COMITATO NAZIONALE PER LE MANIFESTAZIONI CELEBRATIVE
DEL IV CENTENARIO DELLA NASCITA DI GALILEO GALILEI

ATTI

DEL

CONVEGNO SULLA RELATIVITÀ GENERALE: PROBLEMI DELL'ENERGIA E ONDE GRAVITAZIONALI

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Gravitation and Unified Picture of Matter

ESTRATTO

FIRENZE
G. BARBÈRA EDITORE
1965

Gravitation and Unified Picture of Matter.

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1. - Introduction.

The great Galilean Jubilee offers an opportunity of discussing some new possibilities of the unified picture of the world. It is well known that impossibility of the universal classical mechanical picture became clear under some dramatic circumstances at the beginning of 20th century. The programmes of universal electromagnetic or later of unified geometrical picture proved also to be too narrow in view of quantum treatment of multitude of elementary particles. In these days new interest arose in the building of unified theory of « ordinary » matter, including elementary particles and their excited states: « resonons », but usually gravitation is as yet left aside.

We may point here some possibilities of including gravitation in unified picture of matter and also suggest the necessity of taking into account cosmological phenomena when trying to construct not merely « local » but « natural » picture of the world. J. A. Wheeler in his geometrodynamics aims also at the construction of some « natural » unified picture; in some points *e.g.* in emphasizing importance of mutual transmutations of ordinary matter and gravitational field, this programme is not far from ours.

2. - Nonlinear theory.

Starting from ordinary matter we may draw attention once more to non-linear spinor theory, being a generalization of de Broglie's fusion hypothesis. It is especially important for applications to gravidynamics as in both cases nonlinearity is of genuine, primordial character and not solely induced by vacuum effects. There are many other attempts aiming at unified theory *e.g.* Sakata model, octets, rotator model of de

Broglie-Vigier-Takabaysi and others, developed recently by Yukawa and his collaborators (which partly makes use of our idea of fusing minkowskian and isotopic spin spaces), further also dynamical compensational approach (in spirit of Yang-Mills, Sakurai and others), reggesized dispersional treatment, more formal but powerful group approach. Of course there are many links between all them, *e.g.* between compensation and so succesful SU_3 group.

Not entering here into discussion we present a simple derivation of the mass which is one of chief problems of the whole theory. Suppose the vacuum is degenerate in spirality, due for instance to pairing of particles of opposite spirality. This leads to an energy gap in excitation spectrum, which is considered to correspond to the nucleonic mass. To remove degeneracy add to Lagrangian a mass term

$$\mathscr{L}'_0 = \mathscr{L}_0 + m\overline{\Psi}\Psi; \qquad \mathscr{L}_{\text{int}} = \mathscr{L}_{\text{int}} - m\overline{\Psi}\Psi . \tag{2.1}$$

Leaving spirality conservation and introduce anomalous greenians $S_C^{\mathscr{L}R}$ alongside with normal S_C^{RR}, $S_C^{\mathscr{L}\mathscr{L}}$ using perturbation calculation and requiring compensation of diagrams, corresponding to $\mathscr{L}'_{\text{int}}$ with an ingoing φ-line and outgoing χ line, one gets

$$-\tfrac{3}{2}\lambda \,\text{Sp}\, S_C^{RL}\mathscr{L}(0):\varphi^+(x)\,\chi(x): = m:\varphi^+(x)\,\chi(x): \tag{2.2}$$

or in impulse space

$$m = \frac{3i\lambda m}{(2\pi)^4}\int \frac{\mathrm{d}^4 p}{p_0^2 - m^2 - i\varepsilon} . \tag{2.3}$$

Non-trivial solution of this equation happens to coincide up to numerical factor with the result of Nambu and Jona-Lasinio who started from different considerations:

$$m^2 \ln\left(1 + \frac{\mathscr{L}^2}{m^2}\right) = \mathscr{L}^2 - \frac{16\pi^2}{3l^2} . \tag{2.4}$$

Supplementary conditions $\lambda < 0$ (effective attraction), $-\mathscr{L}^2\lambda \geqslant 16\pi/3$ were taken into account. Cutting at $\mathscr{L}^2 = m$, we get $ml = 13,\ 0$; at $\mathscr{L}^2 = 3m^2$, $ml = 5.75$. These results obtained in very simple manner are practically coinciding with conclusions of Heisenberg and collaborators, who used complicated new Tamm-Dancoff method and indefinite Hilbert space metrics. Although we are emphasizing the necessity of introducing new type of propagator for nonlinear equation, one can see that even a simplified quantization leads to qualitatively satisfactory mass spectrum of barions. The gravitons can be constructed like photons and a preliminary evaluation in analogy with derivation of fine structure constant leads to reasonable value of gravitational constant. Of course for classification of particles a non-linear generalization accounting also invariance under SU_3 would seem to be promising.

