now to the primary goal of this paper - being the assessment of efficiency of investment projects. It appears that such an assessment (albeit, for "small projects") should be based on the shadow prices for cash and loan rights arising out of the dual model.

Let's consider Project *P* commencing at the step 1 and coming to an end before step T – such that in year t it would change firm's net operating income by f_t, and the value of its fixed assets - by g_t. If the project is large-scale (i.e. f_t and g_t are too great), its impact on the objective function can only be ascertained by seeking a new solution to the optimizing problem with values for F_t and G_t being changed. However, if f_t and g_t are small, then, on the basis of meaning for dual estimates and owing to (8), implementing the project would result in the following change in the objective function (the optimality criterion): $\sum_t (\pi_t f_t + h\mu_t g_t) + g_T$. Denoting: $\alpha_t = \pi_t/\pi_1$, $\gamma_t = h\mu_t/\pi_t$ for $t < T$, $\gamma_T = 1/\pi_T = 1$,

$$DEI = \sum_t \alpha_t (f_t + \gamma_t g_t). \qquad (11)$$

Then $\sum_t (\pi_t f_t + h\mu_t g_t) + g_T = \pi_1 DEI$. Therefore, project implementation is equivalent to the amount of income *DEI* being received at step 1. In other words, assuming that the optimization of financial policy and the assessment of a project are carried out at the beginning of step 1, the amount of *DEI* reflects "current equivalent" of all cash flows from the project (the use of this acronym shall be explained below). Therefore, a project should be considered effective if and only if $DEI \geq 0$ (value V_T "with the project" should be no less than that value "absent the project"), and the best version of the project out of several is the one indicating the highest *DEI*. It prompts us to consider (11) as a criterion for the efficiency of the project – a specific "substitute" for NPV, and view the parameters α_t in the formula as discounting factors (to the step 1). The rates of decline in these factors lend themselves to interpretation as being the corresponding discount rates:

$$E_t = (\alpha_{t-1} / \alpha_t) - 1 = (\pi_{t-1} / \pi_t) - 1. \qquad (12)$$

Some properties of such discount rates are illustrated in the examples that follow.

Example 4. At step t, a firm puts its cash into the single-step deposit at the rate p. The "price" of 1 dollar of investments at this juncture is equal to the price of cash - π_t, return on the investments - $1 + p$. Besides, as this deposit is opened in the planning period, its "price" $\lambda = 0$. Further to the point, if the firm places its funds on deposits, it means that it has no need for loans, so $\mu_t = 0$. At last, for (9) the following equality should be observed for this deposit: $-\pi_t + \pi_{t+1}(1 + p) + h\mu_{t+1}(1 + p) = 0$, whence we have: $\pi_t = (1 + p)(\pi_{t-1} + h\mu_{t+1})$. But then the discount rate - the rate of decline in the shadow prices of cash - should be no less than the depositary interest. The exact equality obtains only if $\mu_{t+1} = 0$, that is if the firm needs no access to loan at the following step (the firm deposits cash at step t so it has no need for cash funded from loans at this point, but the situation can change at the following step!).

Example 5. Assume that at a step t a firm takes on the loan with 1-step duration at the rate r. Then the following obtains due to (10): $\pi_t = \mu_t + \pi_{t+1}(1 + r)$. Therefore, given $\mu_t > 0$, the discount rate can exceed the loan rate ($\pi_t / \pi_{t+1} - 1 > r$), which is contrary to the prevailing notions.

Assume now that the firm borrows funds (amounting to 1) by issuing m-th liability at a step s ($l^{ms} = 1$). Then Condition (10) results in: $\pi_s = \mu_s + \sum_{t=s+1}^{T} \left\{ \pi_t \left[l^{mt-1}\left(1 + r^{mt}\right) - l^{mt} \right] + \mu_t l^{mt} \right\}$ - the shadow price of cash attracted at the expense of emission of liabilities is equal to the "price" of payments under this liability increased by the price of associated rights to the build-up of indebtedness (meaning that the firm exhausts these rights by increasing its debt).

Example 6. Assume that at some step k a firm purchases some FT, receiving net cash inflows a_t from it at the following steps, and then subsequently sells it at step n generating revenue c_n. The firm doesn't "hit" the borrowing limit over the considered period. Then it readily follows from (9) that $\pi_k = \max_n \left\{ \sum_{t=k+1}^{n} \pi_t a_t + \pi_n c_n \right\}$. Therefore shadow prices of cash, and, consequently, discount rates, are indeed related to the (somewhat originally estimated) returns on investments into FTs for a rational term, however the associated relation is not so primitive as customary presented in various teaching manuals, methodological documents and Valuation standards.

Few words are in order regarding economic substance of γ_t magnitudes. Different from μ_t only by a multiplier, they also reflect a valuation ("shadow prices") of rights to the build-up of indebtedness, but one proportioned not to the termination of the planning period, but to the step t. Thus, as is apparent from the formula (11), *each dollar in fixed assets at step t provides the same contribution to the firm's objective function as γ_t dollars of net cash inflow*. Such a result from the very fact of existence of these fixed assets follows because these assets (acting, as it were, in their capacity for collateral -- reflected by the factor h) expand for the firm the opportunities for attracting borrowings to allow it to implement any beneficial financial projects. Because of this, γ_t can be thought of as the discounted unit (i.e. per 1 dollar of fixed asset value) price of loan rights. In the context of the planning design, it is not sensible to engage in attracting loanable funds at the end of the planning period. Here the equality $\gamma_T = 1$ would mean that the fixed assets which will have "survived" till the end of the planning period will provide a contribution to the objective function to the full extent of their market value (as the firm has nothing left at this juncture but to sell off its fixed assets).

Note. It would be possible to consider situations where a firm maximizes an objective function different from (6) facing the same restrictions (3) - (5). In this case, a criterion for efficiency of "small (scale) projects" will retain the same form (11) as *DEI*, however ratios in the dual model will be different, and the dual prices, as well the resulting discount rates, may lack some properties specified above.

Let's consider now a project providing for the purchase of a unit of FT at some step and its sale at some other step. If the optimal policy of a firm provides for these operations, the *DEI* of such projects will be equal to 0, otherwise it will be negative or zero (this is applicable as well in relation to projects for the issuance of various liabilities). From this it follows that if any small project has a positive *DEI* estimate, implementing it would be more beneficial than any *operations with FTs*.

N**ote**. It will be noticed that only the operation to purchase **a unit** of FT of some series which "had entered into the optimal policy", results in *DEI* of zero, however it should not be concluded that this proposition applies to **all** FTs purchased at this step (foregoing the purchase of such FTs or purchasing them in a different proportion may result in the policy ceasing to be optimal and the value of the objective function decreasing substantially). This emphasizes once again that such indicator as *DEI,* as well as any other indicators of "local efficiency", provide correct estimates only for "small projects".

At the same time, the positivity of *DEI* does not imply that investing into the project should bring more benefits than the placement of funds into any of the alternative *directions of investment*: such directions may include "multistep" operations (for example, extraction from clay deposits, even provided that such deposits are many, all of them identical, can be freely put up for sale and bought, but that they should be developed over, say, 10-30 year period).

Let's note now that the *DEI* indicator (11) can't be referred to as the net present value of income: its formula, unlike the traditional formula for NPV, discounts not merely net cash inflows from the project, but a more complex aggregate (for this reason, we avoid further using the "NPV" notation to denote it). Therefore, the elements discounted in formula (11) shall be designated by a separate name: the **equivalent income**. Substantively, it reflects such an amount of net inflow the receipt of which at a given step would be equivalent to the associated results from the project (i.e. to net inflows received, and the value of fixed assets created, under the project). In this case, the *DEI* indicator is the **discounted equivalent income** (which explains the acronym applied to it[3]). The discounted cash flow method thus transforms itself into the *discounted equivalent income method*. We shall have an occasion to inquire into the robustness of such treatment after further consideration of the problem.

The stated considerations would allow to introduce the following definition of the basic project efficiency indicator (*DEI*) and of the related discount rates entering into its estimation (we shall see further that the validity of these definitions is not impaired under the conditions of uncertainty).

Under the "discounted equivalent income (*DEI*)" from the project is understood "present (today's) equivalent" of monetary effects from the project, i.e. such amount the receipt of which at the initial moment of time is equivalent for the firm to the set consisting of the entirety of fixed assets created under the project and all monetary receipts and outgoings associated with the participation in the project.

***DEI* of the project is determined as the sum of discounted equivalent incomes from the project covering the entire period of its fulfillment.**

[3] It would have been more precise to speak about *Present Equivalent Net Income to the Subject*, however such appellation, as well as its acronym, would have been far too long. It will make no sense applying the customary notation of *NPV* as well: or it will be necessary to additionally stipulate whether we are discussing the conventional or modified criterion every time it is being used. Furthermore, we shall make use of *DEI* in assessments under conditions of uncertainty, discussed in the later sections. Thus, via the NPV acronym we shall continue referring to the conventional criterion which uses time invariant discount rates and fails to account for restrictions on the emission of liabilities.

The discount rate is understood as the rate of decline in the (shadow) prices of cash within the context of the optimal plan for managing financial activity of the firm.

It is possible to regard such treatment of the discount rate and the *DEI* indicator as the **optimization approach**. This optimization approach to discount rate should not be regarded as something new -- in effect, it was in the works of A.L.Lurie [19, 20, 21] (see also [11, 12]) where the approach was first enunciated. To our knowledge, this approach is without parallel in the western literature, the exception being the works [15, 22]. It is important that the rate being thus determined is based on the long-term policy of the firm which, when framed, may well have considered any other restrictions on its activities or any other optimality criteria (however, it is not clear what other reasonable criteria, if any, can be suggested for the given problem). Thus, a discount rate for the firm should be established in the course of its decision-making on **strategic planning**.

Since discount rates for different steps can be different, a delay in the realization of an inefficient project can transform it into being an efficient one (or vice versa). Besides, solving the considered optimization problem given one and the same information about FTs, firms with differing initial capital endowments (N_i) and operational activity results may obtain discount rates different in terms of their magnitude and dynamics. Consequently, **no such things as uniform (and, specifically, time-invariant) general-market discount rates exist** even under the conditions of completeness of information and even with reference to assessing any one single specific project – a sentiment that runs counter to the valuation standards [9].

Before we start discussing the suggested criterion, let's pay attention once again that it is applicable only with respect to *"small"* projects. "Large-scale" projects may cause shifts in the criterion beyond (11), as they may alter shadow valuations of the assets and, thus, the discount rate. Thus, "large-scale" projects in general should not be assessed on the basis of locally-meaningful calculations; irrespective of the chosen discount rate, it would be necessary to conduct their assessment "against the backdrop" of the entire set of activities of the firm, optimizing its financial policy in situations "with the project" and "absent the project" and then comparing the obtained values of the criterion. Intuitively it makes sense, but how the "smallness" of a project is to be understood? Whether it is possible somehow to formalize or measure this "smallness"? To answer these questions, we shall consider an abstract situation wherein we continuously move on from "absent-the-project" situation to a "with-the-project" situation.

In other words, instead of just one project being evaluated characterized by the values of f_t and g_t, let's consider a "family" of projects P_k, characterized by the proportionate values of kf_t and kg_t. The value of k=0 corresponds to "absent the project" situation, whereas k=1 – corresponds to "with the project" situation. However, projects at $0<k<1$ in the P_k set can only be abstractly conceived, as their related cash flows may not have any parallels with the investments into real assets.

Still, we can inquire at a notional level as to the effect for the firm of implementing a particular project P_k. For this, we should solve the optimizing problem (2.3)-(2.5) substituting in it $F_t + kf_t$ and $G_t + kg_t$ for F_t and G_t, respectively. By $V(k)$ we shall denote the resulting value of the objective function (2.6) in this instance. From the linear programming theory it follows that the $V(k)$ function is convex, and its derivative at k=0 is equal to $\sum_t (\pi_t f_t + h\mu_t g_t) + g_T = \pi_1 DEI$. It implies that the change in the objective function of the firm caused by the implementation of the P_k project, at a small values of k, will be approximately equal to $k\pi_1 DEI$. However at k=1 such “equality” can be violated. This may happen if transitioning from k=0 to k=1 would materially alter the solutions of the dual problem, i.e. values for λ_i, π_t and μ_t. Thus, strictly speaking, **that project is considered to be “small” the implementation of which would not materially affect a solution to the dual problem, i.e. shadow prices of the assets and rights to the assumption of debt.**

It is possible to express the same thought in a different way. The estimate of efficiency of a project should, basically, reflect an increment in the objective function of the firm due to the fulfillment of the project. Meanwhile, the use of formulas like *NPV* or *DEI* for these purposes implies, in essence, the substitution of the specified increment by its differential that for "large" projects may lead to an error. Moreover, similar error will be produced by any formula containing only the information on the project (at the risk of crude simplification we can say: one and the same cash flow added to different strategic plans, let alone to the strategic plans of different firms, may receive a different valuation). For this reason alone, "large-scale" projects cannot in general be estimated "locally".

Qualitative distinctions between "large-scale" and "small" projects should not go unnoticed, either. Efficiency evaluations of a project, by definition, reflect its conformity to the purposes and interests of the firm -- expressed, in particular, through certain parameters of its strategic plan (such as discount rates). However, the fulfillment of a "large-scale" project can lead to the purposes and interests of the firm being essentially changed. For example, the firm strategy could have provided for a disposal of some property or a liquidation of "surplus" division, whereas realization of a "large-scale" project may call for the use of this property and a substantial development of this very division. Quite often the realization of "large-scale" projects also necessitates changes in the firm control system and in the area of mutual relations of the firm with its counterparties. Cash flows associated with such changes cannot be estimated “from the former standpoint”: for example, additional monthly salary expenditures for workers totaling $1 million will not be evaluated by the firm in the same way as $1 million earmarked for remunerating a one new top manager. For this reason, realization of a "large-scale" project should generally assume changes of strategy in the development of the firm, with the evaluation of its efficiency being reduced to comparing various versions of such strategy. Such a comparison cannot and should not be done by an evaluating economist –rather it is a matter for the administration of the firm. It is in

this context that the *dependence* between investment decisions and the strategic planning, discussed in Section “Optimization of financial policy in the imperfect market context”, becomes an *interdependence*. Here the task for an evaluating economist becomes that of only revealing the need for revising the firm’s development strategy that arises due to the fulfillment of the project.

Change of the purpose and interests occurs not only when realizing "large-scale" projects. In justifying *DEI* criterion, we assumed that the firm participating in the project will continue in its existence at least until the termination of the project. Meanwhile, projects envisaging splits or merges between the firms are also regularly developed and implemented. Thus there arises a question of how to estimate the efficiency of such a project: from the standpoint of "disappearing" existing firm or from the standpoint of a newly created entity? In [4, ch.33] it is suggested to proceed in such instances from the interests of the respective shareholders. In our opinion, this doesn’t go far enough: other economic entities are also interested in the activity of the firm, and their purposes and interests should likewise be reflected in the fulfillment of the project (it is not accidental that large-scale projects are supervised by the state). However, the structure for the corresponding efficiency criterion remains unclear.

Out of the fact that *DEI* indicator is less crude in estimating project efficiency as compared with NPV flows the desirability of its use in conjunction with **property valuation** under income approach. Indeed, according to the general principle of such valuation, the “integral effect” from the purchase and subsequent use of the property should be set equal to zero. Consequently, property value under the given approach should be estimated as the *DEI* of the "project" to use this property. Subject to this, it is essential that:

1) **The property being valued should be “small”** (in a sense specified above). This condition is not usually violated, as the acquisition of "large-scale" property requires the possession of a substantial amount of readily available equity funds (even in case when the firm takes on a loan to purchase the property, the value of its assets should be large enough to serve as collateral);
2) **Time-variable (i.e. step-dependent) discount rates** could be used to discount cash flows over the planning period.
3) Provision is made that a property created under the project grants to the firm additional rights to the assumption of debt (raises “credit appeal” of the firm). This results in some complication of estimates to be discussed later on.

Let's explain the applicability of the *DEI* indicator to the valuation of FTs. We shall consider an FT which is already in the firm’s possession at step 0. Let a_t be net cash inflows at step t that arise from this FT, c_t - its market value at this step. Then out of (9) we deduce that a valuation for the FT (λ) and the optimal moment for its sale (n) are connected through the following relation: $\lambda = \max\limits_{n}\left\{\sum\limits_{t=1}^{n}\left(\pi_t a_t + h\mu_t c_t\right)\right\}$. Out of this it follows that such FT provides the same contribution to the objective function of the firm as the receipt of the amount:

$$S_1 = \frac{\lambda}{\pi_1} = \max_{n}\left\{\sum_{t=1}^{n}\alpha_t\left(a_t + \gamma_t c_t\right)\right\} \qquad (13)$$

at the step 1. Put differently, the firm would neither gain, nor lose anything from selling the FT at this price -- such price would ensure that the deal is break-even which is characteristic of transactions with “mass property”. Therefore S_1 can be treated as the *value* of an FT to the particular firm. But then it flows from the obtained equality that **the value of an FT is equal to the sum of discounted equivalent incomes (given the discount rates that result from the model) arising from the FT over the optimal term of its use, including the proceeds from its sale at the end of this term, i.e. coincides with *DEI* corresponding to the highest and best use for this FT.** This proposition is also deducible from other optimization models of firm’s behavior, has the general character and is widely used in valuation activities.

Let's assume now that the firm starts implementing its optimal policy, but decides upon reviewing it at step 2. Obviously, it will have found the “prices” for money and loan rights, as well as the discount rates, to be the same for the subsequent steps. However, what will be found altered is the FT’s valuation: $\lambda_2 = \max_n \left\{ \sum_{t=2}^{n} \pi_t a_t + \pi_n c_n \right\}$. Hence, $\lambda = \lambda_2 + \pi_1 a_1 + h\mu_1 c_1$. But the value of the FT estimated as of step 1 will now have become equal $S_2 = \lambda_2/\pi_2$. Hence: $\pi_1 S_1 = \pi_2 S_2 + \pi_1 a_1$, or, after simple transformations: $S_1 = \alpha_1 S_2 + a_1 + \gamma_1 c_1$. In other words, the value of FT at each step (or, to be more exact, at the beginning of each step) is equal to the equivalent income yielded by the FT over this step plus the value the same FT shall have at the next step discounted towards “this step”.

It would seem that the reasoning presented above justifies the applicability of the NPV criterion (or that of *DEI*, being a modified NPV) to assessing the project efficiency (albeit that of “small” projects). However, such criterion is often challenged. For example, a few authors do not consider it proper to use the NPV criterion, as it does not reflect a distinction between the loan and depositary rates. [1, 2] suggest to account for this distinction by discounting (back to the preceding year) positive net inflows at the deposit rate, and negative ones -- at the loan rate. A similar method is, essentially, suggested in [29]. However, our model implies that the need to take on a loan is generally conditioned *not only* by the signs of net inflows. Therefore, it is quite possible that the firm may take on a loan given a positive net inflow from the project and may go without a loan even if this net inflow is negative.

Other authors regard the scope of application for the NPV criterion to be narrower. For example, [5,16] indicate that such criterion does not permit a robust comparison between projects with differing “lifetimes”, and that, for the evaluation to be "correct", it may be necessary to base the comparison on the weighted average of annual net inflows from the project (weighted by the discounting factors). Certainly, no serious justifications are adduced in favor of such a criterion (even though such or similar indicators can be found useful for solutions to other problems, see, for example, [18, 30, 33]).

As the analysis shows, the focus on annual (or average annual) net inflows may lead to gross errors in the estimates for the market value of bonds with different terms of redemption, also when making comparisons of development scenarios for oil deposits with differing terms of development and volumes of exploitable reserves, and, even more so, when comparing different available options for taking on loans with a different time structures. A more detailed exploration of other alternative criteria for investment project efficiency assessments is provided in [33]. It will be noticed, by the way, that none of the opponents to the use of NPV for project evaluation would at the same time object against its use as the indicator for property and business value. The reasons may lurk in the stronger institutional conventions (such as IVSC guidance notes) that are “behind the back” of domestic and international appraisers of property, whereas projects are assessed by designers and research advisors not united into any formal institutional structure and eager for trading criticisms.

Let's notice that the *DEI* criterion allows for estimating the efficiency *of participation of the firm in the project* (as it uses increments in cash flows to the firm arising from such participation). However, in practice the NPV criterion is also employed to estimate the efficiency of a project *“in general”* for which purpose the cash flow due to all holders of interests in the firm (so-called “cash flow before debt”) is considered (similar to as it is done in business valuations). It is represented that *such estimating practice cannot be theoretically justified*, as it would be necessary in this context to be aware of the objectives, interests and criteria for the optimal behavior that inhere in the respective "collective" participant of the project, and also to be aware of his operational activities -- which is in practice impossible.

Similar situation holds in respect of **business valuation** using income approach. We will remind that in the basis of such valuation are cash flows from operating the business acquired. The DCF method interpretation is twofold: it is viewed either from the standpoint of shareholders of the firm (“equity value”), or from the standpoint of "all stakeholders” (“enterprise value”).

The meaning of the former valuation is clear: it reflects a today's equivalent of all subsequent benefits (net cash inflows) expected by the buyer from operations of the business. The same value of equity is sought after when “the asset-based approach” is being used. Indeed, this approach “provides for the stand-alone valuation at current market value of each and every balance sheet asset, whereupon the current value of the totality of liabilities is determined and is, finally, substracted from the current market value of the sum of assets of the enterprise. The result indicates the appraised value of enterprise equity” [6, p. 111]. Thus, this approach also yields equity valuation. In other words, this valuation can be derived on the basis of at least two approaches.

At the same time the concept of "enterprise value” requires a more detailed examination. In essence, the amount of this value reflects the market value of assets of the firm. It would seem it can be found by estimating the value of each asset separately and adding together the obtained results. However it is incorrect:

according to an observation attributed to Jack London, two things added together give not only their sum, but also something third which was not separately present in either of them. In other words, the enterprise value reflects not merely the value of its constituent assets taken separately, but the emergent effect arising from the combination of these assets into a single whole. Therefore, a more sound procedure would be to estimate enterprise value in a different way by summing together the value of equity with the value of enterprise' liabilities. It will be remembered that the value of firm's equity can be estimated as the discounted sum of net inflows due from the firm, and the value of enterprise' liabilities -- as the discounted sum of payments under these liabilities (it is permissible to equate the value of enterprise debt to its nominal face amount appearing in the financial statements only if the enterprise is being liquidated). From this follows that the value of the totality of firm's assets can be found under the DCF method by applying it to net cash inflows of the firm comprised of firm cash inflows inclusive of payments under the liabilities -- "debt-inclusive net inflows".

Now it only remains to interpret the debt-inclusive net cash inflows as inflows accruing to "all stakeholders" in the firm and we arrive at the prepositions enshrined in many business valuation standards. Nonetheless, such leap of judgment is incorrect for two reasons.

Firstly, the net inflow before debt, as it has been specified, does not coincide with the "earnings before interest, taxes, depreciation and amortization" (EBITDA) as defined in the European Valuation Standard [7, GN.23], the reason here being that tax expenses, including the profit tax, should be taken into account in determining the net inflows from the firm. Besides, interest payments under the debt reduce the taxable base for profits and, consequently, the profit tax.

Secondly, it is inappropriate to attribute net inflow before debt to "all the holders of interest" in the firm. We will attempt to explain it in a greater detail. It will be noticed that the aggregate of net inflows from the firm and the payments in settlement of liabilities actually relates exclusively to the firm itself and to the holders of its liabilities ("creditors"). From this point of view, it is proper to consider only the firm (or its shareholders) and "creditors" as "the holders of interest". It is to be made clear, however, what is the "interest" and who are its "holders"? Under an "interest" we understand the expectation of benefits ("financial interest", "a card play for interest"; also note a certain parallelism between legal rights and economic interests, though not all economic interests are vested in the form of legal rights: consider an example of assembled workforce, which is of great value to any firm, but its "ownership" is not a category that can be framed in the language of "rights").

From this standpoint, "the holders of interest" are subjects (i.e. persons, natural or legal) anticipating the receipt of economic benefits from the firm. But their circle is not limited to shareholders and creditors, and also includes:

- *The state*, expecting the harvesting of taxes from the firm. Also, as it has been sardonically noticed, in this context the state nonetheless acts as a "passive investor" to the extent that it agrees to appropriate payments "unto

itself” not instantly, but with some deferment – typically, from a half-month to about a quarter-year;

- *The firm employees* expecting from the firm not only the disbursement of salaries, but also bonus rewards, material aid, grants for medical treatment and holidays and any other payments provided for in the collective employment bargain with the administration of the firm. By the way, in some jurisdictions the accounting source of many such awards is the (net) profit -- so the rights of workers to a share of the profit in this context are no different from shareholder rights. Finally, some firms also self-insure their workers, thereby assuming respective liabilities in their favor not unlike those that relate to the “creditors”;
- *Other firms* that have entered into joint investment ventures with the given firm and thus anticipate incomes from such joint activity;
- *Outsider for-profit and charity entities* and municipal undertakings anticipating the advancement of sponsorship aid promised by the firm (which can be both explicitly reflected in long-term agreements or be tacitly implied).

Such conception about the wide range of "the holders of interests” (stakeholders) in the firm is not shared by many in the area of business valuation theory (trendy sentiments about “corporate social responsibility” have yet to find their manifest foothold there). Some specialists steering the middle course believe it necessary to include among the stakeholders, beside the creditors, at least all such subjects that *invest funds into the enterprise*. But then it would be necessary to rank among them all "passive" investors, such as the state, and also those customers that commit to paying for goods in advance/ by downpayment (for they make investments into the enterprise’ working capital). The Valuation Standard doesn’t quite imply the same. Some prominent business valuation experts, e.g. Y.V. Kozyr’, think it necessary to consider among "the holders of interests” only those subjects to whom the firm owes *interest-bearing* liabilities -- as such liabilities “cost something” on the financial market. It is believed this stance is also flawed: in that case it will be necessary to discriminate between groups of such subjects that have advanced interest-free loans to the firm and those that, say, advanced a loan at 0,001% annual interest. And this is not the last of it! An agreement for the delivery of goods can sometimes provide that the buyer pay for goods in advance, but in the event of delay in delivery beyond a set date the firm is to pay penalties at a certain interest rate per every day (or month) the delivery is delayed. Here the buyer would fall outside the “the holder of interests” category before the onset of the established delivery date and shall qualify as such "holder" after that date. To decide about a circle of similar "holders" as of some date, appraisers of business would have to review all delivery agreements and become aware of interest chargeable under each of them. Similarly, if salaried workers of the firm fall behind the due date to get paid (and in such situations the worker does act as a creditor of the enterprise), the firm is obligated under the law to make good for the late payment to the workers by adding an interest component on top of its

debt to them. As we have the occasion to see, such distinctions of "the holders of interests" are not at all easily implementable in the context of practical business valuation problems and are not in the spirit of business valuation standards where they deal with the cash flow structure and estimates.

The matter is not so much about circle of the "holder of interests" constituents, as much that it includes not one, but *several* subjects. No matter how the group of the several "holders of interests" was formed, the valuation of business from its perspective would mean that all the members of this group share one and the same set of interests. It would be a fanciful exercise to imagine an oil refinery valuation in Grozny under the conditions where the banks are interested in the soonest return of the loans, the state -- in the minimization of expenses for the factory reconstruction, and the proprietors – in the secession of the Chechen Republic from Russia (it is likely that some of them would have consented to a rather modest profit from the business, had it eventually helped to bring about such a secession). Put differently, the unity of shareholders, creditors or any other stakeholder subjects cannot be either the buyer, or the seller of a given business,-- all of its constituents have different objectives and disparate interests, and to sum up monetary receipts flowing onto them (or their expenses) would be like summing up the income of a casino with the income of its players, or incomes of fishermen with the incomes of a chemical plant polluting the river with its waste.

Interests of economic subjects are discussed in [24] where the term "efficiency" is understood in the sense of conformity of the project to the objectives and interests of its participants. However, the sense of "participation" involved here is broad -- such that among the participants of a bakery building project would also be included the inhabitants of nearby houses for whom, in all likelihood, it will become inconvenient to go out on the streets or to take out for a walk their children and dogs. Therefore the circle of participants of the project and the character of their interests is not limited here (but another problem is also gracefully bypassed: how, through what monetary indicators, to reflect in the estimates the diverse interests of different participants). Nevertheless, this document offers a certain analogue to estimating "enterprise value" – the calculation of "the efficiency of the project as a whole". Such calculations exclude a consideration of loans and other interest-bearing liabilities, but this is justified a little bit differently. Namely, it is said that such a calculation would provide an estimate of the efficiency of the project from the standpoint of a real or virtual entity implementing the project completely at the expense of its own funds. What is the extent of the need for such indicators of efficiency – is a different question, but it is clear that they do not characterize the conformity of the project with the objectives and interests of its real participants.