In view of success of SU_3 group one may prefer now to start not from iso-spinors (Heisenberg) but from an unitary spinor $\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} = \psi$, whose 3 components may be considered as « quarks » of Zweig–Gell-Mann and which possess further as components ordinary Weyl or Dirac spinors. We prefer however to take massless quarks and to arrive at masses by the above sketched procedure. Using some relations between bilinear invariants *e.g.*

$$(\bar{\varphi}\varphi)^2 = \tfrac{4}{3}(\bar{\varphi}\lambda_i\varphi)^2$$

(λ_i-infinitesimal generators of SU_3), we get for nonlinear supplementary Lagrangian

$$\mathcal{L}' = (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 .$$

In this way one may hope to obtain by nonlinear « excitation » (or fusion) all real strongly interacting particles with their old spin etc. and new strange etc. properties.

3. - Tetrad theory of gravitation.

The interaction of fermions with gravitation requires introduction of tetrads, moreover introducing of $h_\nu{}^a$ instead of $g_{\mu\nu}$ as potentials means not a mere retranslation of equations of general relativity as some new supplementary conditions are needed and there can arise new invariants. Compensational treatment of gravitation is based also on tetrads. So one is inclined rather to speak about a kind of tetrad generalization of Einsteinian general relativity.

Let us introduce at each point an affine repere (latin indices). Then for Lamé's coefficients

$$h^a_\mu = h_\mu h^a ; \qquad g_{\mu\nu} = h^a_\mu h_{a\nu} ; \qquad \Lambda = \det|h^a{}_\mu| = \sqrt{-g} . \tag{3.1}$$

One considers two groups of transformations: A) arbitrary transformations of co-ordinates, B) local orthogonal transformations of tetrads which connect nonholonomic co-ordinate systems. Combined covariant derivative reads

$$*\nabla_\sigma \mathscr{A}^\mu_{(a)} = \mathscr{A}^\mu_{(a),\sigma} + \Gamma_{\nu\sigma.}^{\ \ \ \mu} \mathscr{A}^\nu_{(a)} - \Delta_{\sigma(ab)} \mathscr{A}^{\mu(b)} . \tag{3.2}$$

Christoffel symbols, Ricci coefficients are equal

$$\Delta_{\sigma,\mu\nu} = C_{\sigma\nu,\mu} - C_{\nu\mu,\sigma} - C_{\sigma\mu,\nu} \tag{3.3}$$

$$C_{\sigma,\mu\nu} = \tfrac{1}{2} h_{a\sigma}(h^a_{\nu,\mu} - h^a_{\mu,\nu}) = h_{a\sigma} h^a_{[\nu,\mu]} . \tag{3.4}$$

Variation of scalar curvature

$$\Lambda R = 2(\Lambda \Delta_{a\cdot\cdot}^{\ \sigma a})_{,\sigma} + \Lambda(\Delta_{a,bc}\Delta^{b,ca} - \Delta_{a,b}{}^a \Delta_{c,}{}^{bc}) = \\ = 4(\Lambda C_{a,\cdot\cdot}^{\ \ \sigma a})_{,\sigma} + \Lambda(4 C_{a,b}{}^a C_{c,}{}^{cb} - \Delta_{a,bc} C^{a,bc}) \tag{3.5}$$

over h^a_μ yields Einstein equations in tetrad form

$$R_{na}{}^{n\mu} - \tfrac{1}{2} h^\mu_a R = -\varkappa T_a{}^\mu . \tag{3.6}$$

As a possible supplementary condition we choose with V. I. Rodičev the « quasi-harmonic » one

$$\Delta_{na}{}^n = \frac{1}{\Lambda}(\Lambda h^\sigma_a)_{,\sigma} = 0 \tag{3.7}$$

with garantees the continuity of nonholonomic orthogonal system of co-ordinates and goes over in known harmonic de Donder condition for holonomic reperes. With this quasi-harmonic condition we get

$$R = -\Delta_{abc} C^{abc} \tag{3.8}$$

which is scalar both for groups A and B satisfying our quasi-harmonicity. Then field equations read

$$\Delta_m{}^{\sigma\varrho}{}_{;\varrho} = \bar{\bar{t}}^\sigma_m + \varkappa T^\sigma_m \tag{3.9}$$

where $\bar{t}^\sigma_m$ characterizes energy of the gravitational field.

In contrast to general relativity which locally does not distinguish 1) fictious field, due to nonstationary co-ordinates, 2) inertial field due to noninertial reference system, 3) gravitational field caused by distribution of masses, now the tetrad formalism suggests a new possible interpretation of inertia.