From this point of view, the logic of approach for estimating "enterprise value" expounded in the Standards would seem erroneous. If such value is to be determined, it is best done as explained above: by subtracting the discounted sum of payments under liabilities of the firm from the value of its equity. Besides, it is not very clear who would need such a value. After all, by buying a firm or its stocks, the respective entity also acquires its liabilities or their corresponding share. The assets of a firm separated from its debts can be subject of a market transaction only when the firm had gone bankrupt, but in this case it is more of a question about the liquidation value of the assets (determinable under different methods), rather than the market value.

Probably, the Standards imply the situation where a large firm owning several enterprises disposes of one of them. However, in this instance it is necessary to give thought to the fact that the issuer of liabilities as well as the taxpayer here is the firm, rather than its enterprise (a firm division). Therefore, liabilities to creditors and the state remain vested in the firm, and liabilities to workers, to local authorities and to partners in other investment projects – in those who accepted them (i.e. they rest either with the administration of the enterprise or the firm management). If the firm has earlier accepted obligations for the enterprise development financing, some of them (depending on the terms of sale) may continue to hold good, others will pass on to the buyer, yet others - will be annulled (no general solution can be suggested here; for an example of this, consider a case of a factory sale during the period when the firm personnel carries out major repairs of one of its producing factory divisions and has not yet completed the construction work and the installation of an equipment). The general principle for the constitution of cash flows for the enterprise valuation is still the same: these flows should relate to the enterprise only, but not to other "holders of interest".

Inexpediency of the "enterprise value" assessment basis is also admitted in [6, p.114-115]: "the use of cash flow to the invested capital model (before-debt cash-flow basis) is barely applicable under the instant Russian conditions and is limited to situations involving large holding companies, due to the extremely rare use by a great bulk of the Russian companies of schemes for attracting financial resources similar to the western ones".

For these reasons, when discussing business valuation further below, we will imply only the valuation of equity under income approach.

Let's consider in a greater detail a situation where the firm ("with the project" and "in the absence of the project") does not brush up against the restrictions on the volume of loans. Here all $\mu_t = 0$ and the criterion (11) fully coincides with the conventional NPV, though it is being derived on the basis of different considerations. This inference, however, will be fair only if the project is *small*, i.e. it influences the valuations of FTs in but a slight measure (although the smallness of the project is objectively needed for any local calculations of efficiency and is not particularly associated with the considered model).

The maters that we discussed allow us to consider critically one of the myths associated with the determination of the discount rate. Quite often this rate is portrayed as the maximum return from *any* alternative investments available to the investor, with the NPV of the project then reflecting the effect received at realizing this project -- as compared with the performance of the best of *all other* alternatives. If we agree with such "primitivized" approach to the determination of the discount rate, then the assessment of any one project should rely on the discount rate taken at the level of return afforded by another, alternative, project (this is sometimes recommended by a number of authors). However, as an example 1 makes clear, we already saw that it leads to errors. Now it is possible to explain the source of such errors in a greater detail.

When assessing a project providing the cash flow $\{f_t\}$, we compare values for the objective function of the firm at its best performance "with the project" vs. "absent the project". There, therefore, arise two optimization problems in which the operating incomes in (4) differ by f_t. If a project is small, the criterion (objective function) will change the same as at the one-time receipt of the lump-

sum $DEI = \sum_t \alpha_t \varphi_t$, and the dual estimates and, consequently, the discount rates (12) will not change perceptibly. But then it follows that the evaluation of any other variant of the project will be predicated on the similar formula with the same values for the discounting coefficients. It is material that these coefficients will depend on the returns from individual FTs, from the firm's financial position, from its operational activities, from the specificities of liabilities issued by the firm, but will not be contingent on what other variants of the project the firm can perform. In other words, discount rates are determined not by returns on all alternative projects, but only at the nexus of those projects which simultaneously constitute firm's "directions of investment". At the same time, Projects A and B in the example 1 do not amount to the investment directions of the firm, for they could only be performed once, at a certain scale and over a certain time. Therefore, the returns from these projects do not influence the discount rate in general.

Accepting the returns from any one project for the discount rate at evaluation of another may present us with some "psychological" difficulties as well.

Example 7. Slapdashbuilding ltd. decides to acquire a parcel of land near a metropolitan area, and to redevelop it into cottages for subsequent resale. After considering the project design documentation, it was found that two variants of the project are feasible. Both variants provide for the duration of the construction period of 1 year, the cottages are expected to sell upon being constructed at - $15 mln. The Variant 1 requires that the investment costs of $10 mln be advanced, the Variant 2 differs only in that a larger playground area is contemplated (the additional costs being $100).

In this case, the returns on Variant 2 shall amount to (15 000 000/10 000 100) -1 = 0,499985. If we employ such a rate for evaluating the Variant 1 project, its NPV will constitute: 15 000 000/ 1,499985 - 10 000 000=100 dollars, which, as a matter of fact, is what we should have expected. However, having learnt that such a "million dollar" project provides an effect of only 100 dollars, the shareholders have shown their discontent and have motioned for the change of the managing director of the project, whereas the project crediting bank has decided not to make haste with advancing the necessary loan funds. From the both quarters it was suggested to the Jerrybuilding ltd. managers to think of developing more effective projects.

What is the matter? The value of 100 dollars does really reflect the effect from implementing Variant 1 project, but only as compared to the **alternative variant 2**. Meanwhile, managers of the firm and the bankers in this case are more interested in learning about the influence of the project on the results of economic (operational) activity of the firm. In other words, an indicator of the performance effect allowing to compare a situation "with the project" with "absent the project" situation is what they seek, instead of comparing "with a Variant 1" vs. "with a Variant 2" situations. Such indicator is afforded by *DEI* (11), and the discounting factors entering into it are independent of the performance indicators under both variants of the project.

Thus, a "primitivized" treatment of the discount rate distorts its content and may result not only in errors during the efficiency evaluations, but also complicate understanding of the results of such estimations.

The examples that follow are focused on the construction of the optimal policy of the firm and estimation of the discount rates corresponding to it. The first of them illustrates that the association between the discount rates and the rates of return on FTs can be complicated enough, and that the rate **may not represent the maximum returns** (in any usual sense of this term) achievable on any of the FTs, as we have had the occasion to note in the example 3.

Example 8. Only one-year and two-year deposits are available for investing into at the FT market. The emission of liabilities (the use of loans) is not provided for. Prolongation of the deposits is not permitted,

and the interest on the deposits is foregone in case of their premature closing. (Time-variant) depositary rates are specified in Table 1.

Table 1.

Years (T)	0	1	2	3,4,...
The rates on 1-year deposits , %	20	18	15	12
The rates on 2-year deposits , %	48	42	34	27

The optimal policy turns out to be as follows. On even *T's* all the funds should be placed into two-year deposits, and nothing should be done in the "intervening" years. At $T = 3$ the funds should be put up first into the one-year deposit, and then reinvested into the two-year one. For odd $T \geq 5$ it will be found best to be re-investing two times into two-year deposits, then – switching to the one-year deposit, and after that re-switching again into the two-years.

The values for the discounting coefficients(factors) and the values of discount rates corresponding to them are calculated based upon our model for different *T's* and are collated in Table 2 (in this instance, the values for α_t recur for any length of the planning period at $T > t$ -- to each length of the planning period T there corresponds its own sequence of discounting factors $\alpha_1,\ldots,\alpha_T$: the value of α_1 will be identical in all such sequences/series with $T > 1$, ditto for α_2 in series in which $T > 2$, … , thus, the value of α_t will be one and the same in all series with $T > t$).

Table 2.

Year (t)	1	2	3	4	5	6	7	8
Discounting factor α_t	0.8333	0.6757	0.5869	0.5042	0.4502	0.397	0.3545	0.3126
Discount rate E_t ,%	20.0	23.3	15.1	16.4	12.0	13.4	12.0	13.4

It shall be noted that for greater *T's* the discount rate would fluctuate periodically, and there would be no "asymptotic" rate at the margin. Probably, this shall always be the case if we limit ourselves to considering a finite number of FT types recurring in time. In real-life the situation would be different, since financial transactions are performed daily, and the values of FTs constantly fluctuate.

Let's pass on now to a more realistic scenario where the firm perceives a need for borrowed resources. Here the second member in the criterion (11) comes into play. The essence of the matter is that the commissioning of additional fixed assets expands firm's opportunities for attracting the borrowed capital so, other things being equal, project variants with a greater volume of fixed asset capacity inputs become more preferable. In other words, the criterion (11) would orient planners towards such project realization options where new asset capacity is pressed into service by the moment the firm feels the need for attracting additional funds. The discount rate increases in such cases and, as we saw, can even exceed the loanable rate of interest, not to mention the depositary rate. In business valuation practice, if the firm needs sustaining its loan funding, this circumstance is quite often reflected through inclusion of a so-called risk-premium into the discount rate [8, 15, 23]. Our analysis shows that such enhancement of the "usual" discount rate would also be motivated by the objective reasons even in the risk-free (deterministic) situation. Therefore, even though the basic premises of the enhancement method applied by the appraisers seem more than disputable to us, their practical actions certainly are aimed in the right direction.

On the other hand, as our analysis shows, the heightened need of the firm to borrow funds is more correctly handled through introducing additional valuations of its liquid property into the framework of efficiency calculations (in our case, this role is played by γ_t) which would reflect neither the "historic costs", nor the

foregone income from selling the property, but the assessment of the right provided by the property to the incurrence of the debt (first and foremost, the right to attract borrowed funds).

Thereupon, we will also notice that valuation of property under the DCF method disregards the capacity of property to serve, for example, as a collateral for the security of the loans and facilitate the receipt of borrowed funds (for any commercial needs -- not necessarily associated with the use of the property). It is represented that this "value component" of the property should be accorded a due attention in the valuation profession. We will further provide an example dealing with the optimization of firm's policy taking into account the loan volume restrictions in borrowing.

Example 9. A 9-year-long planning period is contemplated. The firm enters the period with the possession of $250 in cash. Each year it can place its cash on deposits – a one-year deposit (1D), or a two-year deposit (2D), and can also take on one-year loans (L). Table 3 lists the corresponding rates d_{1t}, d_{2t} and r_t (they are stated in percentages and relate to the year of opening the deposit or receiving the loan). Credit (i.e. loan) volumes are limited to 60% of the available asset value (h=0.6). On top of that, Table 3 contains the optimal solutions to the primary and dual problems: volume of investments into deposits, loan volumes, discount rates (E_t) and, last but not least, the unit valuations (in present value terms) of loan access rights (γ_t). The hoarding strategy is proven inefficient in this case, so zero volumes of the stored cash on hand are not mentioned in the table. To keep the table short, the last year indicators ($\pi_9 = \gamma_9 = 1$) are omitted.

Table 3

t	d_{1t}	d_{2t}	r_t	1D	2D	L	π_t	μ_t	E_t	γ_t
1	20%	48%	22%	0,0	250.0	0,0	3,567	0.000	21.5%	0.000
2	21%	43%	18%	265,0	110.0	375.0	2.935	0.175	25.5%	0.036
3	16%	40%	17%	0,0	785.3	537.2	2.339	0.046	19.3%	0.012
4	14%	32%	16%	0,0	0.0	471.2	1.960	0.089	21.5%	0.027
5	12%	27%	15%	0,0	552.9	0.0	1.613	0.000	13.4%	0.000
6	12%	27%	14%	0,0	0.0	0.0	1.422	0.000	12.0%	0.000
7	12%	27%	14%	0,0	702.1	0.0	1.270	0.000	13.4%	0.000
8	12%	27%	14%	0.0	0.0	0.0	1.120	0.000	12.0%	0.000

The calculation shows that by the planning period's end the firm will have accumulated in equity the amount of 891.72. It is interesting to note that for the first years the discount rates exceed both the depositary and loanable interest and that they are not stabilized in the latter years, as had been the case in the example 9.

Let's assume now that the firm is offered a participation in a project providing the following cash flows and values of the fixed assets (pay attention to the fact that the initial investments exceed the cash funds available in the firm):

Table 4.

Years	1	2	3	4	5	6	7	8	9
Net cash inflow	-310	-70	145	180	150	135	75	50	0
Value of fixed assets	300	375	320	275	200	150	100	50	0

We shall assess the efficiency of this project in three ways.

1. *Direct solution of the optimization problem*. By solving the presented optimization problem having regard to the changed values for F_t and G_t, the following results would be obtained (Table 5):

Table 5.

t	1D	2D	L	π_t	μ_t	E_t	γ_t
1	0.0	0.0	24.0	3.609	0.000	22.0%	0.000
2	66.8	0.0	124.1	2.959	0.177	25.5%	0.036
3	0.0	148.5	161.1	2.357	0.149	24.9%	0.038
4	0.0	94.0	205.5	1.887	0.017	17.0%	0.005
5	0.0	29.5	0.0	1.613	0.000	13.4%	0.000
6	162.7	15.4	0.0	1.422	0.000	12.0%	0.000
7	0.0	250.7	0.0	1.270	0.000	13.4%	0.000
8	40,6	0.0	0.0	1.120	0.000	12.0%	0.000

Thus, the firm's equity "with the project" will have sustained an increase from 891.72 to 914.24 by the period's end, i.e. will have grown by $22.52 -- which result testifies of the efficiency of the project. Despite large enough scale of the project, the solutions of the dual problem have not changed in a pronounced way.

2. Let's estimate the efficiency of the project *on the basis of information available on it* (table 4) *and dual estimates for the optimal plan of the firm* (table 3). It is easy to figure out that *DEI* = 8.09 in this case. Given this, it is possible to provide an estimate of the amount by which the firm's equity will have grown by the end of the period due to its participation in the project. The respective calculation yields: $8.09 \times 3.567 = 28.9$, i.e. a little more than the result indicated in the calculation under par. 1. As a matter of fact, this result should have been expected: if the change in the objective function in a model of linear or convex programming is determined by means of the dual estimates, such analytical process always overestimates the result, and the difference is the more, the greater is the change in the parameters of the model (in this case – the larger is the scale of the project).

3. Assuming that the estimates for the loan access rights are negligible, let's ignore them and try estimating the 'integral effect' from the project in a conventional way or, which is the same, under the formula (11) -- ignoring the *influence of fixed assets*, but using the discount rates specified in table 3. Doing such calculation shows the value of the "integral effect" being negative (-9.56) and, were we to put trust in this approach, we would have concluded about the inefficiency of the project. This precedent thus shows that ignoring the discussed loan access rights effect can lead to wrong investment decisions.

Certainly, it is hardly possible to assert that real investors when "deciding the destiny" of their projects ever consider the inter-temporal change in the discount rate or the capacity of the property to facilitate the assumption of loans, or that they are ever guided by the criterion of a (11) type. Nevertheless, such a criterion allows explaining why:

- Different investors, when evaluating the same project on the basis of the same information, may provide contrasting assessments (with some deciding to take part in it, and others declining the participation);
- Having once declined participation in the project, an investor may reverse his decision after a time without his underlying information set undergoing any material changes (though, seemingly, such a delay should have an effect of reducing the efficiency of the project);
- If a division of a large-scale firm implements a highly efficient project, the firm may sometimes scale down or even temporary discontinue its funding (though, seemingly, it should materially lower the efficiency of the project);
- Decisions on the realisation of large-scale projects are often taken after careful analysis of the balance sheet of a firm.

Optimization of operating activities

When studying economic analysis, it always transpires that the best time to buy was in the last year

An Aphorism from the Internet

In the foregoing treatment of the subject, the operating activity of a firm was always assumed as exogenous, and only its behavior on the financial markets had been considered for optimization. Meanwhile, the strategic planning also envisages the optimization of operational activities (which in our model implies both "the basic production" and the participation in real investment and innovative projects).

An important aspect of this activity consists in the following. It is possible to transition continuously from any one variant of the financial activity to another by changes in the corresponding model variables. The feasible sets of these variables form a convex set, and the objective function appears as a convex function of these variables. The situation is different with the operating activities. Here too there are some guiding parameters that can be adjusted "almost" continuously (volumes of production output, the size of production stocks, etc.). However, given this, some cash receipts and expenses may at the same time experience non-linear changes. Thus, operating costs of an industrial enterprise are subject to non-linear dynamics given the change of its capacity or a change of relative proportions in the mix of its production output [30], costs for rail transportation would also undergo non-linear shifts as the cargo flow fluctuates on the corresponding line [17], etc.

Under these conditions, the optimization problem for operational activities can be stated as follows. We shall assume that a firm can adjust its operational activity by continuously changing some of its guideline parameters (for example, volumes of output for certain kinds of goods or the deadlines for the commencement of certain activities) constituting a vector $\boldsymbol{u} = (u_1, \ u_2...)$. Then, at each step t, the net operating income F_t and the market value of fixed assets G_t become a function of $\boldsymbol{u}$ which we shall assume to be "smooth enough". Whereupon, it is possible to seek solutions to the optimal control of $\boldsymbol{u}$ in the framework of the same model (3) - (6).

It is easy to demonstrate that now some new restrictions on the guideline parameters would be added in the dual model:

$$\sum_t \left(\pi_t \frac{\partial F_t}{\partial u_j} + h\mu_t \frac{\partial G_t}{\partial u_j} \right) = 0 . \qquad (14)$$

Their meaning is immediately apparent. Let's consider a "small" project consisting in the increase of u_j by a small increment. Such a project would result in the increment of net operating incomes and the market value of fixed assets of the magnitude $\frac{\partial F_t}{\partial u_j}$ and $\frac{\partial G_t}{\partial u_j}$, respectively. If the vector $\boldsymbol{u}$ is optimally chosen, such a project would not change the value of the objective criterion (more precisely, - its change will be on the second order of smallness). It would mean that *DEI* of such a

project, calculated under the formula (11), is equal to 0, and this fact is expressed in the resulting equalities above.

Let's consider an interesting special case of the problem at hand. The firm plans on changing its operating capacity at step 0. Therefore, there is only one unique guideline parameter u – the capacity -- which, we assume, can be changed continuously. It is supposed that the optimum of the capacity is positive, that the firm does not “scrape” against the restrictions on the loan volumes at the optimum of the policy being achieved (i.e. all $q_t = 0$), and that the discount rate is one and the same at all steps, so $\pi_{t-1} / \pi_t = 1+E$. Then the condition (14) transforms itself into:

$$\sum_t \frac{\partial F_t / \partial u}{(1+E)^t} = 0. \qquad (15)$$

The magnitudes $\partial F_t / \partial u$ — being the increments of the net cash inflows to the firm attendant on the change in the optimal capacity by a small unit – it is natural to treat them here as net cash inflows from the “incremental” project implying a small change of capacity. Then (15) would imply that the discount rate here coincides with the IRR of the specified “incremental” project and so is characterized not only by the return on the FT investments, but also by the returns on investments into “own production”. The solution thus appears such as if a “project” for enhancing the capacity by a small unit has been added to the set of FTs (just because such a unit is small makes it legitimate to regard the "project" as being a replicable one). In other words, it is possible, in this context, to define the discount rate as the maximal return on the alternative directions of investments, with the projects for a small capacity change (in comparison with its optimal, rather than the existing level) being included in the set of such “investment directions”.

The need to consider investments into the “own production” is intuitively felt both by investors and the appraisers, thus, one quite often comes across an opinion that the choice of discount rate should be informed by the settled level of a firm’s (operating) performance returns.

For example, [14] suggests that when comparing prices for assets distinguishable by different terms of their service life, one can employ for the discount rate “a risk-less rate, if the purchaser has no business of his own, or the rate equal to a projected returns in the own business of the buyer or the seller.”

At the same time it is theoretically inadmissible to equate the discount rate to the profitability of firm’s own business operations: a settled level of *average* firm’s returns (even if stable overtime) can materially differ from the *marginal* returns on investments, even though the big mismatch between them is not likely to be seen too often. At the same time even from the specified incorrect position the author, the known appraiser, does absolutely fair, but running counter to estimation Standards (and consequently courageous enough), a conclusion about **distinctions of rates of discount at different market makers**: “it is necessary to mean that the

seller and various groups of buyers differently estimate for themselves the discounting rate: as a rule, for the majority of buyers the rate of discounting more low, than for the seller (as they or have no own business, or if have, its profitability can yield to profitability of trading object)" [14, p. 188].

Let's now address the following interesting problem. Assume that a firm has somehow selected a discount rate and evaluates under the DCF method the expediency of acquiring a new machine tool. If the NPV (or *DEI*) of the project of such purchasing appears negative, the situation is clear: the purchase is inexpedient. A case where NPV=0, obviously, is unlikely. But how to act, should it happen that the NPV > 0? It would seem that the answer is again obvious here: the machine tool should be acquired. Meanwhile, such an answer *is formally incorrect*! The matter is that the project of purchasing a machine tool is repeatable to some extent, and each new acquisition by way of an additionally commissioned machine tool should improve the financial position of the firm. Therefore, apparently, the firm should purchase *as much of* such machine tools *as possible*. But, after all, this obviously contradicts common sense! What is the matter? It appears, the matter lies in an essential difference of machine tools from FTs: projects to purchase machine tools, unlike FTs, *are not replicable projects*. Assume that the firm has embarked on acquiring the additional machine tools one by one. Its technical and economic indicators of operational performance will, in consequence, undergo a change, but their dependence on the quantity of employed new machine tools will be nonlinear: the more machine tools the firm will purchase, the more changes will it have to introduce into the organization of its production technology, into the activities of its basic and auxiliary units, its sale and advertising system, etc. At last, there will come such a moment when purchasing an additional machine tool would seem to be already inefficient. It will then be notionally possible to say that the "returns" on purchasing the last (marginal) machine tool "even up" with the discount rate,-- which, as a matter of fact, is what is expressed by the formula (15). From here, by the way, it is possible to make an inverse deduction: if the rational policy of a firm provides for the acquisition of machine tools, the discount rate would not exceed the returns on this "acquisition". On the contrary, if the rational policy of a firm does not provide for machine tool purchasing the discount rate would have to be in excess of the "profitability" of this purchasing. These rules may appear useful for practically estimating discount rates for specific firms.

It would be useful to bear in mind that equalities (14) express only the necessary, but not sufficient, conditions for optimality. The essence of the matter is that nonlinearity of the functions $F_t(u)$ and $G_t(u)$ may often be a "bad" non-linearity. The optimization problem may thus happen to be "non-convex", with the multiplicity of local optima often far enough removed from the global one (see also [20, ch.4]).

At the same time, impacts on the operational activity of a firm can be "discrete" (non-continuous), where the firm should choose the best out of several possible variants of decision-making (for example, a type of a raw material used or its supplier, a type of equipment or vehicles to acquire). But it also just means that

there are some different methods for using a property, out of which it is the appraisers' task to choose the best, the most acceptable for the buying firm, and this is also the problem in discrete optimization of the operational activity.

It is essential that in all problems of such a kind it is impossible to pass continuously on from one choice in decision-making to another. In these situations, the "operational" policy of a firm is determined not only by a "continuously changing" vector parameter $\boldsymbol{u}$, but also by a vector parameter $\boldsymbol{e}$, having a finite number of values $\boldsymbol{e}_1$, $\boldsymbol{e}_2$... To each *k-th* combination of these values there, generally speaking, correspond functions $F_{kt}(\boldsymbol{u})$ and $G_{kt}(\boldsymbol{u})$, and, accordingly, - own optimal decision-making $\boldsymbol{u}$ and the value of the optimality criterion. Therefore, there is nothing better left but to consider each possible vector $\boldsymbol{e}_k$ separately, solving the corresponding optimization problem and finding a corresponding value of the objective function-- with that *k-th* combination being chosen where this value is the greatest. It would be difficult to suggest something more simple and convenient by way of a method for the general case, though it is certain that for some specific problems more efficient short-cuts can be found having regard to the specificity of problem-related discretely changing parameters there. But for us the following observation is of a greater importance: Generally speaking, to different $\boldsymbol{e}_k$ there would correspond different estimates for p_{it} and q_t, and, consequently, different α_t and γ_t.

Therefore, even if we were to know the discount rates and valuations of loan access rights attendant on some vector $\boldsymbol{e}_1$, they cannot be used with abandon for evaluating the efficiency of choosing some other vector $\boldsymbol{e}_2$ (especially if the transition from $\boldsymbol{e}_1$ to $\boldsymbol{e}_2$ is associated with some cardinal changes in the functioning of the firm). In other words, if there exist starkly alternative scenarios for firm's development (which is most often the case in strategic planning problems), the choice of the best of them can't be informed by any local optimality criterion (neither NPV, nor any of its modifications).

Accounting for inflation and settlements in foreign currencies

> The prices have been in free float for two hours now, but the economy has not yet stabilized.
>
> Russian TV news, a morning broadcast of Jan 2nd, 1992

Up to this point we have been silent about inflation. It does not mean that we have completely disregarded it. On the contrary, when dealing with asset prices and their exchange rates, we assumed they change overtime, i.e. the presence of inflation. Therefore, strictly speaking, solutions to the primary and dual problems and, consequently, the resulting discount rates, do incorporate inflation. We will consider this in greater detail.

Let it be noticed in this context that, for the purposes of description, the inflation is subdivided into general and structural. General inflation reflects the

overall growth of prices in the country (the change of purchasing power of local currency), whereas "structural inflation"[4] reflects the change of ratios (proportionality) between the prices for various goods and services, i.e. their relative rise in price or reduction in price. General inflation is characterized by chain and fixed-base indices. The fixed-base index of general inflation J_t reflects a price level at step t in comparison with that prevailing at step 1 (so $J_1 = 1$). A chain index j_t reflects a rise in prices that has occurred over step t, i.e. the price level at step t+1 as it compares with the price level instant at step t. Therefore $j_t = J_{t+1} / J_t$. In the model included in section "Optimization of financial policy in the imperfect market context", the structural inflation is considered explicitly: the ratios between the prices of various assets, i.e. their rates of exchange, are set changing in time. The general inflation is reflected in the absolute parameters of the model (net inflows from the operating activities, returns on assets and their market values) which also change over time. What impact does it exert on discount rate?

The need to answer this question is motivated by the developed assessment practice for investment projects. The matter is that it is customary that such calculations are initially carried out in constant prices as at the "basic moment of time", which is assumed to be the date of assessment or the beginning of the year in which the project assessment is made (the constant price method). The change in prices in comparison with this constant basis is accounted for only later. An advantage of estimating in constant prices is the consistency ("obviousness") of the results: the amounts of incomes and expenses expressed at constant prices have a clear meaning to the participants in the project. Similarly, a statement that "*DEI* of the project constitutes $ 147 mln." would be understood by a participant of the project as implying that the realization of the project would be as beneficial to him as the receipt of the amount of $147 mln. at the initial moment of time.

On the other hand, efficiency estimates can't do without accounting for both the general change in the level of prices in the country, and changes of ratios (proportions) between the prices for individual resources, goods and services. It means that to achieve an acceptable level of the accuracy of calculations, all cash flows from the project should be specified at the prices for corresponding years (forecasted prices). However, as we saw, something of a consistency is thereby lost – the parameters pertaining to different years appear to be expressed at disparate prices and their subsequent aggregation may strongly smack of the summation of acres with hectares, or pounds with kgs. Therefore, the existing guideline documents [24, 33] provide for the subsequent re-casting of all cash flows expressed in the forecasted ("nominal") price format into the forecasted but deflated format (the latter operation is done by applying a fixed-base deflator index to the forecasted prices). The deflated prices, generally, would not coincide with the constant price format, but are comparable to constant prices – since 1 deflated

[4] The term "structural inflation" is here used in other than its conventional macroeconomic sense ("organic inflation without particular causes"), rather it implies shifts in the relative "structure" (hence—"structural") of prices in the country. Structural inflation reflects that the inflationary process is non-uniform ("non-neutral") and affects different commodities in a different way.

dollar has the same "purchasing power" as 1 dollar at the “basis moment of time”. Statistical estimates often deal with comparing incomes or expenses for different years. If the indicators being compared are expressed at the prices for corresponding years, their observable dynamics is characterized by its specification as a "nominal" one; if they are expressed at prices deflated to some (but one and the same) moment in time, their trend would be referred to as "real" (for example, we can speak about the real growth of a median salary). In terms of this vocabulary, the recommendations [24, 33], in essence, require the analysis to be carried out on the basis of real, rather than nominal, cash flow dynamics.