Let us consider now the problem of energy, which as well known up to now could not find any satisfactory generally recognized solution. Applying Noether's theorem to complete or abridged Lagrangians one got either Møller-Mitskevič or Einstein expressions for the energy complex; Lorentz complex obtained by variation over $g_{\mu\nu}$ yielded vanishing result. Now one clearly must apply Noether formalism not to $g_{\mu\nu}$ but to h^a_μ and gets

$$(3.10)\quad \begin{cases} \overline{\mathscr{L}}_0 = \frac{1}{2}\Lambda G, \\ G = \Delta_{a,bc}\Delta^{b,ca} - \Delta_{a,}{}^{a}{}_{b}\Delta_{c.}{}^{bc} \\ \bar{t}^\sigma_m = -\frac{1}{2}Gh^\sigma_m + 2\Delta^{a,b\sigma}C_{a,bm} + 4C_{a,}{}^{\sigma a}C^{b}{}_{\cdot bm} - 4C_{a,}{}^{ba}C^{\sigma}{}_{\cdot bm}. \end{cases}$$

On the other hand from complete Lagrangian

$$(3.11)\quad \mathscr{L}_0 = \frac{1}{2}\Lambda R$$

we get another covariant quantity

$$(3.12)\quad t_m{}^\sigma = -\frac{1}{2}R^\sigma_m - 2\Delta^{a,b\sigma}C_{a,bm} + 2C^{\sigma,ab}C_{a,bm} - \\ - 4C^{a}{}_{.ab}C^{(\sigma,b)}{}_m + 2g^{\sigma\varrho}C^{a}{}_{\cdot am,\varrho} - 2h^{(\sigma}{}_a h^{\varrho)}{}_b C^{a\cdot b}{}_{m,\varrho}.$$

We may remark that both energy complexes can be obtained also by usual procedure of separating a divergence in field equations and shifting remaining terms on the right hand side.

Both energy expressions can be obtained by means of superpotentials

$$(3.13)\quad \begin{cases} \Lambda(\bar{t}^\sigma_m + \varkappa T^\sigma_m) = \frac{\partial}{\partial x^\varrho}(\Lambda\Delta_{m,}{}^{\sigma\varrho} + 2\Lambda\Delta_{a,}{}^{a[\sigma}h^{\varrho]}{}_m), \\ \Lambda(t^\sigma_m + \varkappa T^\sigma_m) = \frac{\partial}{\partial x^\varrho}(\Lambda C_{m,}{}^{\sigma\varrho}). \end{cases}$$

Using from beginning the quasi-harmonic condition we get by means of field equations an expression

$$(3.14)\quad \bar{\bar{t}}^\sigma_m = \frac{1}{2}\Delta_{a,bc}C^{a,bc}h^\sigma_m + 2\Delta^{a,b\sigma}C_{a,bm}$$

which results also from $\bar{t}^\sigma_a$ by applying this condition.

The total energy-impulse

$$P_m = \frac{1}{\varkappa c}\int (\mathrm{d}^3x)_\sigma (\Lambda t^\sigma_m + \Lambda T^\sigma_m) = \frac{1}{\varkappa c}\oint (\mathrm{d}x^2)_{\sigma\varrho}(\Lambda C_m{}^{\sigma\varrho}) \tag{3.15}$$

is an affine vector in respect to Lorentz transformations but a scalar in respect to arbitrary trasformations of co-ordinates.

Applying Noether formalism to local 4-rotations we get angular momentum of the field, which with quasi-harmonic condition reads

$$\Sigma^\sigma{}_{mn} = \frac{2}{\varkappa c}\, C^\sigma{}_{mn} \tag{3.16}$$

which explains the meaning of C^σ_{mn}. We see that tetrad formalism led already to some interesting conclusions in gravidynamics and its development seems rather promising removing in particular serious difficulties in definition of energy, well known from all previous investigations.

4. - Non-linearity and torsion.

At this stage one can say that the physical reality seems to manifest itself in dualistic manner, being described by means of a some nonlinear spinor (ordinary matter) and on the other side by a tetrad gravitational potential which also can be put in spinor form (which suggests introducing of a kind of combined universal spinor).

On the other side geometrodynamics prefers to operate with bosonic geometrical fields and in this connection J. A. Wheeler, recognizing the interest of nonlinear spinor theory, pointed the absence of geometrical meaning of nonlinear term, which moreover possessed in his opinion only local character.

But an attention must be paid to Rodičev's theorem which just tries to endow the nonlinearity with geometrical meaning. Let us use a general affine connection $*\Gamma^\mu_{\sigma\lambda}$ which is nonsymmetrical in lower indices. In simplest case when torsion tensor is fully antisymmetric (Galilean metrics, geodesics are straight lines) we have

$$*\Gamma_{\sigma\mu,\nu} = K_{\sigma\mu,\nu} = \Phi_{[\sigma\mu\nu]} \tag{4.1}$$

and curvature scalar

$$*R = \Phi_{[\sigma\alpha\beta]}\Phi_{[\sigma\beta\alpha]}\,. \tag{4.2}$$

For spinors in spaces endowed with torsion we get

$$\Psi_{,\sigma} = \Psi_{;\sigma} - {}^*B_\sigma \Psi \tag{4.3}$$

$$^*B_\sigma = \tfrac{1}{4}{}^*\Delta_{\sigma,ab}\gamma^a\gamma^b + iI\varphi_\sigma \tag{4.4}$$