Estimating the efficiency of the project on the basis of real (deflated) cash flows would require the use of different, *real*, discount rates. The presented model provides for a way of deriving them. Indeed, 1 “real” dollar at step t, by definition, is equal to J_t nominal dollars, so its dual estimate (shadow price valuation) should be equally proportioned by J_t - i.e. be equal to $J_t \pi_t$. But then the real discount rate, i.e. the rate of decline in these valuations, will be determined as $E_{tr} = J_t\pi_t / J_{t+1}\pi_{t+1} - 1 = \pi_t / j_t \pi_{t+1} - 1$. Therefore, nominal ($E_t$) and real ($E_{tr}$) discount rates are related to each other by a ratio:

$$1 + E_{tr} = (1 + E_t) / j_t , \qquad (16)$$

which is similar to the renowned Fisher formula.

It appears, the same values for E_{tr} can be obtained using not nominal, but real cash metrics in the model (3) - (6). To see this, let’s reformulate this model away from relying on nominal and towards real business metrics – for volumes of cash, indebtedness under the liabilities, values of fixed assets and net operating income. This recasting of the model is performed by dividing all meaningful parameters by an index for general inflation that relates to the corresponding step. All the other variables remain unchanged. Presently, we shall introduce a related notation for "real" (deflated) value parameters and "real" (calculated under Fisher's famous formula) interest rates - we denote them with the barred notation:

$$\bar{F}_t = F_t / J_t , \bar{G}_t = G_t / J_t , \bar{l}^{mt} = l^{mt} / J_t , \bar{a}_{ist} = a_{ist} / J_t , \bar{c}_{ist} = c_{ist} / J_t , \bar{r}^{mt} = \frac{1 + r^{mt}}{j_t} - 1 .$$

Aided by this notation, the following is derived out of the system (3)—(6):

$$\sum_s v_{is} = N_i , \quad (i < 0);$$

$$\sum_{i,s} \bar{a}_{ist} v_{is} + \bar{F}_t - \sum_m y^m \left[\bar{l}^{mT-1} \left(1 + \bar{r}^{mT} \right) - \bar{l}^{mT} \right] \geq 0 , \ (0 < t < T);$$

$$\sum_i y^m \bar{l}^{mt} \leq h \left[\sum_{i,s} \bar{c}_{ist} v_{is} + \bar{G}_t \right];$$

$$\bar{V}_T = \sum_{i,s} \bar{a}_{isT} v_{is} + \bar{F}_T + \bar{G}_T - \sum_m y^m \left[\bar{l}^{mT-1} \left(1 + \bar{r}^{mT} \right) - \bar{l}^{mT} \right] \Rightarrow \max .$$

As we have the occasion to see, the derived equations are characterized by the same form as the original ones. Besides, it is the maximization of **real**, rather than *nominal*, volume of cash by the end of the planning period that results as the objective optimality criterion here – which, by the way, may seem more sensible and intuitive to investors. Naturally, the dual model will also be similar to the system (8) - (10) -- with that only difference that its variables will reflect real, rather than nominal, shadow price valuations. The **real discount rate** will thus be determined, as before, **by the rate of decline in the real (dual) valuations of cash**. It is easy to demonstrate that this rate can be derived from the nominal rate by means of the Fisher's formula (16). On the basis of this, it can be concluded that, in case the monetary results of a project are expressed in real (deflated) prices, the evaluation of its efficiency can be produced under the same formula (11) with the use of real, instead of nominal, discount rates. The application of this estimating approach is seen often enough in the project assessment practice, though not universally. At the same time, property and business valuations are usually produced with the use of nominal discount rates. The characteristic errors in this context creep not into the discounting procedure per se, but relate to the forecasting of cash flows (proceeding on the basis of forecasting “from the reached level” it would not be possible, as a rule, to adequately capture time variations in the nominal interest rates and for the rates of price increases, or the rates for foreign exchanges).

Let's discuss now a situation where the project envisions payments in foreign currencies (for example, in the context of importation of goods, or assumption of a loan nominated in a foreign currency). It is asserted in [33] that such cases should necessarily be handled by translating all cash flows into the national (measurement) currency, using a forecasted rate for foreign exchange. However, it remains unclear whether such a resulting *DEI* would really reflect the "value" of the project from the standpoint of its participant, or a "contribution" of the project to the corresponding objective function. It may seem the issue can be settled by modifying the model presented in section “Optimization of financial policy in the imperfect market context” and including the foreign exchange into it as one of its constituent FTs. But such a trick "would not pass": in this model all the cash flows should be expressed only in the national (measurement) currency. Besides, the prices and returns on some of the FTs are denominated in terms of foreign currencies, and they are also often acquired and sold abroad, i.e. *exchanged* into foreign exchange. However, directly exchanging one FT for another is not provided for in the model. Thus, the model has to be re-specified. For reasons of simplification, we shall assume that there exists only one foreign exchange currency. We shall designate it simply as the "foreign currency", and the corresponding accounting units – *foreign currency units*.

All the parameters relating to foreign currency denominated FTs, liabilities and cash flows shall be denoted using an additional index specification "*c*" (for “ currency”). We will also assume that foreign currency denominated FTs and the national measurement currency, say, *dollar* can be bought and sold for foreign

currency units only, whereas dollar-denominated FTs and the foreign currency are bought or sold for dollars only. As the model now specifies transactions involving exchange of dollars into the foreign currency and back, we will have to introduce the following notation:

b_t — market rate of foreign currency exchange at step t (dollars/foreign currency);

b_{ct} — the exchange rate for dollar offers at step t (foreign currency/dollar);

The model will also feature two new groups of non-negative unknowns:

x_t — the amount of dollars the firm is selling (exchanging into foreign currency) at the beginning of step t;

x_{ct} — the amount of foreign currency units the firm is selling (exchanging into dollars) at the beginning of step t.

The major relations in the model will therefore look as follows.

1. The balance of dollar- and foreign- currency denominated FTs in possession of the firm at step 0:

$$\sum_{s\le 0} v_{is} = N_i; \tag{17}$$

$$\sum_{s\le 0} v_{cis} = N_{ci}; \tag{18}$$

2. The balance of dollars and foreign cash at each step t ($0 < t < T$):

$$\sum_{i,s} a_{ist} v_{is} + F_t - \sum_m y^m \left(l^{mt-1} - l^{mt} + r^{mt} l^{mt-1} \right) + x_{ct} b_t - x_t \ge 0; \tag{19}$$

$$\sum_{i,s} a_{cist} v_{cis} + F_{ct} - \sum_m y_c^m \left(l_c^{mt-1} - l_c^{mt} + r_c^{mt} l_c^{mt-1} \right) - x_{ct} + x_t b_{ct} \ge 0; \tag{20}$$

3. It is too complicated in the general case to write down restrictions on the total amount of liabilities, as the terms and conditions for foreign currency- and dollar- denominated loans can differ. To simplify the matters, we shall assume that the debt under all liabilities, expressed in dollars, should not be in excess of the dollar-denominated “mortgage value of assets” -- at all steps, excepting the last (it is implied that the market value of fixed assets is expressed in dollars only). If, given this, the foreign currency indebtedness and the value of foreign currency denominated FTs are translated into dollars at the rate of foreign exchange sales, this restriction will assume this form:

$$\sum_i y^m l^{mt} + b_t \sum_i y_c^m l_c^{mt} \le h \left[\sum_{i,s} c_{ist} v_{is} + G_t + b_t \sum_{i,s} c_{cist} v_{cis} \right], \quad (0 < t < T). \tag{21}$$

4. The firm maximizes the value (denominated in dollars and translated at the offering exchange rate) of equity at step T:

$$V_T = \sum_{i,s} a_{isT} v_{is} + F_T + G_T - \sum_m y^m \left(l^{mT-1} - l^{mT} + r^{mT} l^{mT-1} \right) +$$
$$+ b_T \left\{ \sum_{i,s} a_{cisT} v_{cis} + F_{cT} - \sum_m y_c^m \left(l_c^{mT-1} - l_c^{mT} + r_c^{mT} l_c^{mT-1} \right) \right\} \Rightarrow \max. \quad (22)$$

The dual model will also feature new variables λ_{ci} and π_{ct} – dual estimates for the restrictions (18) and (20), i.e. shadow price valuations for the initial foreign currency FTs and foreign cash. If in addition we agree that $\pi_{cT} = 1$ and $\lambda_{ci} = 0$ for $i \geq 0$, the dual model will take on the following form:

$$\sum_{t=1}^{T} \left[\left(\pi_t F_t + h\mu_t G_t \right) + G_T + b_t \pi_{ct} F_{ct} \right] + \sum_i \left(\lambda_i N_i + \lambda_{ci} N_{ci} \right) \Rightarrow \min ; \quad (23)$$

$$\sum_{t=1}^{T} \left[\pi_t a_{ist} + h\mu_t c_{ist} \right] \leq \lambda_i ; \quad (24)$$

$$\sum_{t=1}^{T} \left[\pi_{ct} a_{cist} + h\mu_t b_t c_{cist} \right] \leq \lambda_{ci} ; \quad (25)$$

$$\sum_{t=0}^{T} \left(\pi_t \left[l^{mt-1} \left(1 + r^{mt} \right) - l^{mt} \right] + \mu_t l^{mt} + b_t \left\{ \pi_{ct} \left[l_c^{mt-1} \left(1 + r_c^{mt} \right) - l_c^{mt} \right] + \mu_t l_c^{mt} \right\} \right) \geq 0 ; \quad (26)$$

$$b_t b_{ct} \pi_{ct} \leq \pi_t \leq \pi_{ct}; \quad (0 \leq t < T). \quad (27)$$

The economic substance of individual relations in this model is about the same as that in the model in the section “Optimization of financial policy in the imperfect market context”. But there is also an important difference. Namely, we shall consider a ratio of "values" of the foreign exchange for the firm and for the market $\chi_t = \frac{\pi_t}{\pi_{ct}}$. From (27) it is easy to infer that $b_t b_{ct} \leq \chi_t \leq 1$. It is apparent, -- for the value $b_t b_{ct}$ reflects a fraction of the dollar amount which the firm will receive, having exchanged this amount into foreign currency and then back into dollars. Thus, χ_t can be different from 1, i.e. the ratio of dual valuations for dollar and foreign currency cash may not coincide with their market mutual exchange rate (except for the last step, where $\chi_t = 1$). This necessitates some additional specification to the criterion for evaluating the efficiency of the projects.

Indeed, implementation of a small project characterized at a step t by the market value of fixed assets g_t, and net dollar and foreign currency inflows - f_t and f_{ct}, will change the objective function (23) by $\sum_t \left(\pi_t f_t + b_t \pi_{ct} f_{ct} + h\mu_t g_t \right)$. It is easily demonstrable that such a project would be equivalent to the receipt, at step 0, of income

$$DEI = \frac{1}{\pi_1}\sum_t \left(\pi_t f_t + b_t \pi_{ct} f_{ct} + h\mu_t g_t\right) = \sum_t \alpha_t \left[f_t + \left(b_t f_{ct}\right)/\chi_t + \gamma_t g_t \right].$$

Naturally, the value of *DEI* here too can be interpreted as today's value of equivalent net cash inflows from the project, however, the estimating formula for it is a bit different – the operating income in foreign currency is discounted here in a somewhat different way. The respective member in the obtained formula ($b_t f_{ct} / \chi_t$) has the form of a fraction. Its numerator represents the foreign currency operating income recalculated in dollars at the exchange rate. It is this amount which, consistent with the provisions in [24, 33], should be added to dollar net inflows from the project for discounting. At the same time, we have inferred that such converted income should additionally be divided by the adjuster χ_t, which can assume values less than 1 (not only because of the involved expenses attendant on converting the currency, but also due to differences in the terms for dollar- and foreign currency- denominated loans and other factors). However, in practice such deviations do not have to be significant, and the adjusting factor should not necessarily be incorporated into *DEI* calculations. Therefore, those guidelines in [**Ошибка! Закладка не определена.**, **Ошибка! Закладка не определена.**] dealing with accounting for foreign transactions by “simple conversion” into dollars at the exchange rate for a corresponding step -- may be considered sound enough.

The highest and best use of property and its diverse values

> And the advantages of economic freedom are never more strikingly manifest than when a business man … is trying experiments, at his own risk, to see whether some new method, or combination of old methods, will be more efficient than the old. Every business man … is constantly endeavouring to obtain a notion of the relative efficiency of every agent of production that he employs.
>
> Alfred Marshall, Principles of Economics (v.viii.9)

> Whatever quality you wish to evaluate, there will always be found at least three contradictory criteria for its evaluation
>
> Internet Aphorism

It has been mentioned above that one and the same property (and business) can be characterized by different values. In this section we will go on to consider some of them and find out, how they relate to each other.

As is known, the income approach to estimation of the market value of a property is predicated on the proposition that market value reflects the benefits from a subsequent *use* of that property (“the price of a cow is the value of milk received from it and/or meat”). Such "benefits" in this context can be interpreted as a "contribution" to the objective function of the buyer, i.e. they can be regarded

from the standpoint of the buyer, rather than that of a “detached onlooker” or even “the market” (roughly, what is the benefit -- everyone decides for himself).

Sometimes, a court rules that the acts of commission or omission of an accused may have been motivated by his desire to obtain certain benefits. It is represented that such a treatment of the concept of "benefits" is based on the opinion of judges, rather than on the objectives and interests of a specific subject -- and as such should not be used in the valuation profession.

Furthermore, according to [7, 9], the market value is determined with reference to certain ("standard") model of the respective transaction. In such a model it is meant, in particular, that:

1) The buyer has motives to buy, but no one compels him to do it, and he is not eager to buy at any price;
2) The seller has motives to sell his property for the best price obtainable in the open market after due marketing, but no one forces him to sell;
3) The seller and the buyer have no special or affine relations with each other;
4) The parties are reasonably well informed about the substance of transaction and property characteristics, its actual and potential uses. Thus, each of the parties acts in their own interests, knowledgably and prudently;
5) The settlement for property is instantaneous and in the cash form, and the market value disregards costs for executing the transaction and the taxes associated with the transaction.

The essence of the last precondition is in the following. Making a property acquisition, the buyer, generally speaking, effectuates some different payments. And so: one of these payments should be a payment in settlement for the acquired property having the monetary (“cash”) form, and this payment should relate to the time of transaction closing. The need for the given assumption is motivated by the diversity of conditions encountered in real transactions. The adoption of the "standard" transaction model allows market makers to speak “in one language” using the same basis for decision-making. To achieve this, a “model transaction” envisaging a one-time single target payment for the acquired property has been uniformly agreed. Other payments which are taking place in the course of transactions (for example, the VAT transferable by the buyer to the seller, other taxes, realtor or broker fees and other transaction costs), the possibility of an installment sale or of a delay in payments, and property exchange on terms other than money (like bartering for another property) are not considered in the market value transaction construct. Thus, market value of property would coincide neither with the cash outflows from the buyer, nor with the cash inflows to the seller generated by the transaction. For example, at the acquisition of a building, the buyer should advance to the seller -- over and above the amount of market value -- the value of VAT (or stamp duty), and also incur expenses related to e.g. realtor’s fee and conveyance of the property right to the acquired building. Besides, if the buyer is a firm, the VAT paid over to it may be set off after a while. On the other hand, the seller even before proceeding with the sale is compelled to incur certain pre-selling expenses to make the building “presentable”, and, having received from the buyer the value of VAT, should transfer it to the budget after a while.

Moreover, if the sale price exceeds the carrying amount of the building recorded on the balance sheet of the seller, the seller should also settle on the realized gains income tax. All such incomes and expenses are not considered in the amount of market value, but they should be considered when evaluating the efficiency of the project involving the purchase or sale a building.

Let’s now consider some replicable project (i.e. a project which can be repeated over again) implementable by a number of market participants. Since some market participants actually do go ahead implementing the project, its effect would appear as a non-negative one to them. At the same time, some other market participants rather choose to invest into different alternative projects, implying that the effect of the project in question is other than positive to them. From here we can also infer the principle of zero effect, this time presented in a different guise: any replicable project realizable by market participants has a zero effect. But, as demonstrated above, the effect from a project would be adequately captured by the *DEI* indicator. Therefore, *DEI* for a replicable project realizable by market participants should be zero.

Let's assume now, returning to the property transaction deals, that the purchase/sale transaction is executed on the standard terms. In such a situation the discounted costs to the buyer (i.e. the market value of property plus expenses to execute the transaction and the associated payments in taxes) should coincide with the ‘integral effect’ of the project involving the use of this property after purchasing (again, the zero effect principle). In a similar way, the market value of property at **cost approach** should coincide (accurate to within the value of expenses and taxes associated with the execution of the transaction) with the integral effect of the project to create such a property (taken with the inverse sign).

Thus, the DCF method of property valuation reduces itself to the determination of the corresponding *DEI*. For that purpose, however, it is needed that the "project" to purchase and use the property be a "small" one. Therefore, **DCF method** (irrespective of the fact which cash flows are being discounted and at what rates) **is applicable only to the valuation of "small" properties**. The use of indicators such as *DEI* and NPV for evaluating "larger" acquisitions can only be seen as the approximating device – the less exact, the larger the acquisition (it is not accidental that many countries legislatively provide for the state control over, for example, large-scale mergers and acquisitions of companies). But *what kind of* property value can be estimated by this method?

To answer this question we shall consider all interest-holders of such a property -- its potential sellers, and the set of all its potential buyers, assuming that for each of them the property in question is "small".

Let’s take one of the potential buyers of the property. Making a purchase payment for the property, he acquires the right to receive an income from it -- such that the efficiency of acquiring this property is determined by a difference between the *DEI* for the future use of the property (i.e. *DEI* of the "project" involving the highest and best use *of the acquired* property) and the total purchase costs (costs of acquisition, plus taxes and transaction costs). It means that such a buyer has some

reservation price for purchasing the property equal to the mentioned *DEI* reduced, if necessary, by the total sum of transaction costs and taxes payable in the context of transaction (at the higher price than that the acquisition would appear unprofitable – the purchase costs incurred "would not pay off" and the *DEI* of the project involving both the acquisition of the property and its subsequent use would appear to be negative). We shall designate this price as the "**net acquisition value"** (a concept somewhat similar in meaning to that is referred to in some countries as "the special value to purchaser", as explained in the International Valuation Standards [9, GAVP, sub para. 9.3.2]). Such net acquisition values will characterize each potential buyer, and, generally speaking, different buyers will be characterized by different acquisition values (both because of the differences between their discount rates, and because of a different choices they make in their property use methods). The highest value out of all of them, corresponding to some "closing" (marginal) buyer, shall be denoted by us as S_a and designated as the **"market acquisition value".** Therefore, none of the market participants freely and prudently acting in their own interest and consistent with their objectives will ever choose to spend a greater amount for purchasing the property than S_a. This concept can thus be given the following definition:

The market acquisition value of a property is a maximum monetary amount, such that there will still be found at the market an interested, well informed and prudently acting buyer standing ready, without compulsion, to acquire the possession of the property in exchange for a one-off payment of the said amount. The market acquisition value is based on the contribution that the property in question would make to the enterprise, of which it becomes a part upon acquisition, and reflects the present (discounted to the moment of acquisition) value of the future net cash flows that would arise subsequently to the acquisition of said property consistent with its use in the highest and best manner and its disposal (scraping) at the end of its (optimal) term of useful service life.

Let's consider now one of the potential **sellers**. Selling the property, such a seller would forego a benefit from the continuation of its use. This benefit can be estimated as the *DEI* of a "project" to continue on with the most effective utilization of the property (such a "project" may not require any investments). Therefore there would be some minimum of the reservation selling price still acceptable to the seller in question and equal to the said *DEI* adjusted upwards, if need be, for the amount of transaction costs and taxes attendant on the sale (at the lesser prices being offered to the seller, he will find it more beneficial to himself to continue with the use of the property, rather than contemplate its disposal). We shall designate this amount as the "**gross realization value"**. Such gross realization value will attach to each of the potential sellers, and will, generally, be found different with each specific seller. The least of all such values will be denoted by "S_r" and termed "**market realization value"** (it will be coincident with the seller's gross realization value for a unique property). Thus, no rational and

uncompelled market participant will ever choose to sell the property for less than S_r. This amount can be defined as follows:

The market realization value of a property is the minimum monetary amount, such that would still attract to the market an interested, well-informed seller, acting prudently and without compulsion, which stands ready to dispose of the property in exchange for the one-off receipt of the said amount. The market realization value is based on a contribution which the property makes to the enterprise, of which it forms a part, and reflects the present (discounted to the moment of valuation) value of the future net cash inflows that would arise from continuing to use the property in a best way and from its subsequent disposal or scraping at the end of its (optimum) term of useful service life, increased by the amount of costs and taxes associated with the sale of the property.

Let's reflect on a number of the important features of the values just described and their distinctions from the market value:

- Unlike “usual” market value which predicates itself on the availability of the seller and the buyer agreeing to the execution of the transaction at such a price, the specification of net acquisition value does not presume the presence of the respective seller; likewise, the specification for gross realization value does not envision the availability of the respective buyer;
- Net acquisition value “attaches” to a specific buyer, gross realization value -- to a specific seller. Therefore these values are in “other than the market” category. At the same time, the market acquisition and realization values are by their nature in the **market** category (and not just because we so named them). They do not “attach” to a specific buyer or seller, or, to be more precise, "attach" to the corresponding fringe (“marginal”) market participants and consequently reflect the cutting edge of the market demand and supply.

The International Financial Reporting Standards still carry the term “net selling price” defined as the amount which can be received from selling assets in a commercial transaction less the costs associated with the sale. From what we described above, it is clear that this value is close in its substance to the *DEI* of the "project" to continue with the best use of the property by the marginal seller.

Source [7] makes reference to yet another basis of value, which is also akin to those bases that we introduced above: subjective value (value to the owner). It is the estimated value which the current owner or prospective purchaser ascribes to the right (interest) in the property, having regard to his personal circumstances, e.g. sentiment and tax position. In our approach this value would be different as between “the current owner” and “prospective purchaser”, and the “personal circumstances” would include all factors enumerated in “Optimization of financial policy in the imperfect market context” section, which, in the end, determine the discount rate and valuation of loan access rights, but it would stretch their meaning to call them sentiments.

Let's stop to consider in greater detail the indicators of gross realization value. Here it is important to note that gross realization value of FTs can be inferred from the dual model considered in the “Optimization of financial policy in the imperfect market context” section. Indeed, according to this model any FT in possession of the firm provides a contribution to the objective function equal to the discounted

sum of equivalent incomes from the FT over the optimal term of its holding. In other words, if we were to substitute the specified sum for a given FT, the value of the objective function would stay the same. But then we would be justified in claiming this sum to represent the minimal net disposal proceeds that would be found acceptable to the firm, i.e. gross realization value.

It shall be noted that knowing gross realization value for each of the FTs in firm's possession, the firm can estimate in value terms its entire FT portfolio with which it enters the planning period. A value thus obtained would, of course, be not a "market" value, but the "internal" one, i.e. estimated from the standpoint of the firm, rather than that of outsider market participants. It is represented that estimates of such kind should be found useful by firms for "internal" purposes, for example, within the system for operational administration of financial activities and in the context of firm's value management.

Close in meaning to the net acquisition value and gross realization value are the terms used in valuation profession and accounting measurements, namely **value-in-use** and **investment value**. The first of them is described and explained in [9]:

"3.27 *Value in Use.* This value type focuses on the value that specific property contributes to the entity of which it is a part without regard to the property's *highest and best use* or the monetary amount that might be realised upon its sale. Value in use is the value a specific property has for a specific use to a specific user and is, therefore, non-market related." (GN 4)

"Value in Use -- the present value of estimated future cash flows expected to arise from the continuing use of an asset and from its disposal at the end of its useful life." (IVS 2007 Glossary, consistent with IFRS 5)

The notion of investment value is also described in the International Valuation standards:

"3.2. Investment Value, or Worth. The value of property to a particular investor, or a class of investors, for identified investment or operational objectives. This subjective concept relates specific property to a specific investor, group of investors, or entity with identifiable investment objectives and/or criteria." (Standard 2, IVS 2007)

More careful and detailed treatment of investment value is given in [7], although it contains no full definition of the concept:

"S7.02 The concept of value in use is based on a subjective, non-market derived, assessment of economic utility of an asset to an undertaking. It is sometimes referred to as 'investment value'. The latter term is potentially liable to mislead as it implies a market led assessment which could be confused with Market Value. It is fundamentally different from an assessment of Market Value, but is a component that, in the aggregate, drives market activity and is based upon individual estimates of value in use by the market participants.

S7.03 The calculation of value in use may take the form of a subjective appraisal of estimated cost and benefits accruing to the investor over time, discounted in accordance with the investors internally derived criteria and based on a mix of different elements including, for example, an evaluation of economic trends, residual values, financial targets and risk analysis.

S7.04 Calculations of value in use, although not strictly valuations, are very often part of the vital process of evaluating a project or comparing alternative investment opportunities…

S7.05 The evaluation by an undertaking of an investment portfolio using internally generated, but consistent criteria which is applied to anticipated cash flow, can be argued to represent a more satisfactory long term judgment of a groups' performance than Market Value which is based on spot figures derived from markets in rapid transition. It is not however a basis that can be justified objectively, or is capable of validation, excepting over time."

It would seem, the *value in use* basis is really not a market basis and assumes the existing, rather than the most efficient, method of property utilization. This is not the case! The emphasis placed in the definition on the words "future" and "expected" (evidently implying that because of the use of these words the "value in use" becomes a non-market basis) only refers to the risks associated with the use of subject assets -- and these risks are no greater, nor less than the risks involved in the use of any other assets which are always considered when determining market value as well.

Moreover, just continuing with the current use of assets is usually less risky than redeploying them to some other (the best, the most effective) use, and because of this fact the value-in-use may appear to be above market value. Besides, there is nothing in the cash flows associated with the continuation of asset's use that makes them less market determined than the cash flows corresponding to the most effective utilization. Then what is the matter?

The standards would maintain that the point in question is searching for the best method of assets' use which would give them the highest value. And it is quite reasonable. No use to be engaged in the evaluation of "bad" property utilization methods. But is it a valid point that "value in use" corresponds to such a "bad" method? In our opinion, no. And this is connected with the issue which has been artificially bypassed in the definition. What "continuing use" is implied in the definition? "Use" is not a single act, nor an activity discharged under the terms of some order. Say, what is a "continuation of use" for an oil deposit? After all, here we have a whole cornucopia of options: we can drill new wells, or suspend the production on existing ones, and it is possible to redeploy wells from one layer to service another, on top of that, all this can be done today or in 10 years' time. A truck is used for transporting a cargo. But today it can carry one type of consignments, tomorrow – another, today it takes one route, tomorrow -- another. We can compare, for example, such options: the truck continues to transport cargo in crates but not in bulk; as the firm seldom uses it, it is rented out to the driver on those days when the transportation of cargo is not required; it is put on sale, and a specialized cargo transportation company is hired, whenever a need to transport

cargo arises. In this situation any option can turn out to be the best, although the first one approximately corresponds to the “value in use”, whereas the last one – to the market value. And it is hardly possible to find a “place” to draw the line saying “value in use” comes to an end here and the market value begins on the other side! Therefore, the “value in use” concept (in its technical operational sense) has no economic or technical meaning, unlike the market value relating to the most effective option for the subsequent utilization of property.

Distinctions between investment and market values are illuminated in the textbook [6, p. 56]: “Unlike market value which is determined by behavioral motives of the typical buyer and seller, investment value depends on the individual requirements for investments presented by a specific investor”. Put differently, the crucial difference here is whether an appraiser has "typical" or "specific" investors in view. IVSs also contain a provision to be guided by typical market participants: In the *sales comparison approach* the limits to *Market value* for real property are established with reference to the analysis of prices usually paid for property competing on the market for the buyers of subject property. Comparable sales are analyzed to ascertain that the transacting parties had been typically motivated [9, Property Types, sub para. 2.7.1]. At the same time, no matter what discount rate has been assumed by the appraiser in his estimates or what risks are considered, they will always relate to some group of real or virtual investors, and by no means to their entire set. It shall be noticed that what matters in valuation is not *the number* of market participants, but their "motives" (to be more exact - their objectives and interests). For example, there can be one “typical buyer”, and the overwhelming multitude of “specific investors” with identical motives. So, it is not so important what name and specification are used to distinguish the group of investors, whose objectives and interests determine the value of a property or a business: what is significant is that it is some group of market participants, but by no means their entire set. But if we agree with the stated position from the above textbook, it will be necessary to regard the "market value" of property as being, too, an investment value -- for it relates to some group of “typical investors”.