generalizing well known coefficient of curved space. In our simple case

$$^*\Delta_{\sigma,\mu\nu} = \Phi_{[\sigma\mu\nu]}\,. \tag{4.5}$$

Using spinor Lagrangian and passing to pseudo-vectors we get for action

$$T = \int \{\mathscr{L} - b^*R\}\,(\mathrm{d}x) \tag{4.6}$$

$$I = \int \left\{\frac{1}{2i}(\Psi^+\gamma_\alpha\Psi_{,\alpha} - \Psi^+_{,\alpha}\gamma_\alpha\Psi) - (\Psi^+\gamma_\alpha\Psi)\varphi_\alpha - (\Psi^+\gamma_\alpha\gamma_5\Psi)\tilde{\varphi}_\alpha + \frac{1}{2\lambda_0^2}\tilde{\varphi}_\alpha^2\right\}(\mathrm{d}x)\,. \tag{4.7}$$

Putting for simplicity electromagnetic potential $\varphi_\lambda = 0$ one gets after variation over $\tilde{\varphi}_\lambda$ and Ψ^+

$$\gamma_\alpha\Psi_{,\alpha} + i\lambda_0^2(\Psi^+\gamma_\alpha\gamma_5\Psi)\gamma_\alpha\gamma_5\Psi = 0\,,$$

i.e., the fundamental nonlinear spinor equation (but as yet without eventual SU_3 or other analogous refinements) moreover just with pseudo-vectorial term chosen by Heisenberg from the point of view of group properties among various nonlinearities pointed by us. Anyhow a more systematic study of torsion is suggested by this result even for the better analysis of the ordinary curved space, where Riemannian connection can be divided in two mutually compensanting parts both endowed with torsion.

5. - Gravitational transmutations of elementary particles.

Gravidynamics in tetrad form can be quantized along the same lines as ordinary form based on $g_{\mu\nu}$ and leads to canonical and covariant commutation relations between h^a_ν and Ricci's coefficients plying the role of momenta *e.g.*

$$(h_\varkappa(\varepsilon)\Delta^{0\varkappa}(\varepsilon) - \Delta(\varepsilon)^{0\varkappa}h_\varkappa(\varepsilon)) = \tfrac{1}{2}h\,. \tag{5.1}$$

One can establish also relations of the type of «individual» errors discussing some « Gedankenexperimenten » in the spirit of previous considerations (Bronstein, Regge, Treder, de Witt).

Without entering into these developments, we may apply the results of quantization in linear approximation to some important processes above all to calculation of probability of transmutation of a pair of fermions in 2 gravitons. The possibility of such transmutations suggested by us was discussed later by many authors (A. A. Sokolov, I. Piir, J. A. Wheeler, D. Brill, M. Korkina, J. S. Vladimirov, J. Feynman, J. Weber). There arise in gravidynamics some supplementary diagrams due to nonlinearity, when comparing with electrodynamics.

In nonrelativistic limit $E^2 \sim m^2 \gg p^2$

$$d\sigma = \frac{m^2 \varkappa^2}{64(8\pi)^2} \frac{C}{V} d\Omega . \tag{5.2}$$

Applying qualitatively such calculations also to high energies we obtain

$$d\sigma = \frac{\varkappa^2 E^2}{128(8\pi)^2} (3 \sin^2 2\theta + 2 \sin^4 \theta) d\Omega . \tag{5.3}$$

We remark that cross-section of photon-graviton annihilation tends to constant limit (at such extrapolation)

$$\sigma \sim 10^{-68} \text{ sm}^2 \tag{5.4}$$

at the energies of the order

$$E \sim \sqrt{\frac{r_e}{r_g}}\, mc^2$$

the cross-section of photon graviton, two-graviton and two-photon annihilation of a pair of particles are of the same order.

In view of the some renewed interest in torsion we have quantized this field, putting the torsion tensor in the form:

$$S_{\alpha\beta\gamma} = \frac{1}{2}\left(\frac{\partial \varphi_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial \varphi_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial \varphi_{\gamma\alpha}}{\partial x^\beta}\right) \tag{5.5}$$

getting the corresponding quanta: « torsions » and investigating their interaction with spinor and other fields.

6. - Compensational treatment of gravitation.

We draw attention to the fact that gravitational field can be naturally connected with other fields allowing « compensational » treatment and realizing dynamically some conservation laws. Requiring invariance under some group *e.g.*, of isotopic rotations, but with parameters depending from co-ordinates, one must introduce a compensating field to restore invariance (Yang-Mills, Sakurai and others). In the same sense invariance under localized gauge group

$$\Psi \to \Psi \exp\left[ie\omega(x)\right] \tag{6.1}$$

required introduction of electromagnetic vector potential

$$\mathscr{A}_\mu \to \mathscr{A}_\mu + \frac{\partial\omega(x)}{\partial x^\mu}\,. \tag{6.2}$$

It seems that some predicted in this manner particles were recently discovered as « resonons ». The whole method seems to bring new arguments in favour of more narrow connection of Minkowskian and isotopic spin spaces (external and internal degrees of freedom) (suggestion of Pais, Brodski, Sokolik and ourselves, Yukawa developed by de Broglie-Vigier *et al.*).

Without entering into instructive history of compensational treatment of gravitation (Utiyama, Brodski-Ivanenko-Sokolik-Frolov, Kibble, Schwinger) we may consider the full inhomogeneous Poincaré-Lorentz group. For an arbitrary group Γ connected with co-ordinate transformations, the compensating derivative has the form

$$Q^{\mathscr{A}}_{|a} = h^\sigma_a Q^{\mathscr{A}}{}_{,\sigma} - I^{\mathscr{A}}_{Bm}\mathscr{A}^m_a Q^B \tag{6.3}$$

$I^{\mathscr{A}}_{Bm}$-generator of that representation of Γ under which is transformed the field $Q^{\mathscr{A}}$. In general case one needs for compensation two independent compensation fields $\mathscr{A}^m_a$ and h^σ_a which will be connected by means of equations of motion.

$\mathscr{A}_a{}^m$ compensates terms arising at local transformations of $Q^{\mathscr{A}}$ and the field h^σ_a compensates term due to local transformations of coordinates under Γ.

One constructs locally invariant Lagrangian by multiplying with $\Lambda = \det|h^a{}_\mu|$ and extending the ordinary derivative by compensating one

$$\mathscr{L}_{(Q)} = \Lambda L_{(Q)}(Q^{\mathscr{A}}, Q^{\mathscr{A}}{}_{|a})\,. \tag{6.4}$$

For Lagrangian of the compensating field itself we have

$$\mathcal{L}_0 = \Lambda L_0(\mathcal{F}_{ab}{}^m) \tag{6.5}$$

with

$$\mathcal{F}_{ab}{}^m = h^\nu_a \mathcal{A}_b{}^m{}_{,\nu} - h^\nu_b \mathcal{A}_a{}^m{}_{,\nu} + \mathcal{A}_c{}^m (h_a{}^\sigma{}_{,\tau} h^\tau_b h^c_\sigma - h_b{}^\tau{}_{,\sigma} h^\sigma_a h^c_\tau) + C^{\ m}_{q\cdot n} \mathcal{A}_a{}^q \mathcal{A}^n_b \,. \tag{6.6}$$

One gets the field equations by variation over potentials h^σ_a and $\mathcal{A}_\sigma{}^m$

$$\frac{\delta \mathcal{L}_0}{\delta \mathcal{A}_\sigma{}^m} = -\varkappa \frac{\delta \mathcal{L}_{(Q)}}{\delta \mathcal{A}_\sigma{}^m} \tag{6.7}$$

$$\frac{\delta \mathcal{L}_0}{\delta h_\sigma{}^a} = -\varkappa \frac{\delta \mathcal{L}_{(Q)}}{\delta h_\sigma{}^a} \tag{6.8}$$

with our well known coefficients Γ: (Fock-Ivanenko).

Hence

$$\frac{\partial}{\partial x^\sigma} \frac{\partial \mathcal{L}_0}{\partial \mathcal{F}_{\sigma\mu}{}^m} = \Lambda \mathcal{I}^\mu{}_m = \Lambda \mathcal{I}_0{}^\mu{}_m + \varkappa \Lambda \mathcal{I}^\mu_{(Q)m} = \left(-\frac{1}{2} \frac{\partial \mathcal{L}_0}{\partial \mathcal{A}_\mu{}^m} - \frac{\varkappa}{2} \frac{\partial \mathcal{L}_{(Q)}}{\partial \mathcal{A}_\mu{}^m} \right). \tag{6.9}$$

$$\mathcal{F}_{\mu\nu}{}^m \frac{\partial \mathcal{L}_0}{\partial \mathcal{F}_{a\nu}{}^m} - \frac{1}{2} h^a_\mu \mathcal{L}_0 = -\varkappa \Lambda T_\mu{}^a = -h^a_\mu \mathcal{L}_{(Q)} + \frac{\partial \mathcal{L}_{(Q)}}{\partial Q^\sigma{}_{/a}} Q^\sigma{}_{;\mu} \,. \tag{6.10}$$

In the case of inhomogeneous Lorentz group $h_a{}^\nu$ are Lame's reperes, $\mathcal{A}_\sigma{}^{ij}$-Ricci's coefficients. Putting in (6.3) the generators of corresponding representations of the Lorentz-group we get covariant (compensating) derivatives for vectors etc. For spinor one gets in this manner a new derivation

$$\Psi_{/a} = h^\sigma{}_a \Psi_{,\sigma} - \Gamma_a \Psi \tag{6.11}$$

with our well known coefficients Γ_a (Fock-Ivanenko).