The presented definitions envisage that we estimate the investment value of property on the basis of cash flows to a specific investor (the actual or potential owner of property). It is, however, not specified whether we should deal with the cash flows which the investor will receive should he continue using the property, or with the cash flows which will arise should he acquire such property. In the first instance these definitions would approximately (to the accuracy within the value of expenses and taxes associated with the execution of purchase/sale) reduce to gross realization value, in the second – to the net acquisition value.

Let's now pay attention to the fact that when estimating gross realization value and net acquisition value, it was supposed that subject property can in principle be used in different ways, but the sellers and buyers should choose the best, *most efficient*, of them. However, given that different market participants have different discount rates, a use which is best for one market participant can be other than that for another. Therefore, appraisers should not merely specify some use declaring it

to be the best one, but should demonstrate by their calculations that such and such (generally, different!) use methods would prove to be the most efficient for the "closing" (marginal) seller and the buyer. Alas, there are almost no valuation reports which contain valid economic comparisons of the property use options and substantiate justifications for the choice of the best of them (for the buyer or the seller).

Example 10. A property can be put to use in two different ways. The first option, used almost by all owners, provides the annual income of 500 over 8 years, while the second one -- annual income of 850 over 4 years. There are two potential buyers for the property - "Distance light" ltd. and "Dim light" ltd. that employ (time-invariant) discount rates of 10 % and 12 %, respectively. Then the following economic effect would be registered for the stated use options: for the "Distance light" ltd, accordingly $\sum_{t=0}^{7}\frac{500}{1.1^t}=3234$ and $\sum_{t=0}^{3}\frac{850}{1.1^t}=3113$, and for the "Dim light" ltd. - $\sum_{t=0}^{7}\frac{500}{1.12^t}=2782$ and $\sum_{t=0}^{3}\frac{850}{1.12^t}=2892$ (in this instance, we ignore the distribution of income inside of the years and assume that it accrues at the beginning of each year). Therefore, "Dim light" ltd. will have a preference for the second utilization option, while "Distance light" ltd. will opt for the first utilization option that provides smaller income but over a longer period of time.

To put matters briefly, **generally speaking there is no such thing as "the" highest and best use of property from the standpoint of the market** (even if we discuss buyers and sellers separately). By the way, this is justified by situations which we observe daily in real life in respect of, say, cars, warehousing and industrial space, computers, etc. So, no one will ever think of posing a question regarding the best method for driving a car or using a personal computer -- as we all understand that such methods will be different with each driver or the user.

Let's now return to the analysis of situations arising in the course of property valuations. We have so far presented two types of value for the property – market acquisition value (S_a) and market realization value (S_r). In this context it is possible to describe three combinations:

1) $S_r = S_a$. This result would be an equilibrium value, which can be termed as "fair" or "market" value by appraisers, since no other value would be acceptable for either the plurality of the sellers, or the plurality of the buyers;
2) $S_r < S_a$. Under this combination, there will always be found buyers willing to acquire the property paying anything up to S_a for it, and the seller(s) willing to sell it for the amount of S_r. So in this context, a transaction with the property can take place on the market at any price lying in the interval between S_r and S_a;
3) $S_r > S_a$. Then, whatever the price P in the range between S_r and S_a, no buyer will ever agree to purchase the property at this price (since it would be greater than S_a), and no seller will ever agree to sell it at such a price (for it would be less than S_r). No purchase/sale of the property is at all possible in this context.

It can be thought that in the second case (where the transaction is possible at any price within the set limits), as well as in the third case (where no transaction

can be contemplated), appraisers, taking the lead from [6], will determine the market value of property in the amount of S_k (to the accuracy within the value of expenses and taxes associated with executing the transaction), which would be its **market acquisition value** (according to the valuation standards, property is to be valued from the viewpoint of its owner, in this case -- its future owner). Meantime, according to our position, property should be valued separately from the viewpoint of its specific seller and from the viewpoint of its buyer (this, by the way, would not be contradictory with the valuation standards, as at the moment of valuation both parties decide whether to be the property owners or not), which can be done approximately as follows.

From the viewpoint of the owner (potential seller) the property can either be sold or continued in use. In this context, it would be necessary to consider (as special-type investment projects) various scenarios for the continuation of its use, and evaluate *DEI* for each of them. The highest of these *DEI* estimates would correspond to the most efficient utilization of the property. Adding taxes and transaction costs on top of it, we shall obtain the corresponding minimum selling price of the property (attention should be paid to the fact, that it would *include any of all possible expenses and taxes associated with the sale*; at the same time, according to EVS [6, sub para. S4.24 and S4.57], both market and investment values are determined "*without regard to the costs of sale or purchase and without offset for any associated taxes*"). Thus,we if a sale of the property is contemplated through an auction mechanism, the minimum (starting) price of the auction should be set at this level (if we set this price at some higher level, the general chances that somebody will show interest in purchasing the property would decrease). If no buyer is found at this price, the property should continue to be retained, for disposing of it at a smaller price would render such transaction an inefficient decision.

If we estimate the value of property from the viewpoint of a potential buyer, it would be necessary to consider at the beginning the set of all feasible scenarios for putting the property into use, then to estimate *DEI* for each of them, and, finally, to choose the highest *DEI* corresponding to the best option. This amount, after taking into account all possible expenses and taxes that could be associated with the purchase decision, will determine the maximum acquisition value acceptable to the buyer.

Example 11. The value of an amusement park fun ride that has been in operation for 5 years is to be determined. Its initial acquisition cost = 2500, overall term of useful service life = 10 years, therefore the annual depreciation charges will be equal to 250, and the residual carrying amount value at the present date of valuation = 1250. Over the next 5 years of its service the annual revenue from its use is expected to decrease from 600 to 500, and the annual net operating costs -- to go down from 190 to 170. The net disposal proceeds at the end of its useful service life (retirement value) would be = 70. The owner's discount rate is time invariant at 12 % per annum. Credit restrictions are ignored for the sake of simplicity.

The estimates of cash flows to the owner from continuing to operate the fun ride and of the related *NPV* (in this case it coincides with *DEI*) are presented in the table 6.

Table 6.

№ Line	Indicators	Years				
		1	2	3	4	5
1	Carrying amount residual value	1250	1000	750	500	250
2	Disposal value	0	0	0	0	70
3	Revenue proceeds	600	575	550	525	500
4	Net operating costs	190	185	180	175	170
5	Depreciation charges	250	250	250	250	250
6	Property tax (rate = 2%)	25	20	15	10	5
7	Taxable income (line 2+line 3-line 4-line 5-line 6)	135	120	105	90	145
8	Income tax (rate = 24%)	32.4	28.8	25.2	21.6	34.8
9	Net cash inflow (line 2+line 3-line 4-line 6 - line 8)	352,6	341.2	329.8	318.4	360.2
10	Discount rate	12%	12%	12%	12%	12%
11	Discounting factor	1.000	0.8929	0.7972	0.7118	0.6355
12	Discounted net cash flow (line 9 × line 11)	352.6	304.6	262.9	226.6	228.9
13	*NPV*	1375.7				

Thus, the owner's effect from continuing to operate the fun rides will amount to 1376. The effect from its sale would depend on the value and terms of sale.

Let's consider one of the possible sale options. Under this option the selling (transaction) price is 1450, execution-of- sale expenses - 2 % of the price, i.e. 29 , net proceeds from the sale -- 1450-29=1421. The taxable income from the sale will thus be equal to 1421-1250=171, the income tax – 0.24×171=41,0. Therefore the net economic effect accruing to the seller will amount to 1421-41 = 1380 > 1376. Consequently, in this case it would be expedient for the owner to agree to the sale on these terms. Back-solving the case by trial and error interpolation, it would be possible to establish that the minimum price still beneficial to the seller (gross realization value) will approximately be 1444.

But will such a price be found suitable to the buyers? To find it, we shall consider one of the potential buyers. To simplify the estimates, we will assume that his discount rate is constant over time and equals 10%. *NPV* calculations (here they would be equal to *DEI*) that would result from putting the rides purchased for 1450 into use, -- assuming that its chosen use option is the highest and best one, and that transaction costs to the buyer are zero, -- are presented in table 7.

Table 7.

№ line	Indicators	Years				
		1	2	3	4	5
1	Residual carrying amount value	1450	1160	870	580	290
2	Proceeds on disposal	0	0	0	0	70
3	Revenue	600	575	550	525	500
4	Net operating costs	190	185	180	175	170
5	Depreciation charges	290	290	290	290	290
6	Property tax (rate = 2%)	29.0	23.2	17.4	11.6	5.8
7	Taxable income (line 2 + line 3 - line 4- line 5- line 6)	91.0	76.8	62.6	48.4	104.2
8	Income tax (rate = 24%)	21.8	18.4	15.0	11.6	25.0
9	Net cash inflow (line 2+line 3 - line 4- line 6- line 8)	359.2	348.4	337.6	326.8	369.2
10	Discount rate	10%	10%	10%	10%	10%
11	Discounting factor	1.000	0.9091	0.8264	0.7513	0.6830
12	Discounted net cash flow (line 9 × line 11)	359.2	316.7	279.0	245.5	252.2
13	*NPV* from the use	1452.5				
14	*NPV* from the use (line 13 less the transaction price)	2.5				

So, it is probable that a buyer will be found given this transaction price (1450). Fig 1 illustrates the dependence of buyer's and seller's *NPVs* on the transaction price C at different discount rates for the buyer (*E*).

As we see, at *E*=11 % no transaction would be possible at any price, and as the rate of *E decreases*, the interval of possible transaction prices widens itself. So, at *E*=10 % the rides can be sold at a price limited by the range of approximately from 1445 to 1455, whereas at *E*=8 % the feasible transaction price range becomes 1445 to 1516. Naturally, it may be possible to regard 1445 as the "market value", however there are no guarantees that the market will contain buyers with the discount rate of 10,5% or less. On the other hand, if the "marginal" buyer's discount rate would be 8 %, the market value of acquiring the ride will constitute 1516.

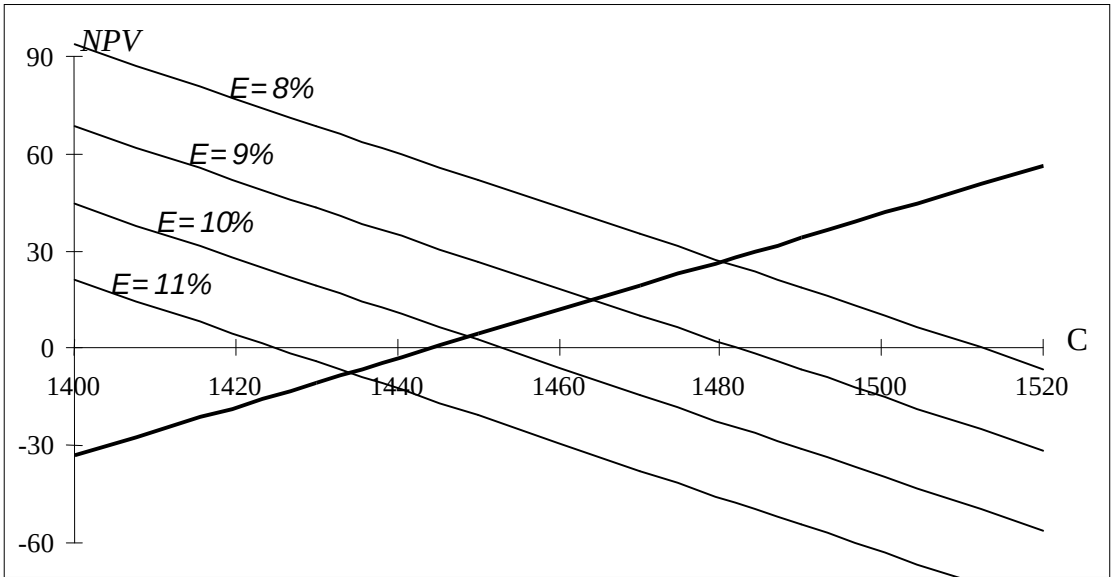

Fig. 1. Seller's and buyer's *NPV* at different transaction prices

A consideration of this example allows to cast a look at the curious feature of property value calculations. Indeed, the usual requirement to approach estimating the value of property on the basis of *NPV* from its use can sometimes be taken too literally: we calculate *NPV* and claim it to represent an estimate of market value (as can be well seen from an estimating example in [6, pp.388-411]). Meantime, such estimating procedure fails to consider the fact that cash flows from the property use (for example, the amounts of depreciation charges and the income tax) depend on the amount of purchase (transaction) price. Therefore the estimating schema should be different: in the beginning we set the "approximate value" of property, which serves as an input for specifying the cash flows associated with its subsequent use, and make an *NPV* estimate. After that "the approximate value" of property is adjusted as an input for feeding into the next iteration until it coincides with *NPV*. The same pattern of calculations fits in with the cost approach to property valuation (here it would be necessary to consider expenses associated with the sale and the taxes which are not included in the market value but depend on it).

Discussing above about the acquisition and realization values of property, we implicitly assumed that property purchase should be efficient for the buyer, while its sale should be efficient for the seller. However, in certain cases this assumption would not be valid. We will take a look at what happens then.

If the property sale is motivated by some earlier adopted decisions or resolutions of state authorities, the interests of the seller should not be considered in such a context. The selling price will be determined by that maximum net acquisition value which is still acceptable to the buyers. However, such value can't be considered the "market basis" under these circumstances of sale.