For Lagrangian

$$\mathcal{L}_0 = \tfrac{1}{2} \Lambda R_{ij} R^{ij} \tag{6.12}$$

one arrives with (6.10) at tetradic form of Einstein equations, and (6.9) gives an expression of torsion tensor by means of the spin of the external field

$$K^\mu_{ab} = -\frac{\varkappa}{2} (S^\mu_{ab} + h^\mu_{[a} S^\sigma_{b]\sigma}) \,. \tag{6.13}$$

For longitudinal part of the compensating field we have

$$\mathscr{A}_a^{(l)m} = -(1/\alpha) h_a^\sigma \tilde{\mathscr{U}}_c^\varkappa I_{\mathscr{A}}^{cm} \mathscr{U}_{\varkappa,\sigma}^{\mathscr{A}} \tag{6.14}$$

α-some number depending from a given representation of $\mathscr{U}_\varkappa^{\mathscr{A}}$. In the case of Lorentz group one naturally takes tetrads $h^a{}_\sigma$ for $\mathscr{U}$.

Although the development of compensational treatment of gravitation is by no means finished and raises many problems, for instance of eventual generalization for torsion, we see that the basic theory is powerful enough to give an important reinterpretation of ordinary General Relativity and to contribute to its present day tetradic generalization.

7. - Cosmological remarks.

Aiming at a construction of some kind of « natural » unified picture of the world and not only of local theory one must carefully take into consideration all possible links between cosmology and elementary particles.

In this connection starting from well known empirical fact that

$$\frac{m_e}{\alpha_e} = \frac{m_{\mathscr{N}}}{\alpha_g} = a \approx 1{,}4 \cdot 10^{-25}\ \text{g} \tag{7.1}$$

(m_e, $m_{\mathscr{N}}$-masses of electron and nucleon, α corresponding fine structure constants) one may try to proceed further and build with D. Kurdgelaidze corresponding masses for the case of Fermi interaction ($m_F = a\alpha_F \approx 10^{-40}$ g) and for the gravitation

$$m_g = a\alpha_g \approx 6 \cdot 10^{-66}\ \text{g}\ . \tag{7.2}$$

If one supposes gravitational field in some manner endowed with such mass one can compare the wave equation

$$\left[\Box - \left(\frac{m_G c}{\hbar}\right)^2\right] \eta_{\mu\nu} = 0 \tag{7.3}$$

with linearized form of Einstein equations supplemented by cosmological term λ_0. One gets then for Hubble constant

$$h_1 = \lambda_0^{\frac{1}{2}} = \frac{1}{\sqrt{2}} \frac{m_g c^2}{\hbar} = \frac{\varkappa a^2 c}{\sqrt{2}\,\hbar} \simeq 4 \cdot 10^{-18}\ \text{sec}^{-1}\ , \tag{7.4}$$

in good agreement with observational value (cf. also Gomide).

We may draw attention further to a tempting possibility to connect the observed overhelming concentration of particles in our part of universe with its expansion, guessing that for contracted model one must pass to anti-particles. Among further as yet scarse links between cosmology and microphysics one must keep in mind also interesting considerations about the arrow of time (Bondi, Hoyle, Hogarth) and Wheeler and Hoenl-Dehnen treatment of boundary Machian conditions.

We see that both nonlinear theory, compensational viewpoint and mutual transmutations all seem to be promising in establishing funda-mental new relations between ordinary matter and gravitation, preli-minary considerations point even on the links with cosmology.

May be an unified picture of ordinary matter will even prove to be impossible without account of gravitation and cosmological features?

Note added in proofs.

Developping these ideas we may suggest that T-parity and combined CP-parity would be in general not conserved (if Lüders theorem holds in gravidynamics), the expansion acting, as a kind of effective very weak cosmological fore, differently on particles and anti-particles. This conclusion can be contrasted with recent discovery by Fith-Cronin and their collaborators of the apparent violation of combined parity at K_2 mesons decay. We believe with D. KURDGELAIDZE that the recent interesting hypothesis of CABIBBO, LEE *et al.* to explain this anomaly by introducing a new field produced by the hypercharge of our Galaxy, anyhow must be supplemented by cosmo-logical viewpoint as otherwise the global effect of all galaxies taken as static not expanding distribution would lead to infinity analogously to well known paradoxon of Newtonian static cosmology.

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INTERVENTI E DISCUSSIONI

— J. Treder:

The question « What the gravitational potentials are?—the 10 $g_{\mu\nu}$ or the 16 components of the tetrads? » is very important for general relativity. If the 16 tetrad-components are going in the gravitation-theory, free rotations of the tetrads are not possible. The relative orientation of the tetrads are fixed by the physical conditions. Weyl has pointed out that no physical meaningful causes existed for this fixation. But for the theory of the interactions between gravitational fields and Fermi fields, the introduction of tetrads is necessary (*). But, if we have no causes of the meaning that only the $g_{\mu\nu}$ have a physical meaning, the Einstein-Lagrangian is only a degenerate combination of the 3 Lagrangians which are giving second-order-field-equations for the « gravitational potential » h^a_μ. These three Lagrangian are the Lagrangians of Weitzenböck (Einstein and Mayer have worked with these Lagrangians). Generally, the Weitzenböck Lagrangians are giving 16 field equations. Only the Einsteinian combination is giving only 10 equations for the 10 $g_{\mu\nu}$.

— A. Lichnerowicz:

Je crois que nous tombons dans le qui-pro-quo parce que deux choses différentes ne sont pas suffisamment distinguées.

1) L'introduction de repères orthonormés (ou tetrades) en un point est nécessaire. Ces repères donnent l'interprétation de grandeur physique, en un point de l'espace-temps relativement aux directions d'espace et de temps qu'ils définissent. Les cordonnées ne servent qu'à la cartographie. Ces repères sont utiles pour une bonne définition des objects spinoriels (*).

2) L'introduction de champs privilégiés de tetrades est une chose différente, grave et sur laquelle il convient de refléchir. Personellement, je n'y suis pas favorable pour beaucoup de raisons. En fait la théorie correspondante est fort éloignée de la relativité générale. Elle est trop riche; elle est une particularisation et pas une généralisation.

— H. Bondi:

Do I correctly understand your attitude to tetrads if I say that two spaces

(*) *Remark of Ivanenko*: as shown by A. Fock-D. Ivanenko and H. Weyl.

with the same metric, but different tetrad systems are regarded as physically distinct by you, but are regarded as physically identical by several others who also use tetrads? And how does the difference show up?

Furthermore, does the greater fixing of the space at infinity mean that your theory comes closer to explaining the definition of inertial frames in terms of boundary conditions at infinity (Mach's principle) than ordinary general relativity?

— P. G. BERGMANN:

1) Formally, one can introduce spinors without tetrads; but the spinors require the unimodular group, which is locally isomorphic to the Lorentz group. Hence, in the presence of Fermian fields, the introduction of tetrads is a matter of convenience, not of principle. Incidentally, I agree with the remarks by Prof. Lichnerowicz.

2) I should like to request further elucidation of your reasons for permitting nonsymmetric affine connections, in view of the considerable formal complications.

— J. A. WHEELER:

The discussion of Professor Ivanenko is useful in reminding us of many important questions on which physics has yet to take a final position. Among these issues it is difficult to name any one which is more important than this: how can the existence of *spin* in nature be reconciled with Einstein's long term dream for a purely *geometrical* description of all of physics? We know very well that there is no such thing as spin $\frac{1}{2}$ in Einstein's standard theory of relativity. Gravitational waves have spin two. Electromagnetism can be described in Einstein's theory as an aspect of geometry (Rainich and Misner), and electromagnetic waves have spin 1. But there is no spin $\frac{1}{2}$. To be sure, one can introduce tetrads, as a matter of convenience in describing the geometry. Tetrads also help in describing a spinor field when such a field is introduced as a foreign entity moving about in the spacetime geometry. But the use of tetrads in formulating Einstein-Maxwell theory in no way changes the conviction that theory deals solely with fields of integral spin. It is completely incapable of explaining the presence of spin $\frac{1}{2}$ in nature whether formulated with or without tetrads. Therefore some fundamental change is required in the theory if it is to embrace spin $\frac{1}{2}$. One suggestion about the direction for such a change is suggested by recent results of de Witt, kindly communicated in a letter of 19 June 1964 and a personal discussion of 8 August 1964. In brief, he shows that the wave equation for the state function of gravitation theory, $\psi = \psi({}^{(3)}\mathcal{G})$, has points of similarity to the Klein-Gordon wave equation for the state function of a particle, $\psi = \psi(x)$. In both cases the differential equation is of second order. In the case of the Klein-Gordon equation Dirac « took the square root » to arrive at a first order wave equation.

This equation is *not* equivalent to the Klein-Gordon equation but it is compatible with experience. Similarly de Witt suggests it will be interesting to « take the square root » in the sense of Dirac of the functional wave equation to which he has arrived. So much for the idea in broad outline the details are summarized in Table:

TABLE – *Analogy between the Klein-Gordon equation and the de Witt equation.*

	Klein-Gordon	de Witt
System under consideration	Particle	Geometry
Configuration described by	x	${}^{(3)}\mathscr{G}$
Key equation in classical theory	$p_\mu p^\mu + m^2 = 0$	$(\mathrm{Tr}\,\boldsymbol{k})^2 - \mathrm{Tr}\,\boldsymbol{k}^2 + {}^{(3)}\mathrm{R} = 0$
Quantum wave equation	$\square\psi + m^2\psi = 0$	See below
« Square root »	$\Gamma^\mu \partial_\mu \psi + m\psi = 0$	To be written out by analogy

The de Witt equation has the form:

$$g^{-\frac{1}{4}}(\delta/\delta g^{(ij)})\, g^{\frac{1}{2}} g^{(ij)(kl)}(\delta/\delta g^{(kl)})\, g^{-\frac{1}{4}}\varphi - \lambda^{\frac{1}{2}\,(3)}R\psi({}^{(3)}\mathscr{G}) = 0\,.$$

To take the square root of this equation in this sense of Dirac will lead, not to a spinor *particle* of mass m, as in the case of Klein-Gordon equation, but to a spinor *field* of zero rest mass. It is of great interest and importance to know whether this spinor field has any correspondence with what we know of neutrinos.