Another possibility: a property sale takes place because cash funds are urgently needed by the seller. This situation can be quite real, however in such a case seller's dual estimates (shadow prices) of money undergo a radical change, so, his discount rate changes as well. In this case, *DEI* calculations should be performed at a higher discount rate, and they will result in a reduction of the realization value minimally acceptable to the investor. To invert the situation, if the market analysis has allowed to find out how the prices of similar properties are diminished in the "fire sales", this information can be used to evaluate the extent by which the occurrence of an urgent need for money raises the discount rate. In a similar vein, when efficiency calculations demonstrate the inefficiency of acquiring a property, and such purchase is still necessary to the buyer for "some reasons", it may indicate that he has incorrectly established his discount rate, for example, has overestimated the opportunities for receiving income from alternative investments into FTs.

Notice that in the course of estimating realization or acquisition values the estimator may alter the discount rate. In this connection it will make sense to consider one of IVS's positions [9, para. GN-9, sub para. 3.4]. Following (hardly the brightest!) definition of the internal rate of return metric (IRR), it is stated "The IRR reflects both the return on the invested capital and the return of the original investment, which are basic considerations of potential investors. Therefore, deriving the IRR from analysis of market transactions of similar properties having comparable income patterns is a proper method for developing market discount rates for use in valuations to arrive at Market Value". In short, the meaning is that in establishing the discount rates it is recommended to take account of IRR values for "projects" involving the purchase and use of similar properties. Here, naturally, it is assumed that such projects do have an IRR.

Let's, however, assume that IRRs exist and are determinable for the projects to purchase similar properties and put them to use, and that such IRRs are close to each other. We shall, for example, assume that all of them are approximately 10 %. Let's find out what bearing does this information have on the discount rate. For this purpose, it shall be noticed, first of all, that we should be dealing with IRR calculations based on the information about already executed transactions (rather than those transactions which are still being "designed" by someone). But, if the "similar transactions" had taken made, the price of purchasing the property in these transactions will have provided a non-negative NPV to the buyers (since in this case the project net cash inflows implied in such IRR calculations would relate to the buyers, and not to the sellers). But then the discount rate of the buyers couldn't have been more than 10 %. If the sale had occurred on a competitive basis, it is likely that the actual buyer's discount rate would have amounted to approximately

10 %[5]. However, it is by no means possible in this situation to estimate the discount rate of the seller (after all, his cash flows are in no way accounted for in the IRR indicators).

It is a notable fact that our analysis doesn't require a concept of property "market value". And is it so impossible to manage without the one? To answer this question, it would be necessary to comprehend reasons for the emergence of this concept. It seems its appearance owes itself to the following. By observing purchase/sale activities occurring on the market, people seem to have noticed that, over certain time intervals and under certain conditions, similar goods are bought and sold at close enough prices. This has compelled them to hypothesize that underlying the actual prices of transactions with every good there is a certain characteristic of these goods not observed directly, a certain uniform reference point for all market participants. A formalization of this reference point has resulted in the emergence of the market value concept. The existence of different lines of thought and methods for estimating the market value simply implies a variety of methods for the approximate measurement of this directly unobservable underlying quantity on the basis of other original and available market information[6]. Thus, market value is a characteristic answering this question: "what lies behind the observed or anticipated transaction prices?"

A situation with the investment value, or more precisely, with its two "incarnations" -- gross realization value and net acquisition value — is starkly different. These characteristics have made their appearance in this paper not out of some desire to analyze any actual information or to demonstrate originality. Moreover, at the beginning of my research, I also did not think that any such value indicators would be needed and simply made it my point to find out under what conditions the realization of investment projects would be beneficial to specific entities (investors). An answer to this question has led to a certain generalization for the NPV measure which is also utilized in the course of property and business valuations. Then new questions presented themselves: how can we evaluate whether it would be beneficial for a given entity (market participant) to purchase/sell property at such and such price? At what prices is a purchase/sale of the property generally possible on the free market? And answering these questions also brought about the need for the above value bases.

Thus, it is futile to talk about "utility" or "uselessness" of the market value and investment value bases, as they correlate with disparate questions. The former base relates to the market as a whole and is simultaneously a uniform reference point for all market participants, as well as an element (if not the engine) of the "market

[5] This, by the way, suggests a simple means for verifying the "fairness of the auction": its participants should be characterized by approximately similar discount rates. If the majority of the auction participants have approximately similar discount rates, but there is one with outlying significantly reduced discount rates (it is not hard to fabricate them "artificially"), the latter will turn out to be the winner.

[6] Here a pertinent analogy is suggested in the application of different statistical methods to infer, on the basis of sample data, certain not immediately observable economic parameters (such as income demand elasticity, or the Cobb-Douglas function parameters).

mechanism", whereas the latter is a tool for specific decision-making by a specific market participant acting under specific circumstances. As a very crude notion, it is possible to compare market value with a compass pointing in a proper direction, and investment value -- with a high-definition ordnance map which makes it clear what footpath should be taken at the present in order to (most likely) achieve the destination /objective with the least difficulties.

Discussing about market value, it is fit here to mention a general concept of its determination which has been set out in detail in a monograph [25, pp.33-34]. There a point is made that it is possible to approach the subject of property valuation from two vantage points, considering it "*from the outside* and *from within*. The first point of view is looking at the property being valued through the "eyes" of the market or, more precisely, a related institutional-economic environment. … Thus, a "subject making evaluation" **under objective criteria** is the market expressing a collective or public interests and circumstances on the basis of which *the best and most effective utilization* of the property is established. A different standpoint is assumed by property occupiers, by the management of a going concern enterprise, investors into a newly implementable project, i.e. by all those persons who make financial and economic decisions in line with their individual (and in this sense, subjective) objectives and/or criteria. ... Thus, the very entity making evaluation according to **subjective criteria** is a decision-maker ... managing on the basis of his own notions about the current property use and business risk" . Let's consider such a position more closely.

There is little doubt that market participants make their financial and economic decisions on the basis of their individual objectives and/or criteria (we shall say - aims and interests). The information necessary for them, it seems to us, is furnished by the indicators of gross realization value and net acquisition value. Besides, as is apparent from the foregoing analysis, it is the highest and best use of the property (**for themselves**) that the seller and the buyer are simply compelled to take into account when valuing a property, and not simply some current use of the property. At the same time, the gross realization and net acquisition values are by no means equivalent to the "value in current property use" which is essentially discussed in the quotation above (from [25]). Thus, while valuation under the objective criteria results in the indications of market value, the valuation under subjective criteria leads to the indicators for gross realization value and net acquisition value, not the value in current use.

Let it be noticed that it is not so simple to operationalize the idea of valuing a property from the standpoint of the market. The matter is that the whole "perception"/"view" of the market should be captured in the *NPV* calculation procedure. In particular:

- Cash flows should relate to such property use which is the most efficient from the "viewpoint of the market", rather than that of a specific property owner. For this purpose, it is obviously expedient to supply appraisers with some tools enabling them to select such a method of use and prove that it ensures the highest efficiency;

- Estimates should incorporate only those costs and results which are acknowledged by the market, rather than those items which are considered necessary by the owner of the property;[7]
- a “market" discount rate should be used for discounting, instead of the rate associated with a specific owner of the property. Certainly, it follows that a definition should be given of such a rate and the methods for determining it be indicated.

Let's move on further. Needless to say, it would be interesting to cast a look at the property through the eyes of the market. But what for? Who is to use such market value parameters, how and for what purpose? Alas, we shall look in vain for answers to these questions in the indicated monograph. Without making claims to a thorough discussion, it is possible to indicate one such possible direction of use for the market value concept. It would appear that investment project efficiency calculations – even gross realization and net acquisition value estimates -- would be untenable without market value parameters. The matter is that, for these calculations to take place, it would be necessary to specify at the very inception the project net cash flows (or the cash flows that arise from a property use). For this purpose it is necessary to specify, in particular, the values of associated consumed resources -- the goods and services. It is usual to know the volumes of resource consumption in physical terms. But what about the prices for these resources? It is often the case that, at the moment the estimates are made, their suppliers (sellers) are unknown, unknown are also the prices demanded by them. Therefore here it would be necessary to proceed on the basis of “average market” information, and the market values for goods and services would represent just such information. For that reason such estimates would primarily rely on the market values, and only in the exceptional cases would "individual" prices for goods and services originally agreed upon with their suppliers be featured in the process. Thus, the need for market values arises “at the border” between the project and its economic environment: at the fringe where the produced goods “leave the project” and consumable resources "enter" it, i.e. where the purchase/sale transactions between the participant of the project and a non-participant are made. In a similar fashion, market values make their appearance in the property and business valuation context where incomes and expenses associated with the use of property or the functioning of a business are subjected to evaluation. However here, so it seems to us, they assume a somewhat different and more specific meaning. The matter is that, in this context, they would relate to "mass-produced" homogeneous property freely circulating on the market in a considerable quantity. Let’s consider, for example, some FT. It follows from the above described model that if an optimal policy of the firm envisions purchasing such FT, the *DEI* of the

[7] So, if the market recognizes a need for bribing officials or a certain volume of advertising expenses, these, in the appropriate extent, should be reflected in the cash flows, even if the property owner would disagree with these costs. On the contrary, if the market ignores the need for environmental regulation compliance costs, the cash flows should reflect only the consequences of environmental non-compliance, rather than the environmental protection or remediation costs that the owner of the property wishes to make.

project involving its purchase and subsequent ownership will be equal to zero. Let’s imagine now for a second that the acquisition value for the given FT (it has been imputed in the model) is too high. Then neither this, nor any other firm will ever include purchasing any of these FTs into their optimal policy. But then these FTs will cease circulating on the market! This does never happen in reality, as in such situations the sellers will proceed to reduce their prices until the supply and demand for these FTs is balanced again. The corresponding price will also be the "true" market value of the FTs. A similar situation takes place in respect of other mass produced goods, though it may not always be possible to measure the benefits expected from their use in monetary terms (office space, cars, process equipment, computers, etc.). If the item is unique, or there prevails a monopoly or monopsony in its respect, the situation changes. Strictly speaking, "equilibrium" prices would also exist here, however there would be discontinuous variables in the associated “market equilibrium models”, with a potential for multiple “equilibrium points”.

As far as investment projects are concerned, it is not at all sensible to talk about their "market value" or to treat their *DEI* measure as an assessment of the project *from the viewpoint of the market*. It is connected with the fact that, as had been noted before, there is no such thing as “equilibrium” in the “market for projects”: each real investment project is exclusive in its own way, it “attaches” to its specific participants, and there are many more projects than participants willing to take financial part in them, moreover, *DEI*s for real projects (unlike for financial ones), as a rule, are different from zero. Under these conditions, a desire to evaluate such a project as if it represents some FT available to every market participant would be akin to a readiness to appreciate the beauty of your aunt on the basis of results for an all-American beauty contest. There are some objects which everyone should evaluate for themselves, without regard to the opinion of the relatives, and even more so without regard to the opinion of the market. For that reason, only you can assess whether it would be necessary to undertake a de luxe quality renovation work in your apartment, but the market would not and should not be able to assess such a project.

Let's mention yet one more important distinction between the considered values. If an appraiser evaluates some property from the point of view of its specific buyer, i.e. calculates net acquisition value for it, his estimates can be verified later on: given a proper estimate, the buyer’s costs of purchase should be on the same order or less. An estimate of market acquisition value S_a can be verified in a similar way: if properly made, a given or similar property will be purchased (by someone) for S_a or less. Similar verification principles would hold in respect of gross and market realization values. At the same time, strictly speaking, it is hardly possible to verify market value estimates performed by an appraiser by comparing them with the actual prices: in many instances identical copies of an asset are bought and sold at a different prices at the same time, even though the market value for all such copies should hold to be one and the same,

and, as a rule, the obtained valuation result would exactly coincide with neither of the actual selling prices.

It would seem that the market valuation results can be compared to average prices for a given kind of property. However, it would then be necessary to acknowledge that the prices of specific transactions are random variables, and that the market value reflects their average magnitudes, i.e. corresponds to an average (rather than “marginal”) buyer, who can use property in other than the best way. Moreover, estimating an average price on the basis of sample data is an inexact exercise (i.e. such estimates would each have a variance), therefore it would be necessary to require of appraisers that each time they specify confidence intervals, at a standardized significance level, for the values calculated by them. It seems it would be a too high price to pay for such a “verification method”.

We discussed above the application of DCF method to valuing property and business under the income approach. At the same time, this method is also applicable to valuing under the cost approach.

Here the underlying assumption would be that the project to create a property for its eventual sale should provide a zero *DEI* to the seller. Thus, as the construction of property requires only the incurrence of costs, the value of property estimated under this method should equal to the sum of the costs to create the property brought forward (compounded) to the time of sale using discount rates E_t relating to the construction period. This statement, essentially, can be viewed as the basis for the cost approach to valuation of property, however, it would only be valid in respect of replicable projects and the related property.

This sum would be approximately equal to the simple (undiscounted) sum of the costs to create the property plus a foregone income from alternative investments of this amount (or "normal" entrepreneurial profit on the capital invested into creating the property). So, in practice such estimates are whittled down to calculating the overall (unadjusted) sum of the specified costs. The attendant margin of error is greater, the longer is the time required for constructing the property, so the largest error creeps into the value for only large-scale buildings and structures.

Another problem that arises here should be noticed. Assume we have to value a building as at 4/1/2012. The duration of constructing such a building is 2 years. To provide valuation of the building in this instance it would be necessary (see above in this section) to assess the construction cost flows as they are distributed over time and time-value them to an aggregate amount (as at 4/1/2012). It should be believed that the value of the building as of the named date should correspond to some virtual purchase/sale transaction as of 4/1/2012. It means that such "virtual" building should have already been constructed and put into place by that moment. Therefore, its construction costs should relate to some previous period beginning 4/1/2010. But then such valuation should have its basis in the market situation extant at the decision-making point leading off to the commencement of the construction (i.e. even relate to some earlier date preceding the commencement of the construction) and account for those risks which had attended construction projects back then, rather than as at the valuation date. In other words, these calculations would require the information pertaining to market conditions and the systematic risk effective not as of the valuation date, but going back to often much earlier times, but appraisers commonly do not turn their attention to this fact. The similar problem is absent in cases of investment projects evaluations -- here all the cash flows relate to the future, instead of the past.

The above discussion should not be taken to deny importance to valuations rooted in the methodology for market value, but rather implies the need, on the one hand, for continuing to elaborate and refine this prominent methodology and, on the other, -- to continue advancing and giving full scope to market realization and acquisition value concepts (as well as entity-specific

gross realization and net acquisition value approaches) without being constricted by the towering predominance of the grooves of thought associated with the market value based mode of thinking.

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