— C. MØLLER:

I should perhaps make a few remarks about the difference between Ivanenko's point of view and my present view regarding the use of tetrads in general relativity. Up to the fall of 1963 I tried to find an expression for the distribution of the energy in gravitational fields and for this I found it necessary to regard the tetrads as true field variables which should be determined uniquely up to a constant Lorentz rotation. Since Einstein's field equations determine the metric, only, they had to be supplemented by six further covariant equations. In view of the arbitrariness involved in the setting up of these supplementary conditions and also because it seemed so difficult to imagine how one could measure the gravitational energy content in a small part of space I came to the convinction that Einstein's old point was right and that only the *total* energy and momentum of an insular system can be regarded as a measurable quantity. From this point of view we do not need to fix the tetrads completely and the problem of setting up supplementary equations does not arise. Only in the case of a completely empty space one has to assume that the tetrads form a stationary vector field in order to avoid the Bauer difficulty which is the most disagreable feature of Einstein's energy-momentum complex. Moreover, in the case of an insular system one has to assume certain boundary conditions for the tetrads at spatial infinity. Apart from that we can freely rotate the tetrads indipendently throughout space-time and it can be shown that the resulting total four-momentum P_i as well as the asymptotic expression for the complex T_i^k at large spatial distances are invariant under these rotations of the tetrads. In the case of the Bondi and Sachs solutions the expression for the total energy is in accordance with Bondi's definition of the total mass and moreover, for a nonradiative system, P_i can be shown to be a free 4-vector under arbitrary space-time trasformations. From this point of view the tetrads play the role of auxiliary variables similarly as the electromagnetic potentials in electrodynamics and the Lorentz rotations of the tetrads are somewhat analogous to the usual gauge transformations. Maybe I have resigned too early and it is not impossible that the point of view of Ivanenko, which tries to connect gravitational phenomena with others physical phenomena, at the end will turn out to be more fruitful.

— D. Ivanenko:

On the whole we had a very stimulating discussion with many interesting remarks, but I may concentrate here only on some few questions. Although different authors treating spinors in general relativity use such expressions as: convenient, helpful, useful etc., it is important to stress that the description of gravitational interaction of fermions requires introduction of tetrads quite compulsory (Fock-Ivanenko, Weyl). This was emphasized recently also by Ch. Møller, A. Lichnerowicz, H. Treder, J. A. Wheeler and others. So for a spinor tetrad components represent gravitational potentials and may be considered as a field. It seems therefore that remarks of P. Bergmann which run counter this general opinion are connected with some misunderstanding. Above this tetrads are connected in natural way with powerful method of compensating fields (Jang-Mills) and seem to be able to lead to a reasonable energy expression (Møller, Rodičev) perhaps contributing also to further clarification of inertia notion. Whether one prefers to speak in this connection about a «rounding» or a «revision» or a kind of tetradic «generalization» of Einsteinian theory based solely on $g_{\mu\nu}$ metrical and gravitational potentials, si a matter of personal taste.

At present state of the tetrad theory of gravitation many important questions need further investigation as was rightly pointed here by H. Bondi, A. Lichnerowciz, G. Wataghin, M. A. Tonnelat. Not only the best choice of supplementary conditions (old Møller?, Rodičev, Schwinger-Deser?) is not settled, but Prof. Ch. Møller even tentatively proposes now to dispense with such conditions (using instead some boundary conditions). We try to selec Rodičev's quasi-harmonic conditions, expressing the conservation of th 4-volume corresponding to some given configuration of events somewhat analo gously to Liouville's theorem. Then only 4 tetradic components are fixed, th other 2 being determined by the nature of a given concrete problem. In th respect we are more inclined to Møller's older view point or Schwinger-Des proposals where some supplementary conditions were used, which restrict to some extent the free rotations of tetrads at different point. Anyhow t various results already obtained on a difficult road of classical and quant tetradic theory of gravitation seem to permit a rather optimistic attitu

As to P. Bergmann's question about nonsymmetrical part of connect coefficients, which correspond to torsion I wish to point that torsion or ticularly distant parallelism really appear in tetrad theory, also in com sational treatment, in a natural way, though it is not necessary. Anyhow one is led to account explicitly the absence of torsion as does *e.g.* Ch. Møller in his report. Also we may draw once more attention to Rodičev's theorem showing that parallel displacement of spinors in a space endowed with torsion leads to a nonlinear term in Dirac's equation (cf. also R. Finkelstein, A. Peres, Sciama).

In view of importance of some kind of nonlinear spinor equation in the unified theory of matter this result must be considered with some attention.

Perhaps one can even guess proceeding in this way some preliminary link between unified theories of nonlinear or fusionist spinor type and geometrized theories of J. A. Wheeler.

[Alla discussione partecipò anche G. WATAGHIN.]