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https://doi.org/10.1007/s11787-018-0212-9

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# Generalized Correspondence Analysis for Three-Valued Logics

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**Abstract.** Correspondence analysis is Kooi and Tamminga's universal approach which generates in one go sound and complete natural deduction systems with independent inference rules for tabular extensions of many-valued functionally incomplete logics. Originally, this method was applied to Asenjo–Priest's paraconsistent logic of paradox **LP**. As a result, one has natural deduction systems for all the logics obtainable from the basic three-valued connectives of **LP** (which is built in the $\{\vee, \wedge, \neg\}$-language) by the addition of unary and binary connectives. Tamminga has also applied this technique to the paracomplete analogue of **LP**, strong Kleene logic $\mathbf{K_3}$. In this paper, we generalize these results for the negative fragments of **LP** and $\mathbf{K_3}$, respectively. Thus, the method of correspondence analysis works for the logics which have the same negations as **LP** or $\mathbf{K_3}$, but either have different conjunctions or disjunctions or even don't have them as well at all. Besides, we show that correspondence analyses for the negative fragments of $\mathbf{K_3}$ and **LP**, respectively, are also suitable without any changes for the negative fragments of Heyting's logic $\mathbf{G_3}$ and its dual $\mathbf{DG_3}$ (which have different interpretations of negation than $\mathbf{K_3}$ and **LP**).

**Mathematics Subject Classification.** Primary 03B50; Secondary 03B22, 03F03.

**Keywords.** Natural deduction, Correspondence analysis, Three-valued logic, Proof theory, Paraconsistent logic, Paracomplete logic.

## 1. Introduction

The primary aim of this paper is to extend Kooi and Tamminga's correspondence analysis for a larger class of three-valued logics. The secondary aim is to generalize the results from the papers [66–68] which were submitted to Vasiliev Logic prize and from the other papers by the author regarding natural deduction systems for three-valued logics [60, 70–73].

Presented at UNILOG'18, Vichy, France as the winner of the first VASILIEV LOGIC PRIZE (Russia) and a runner-up for the UNIVERSAL LOGIC PRIZE 2018.

Published online: 10 October 2018

Birkhäuser

Correspondence analysis was invented by Kooi and Tamminga [48] as a tool for a formalization of all the truth-functional unary and binary extensions of Asenjo [2] and Priest's [78] logic of paradox **LP**. Then Tamminga [90] expanded this method for the case of strong Kleene logic $\mathbf{K_3}$ [44,45]. In [59,63], automated proof-searching was developed for Kooi and Tamminga's calculi. Their method was also applied for Halldén's [35] three-valued logic **PWK** (paraconsistent weak Kleene) [60] as well as for four-valued logics **LRA** (Kubyshkina and Zaitsev's logic of rational agent [47]) [65] and for **FDE** (Belnap and Dunn's [1] logic called first degree entailment) extended by Boolean negation [64]. In [61], correspondence analysis was used to syntactically characterize Tomova's natural logics [92].

In Sect. 2, we present semantics of some three-valued logics, including **LP**, $\mathbf{K_3}$, $\mathbf{G_3}$, and $\mathbf{DG_3}$, and emphasize some important connectives which can be added to the negative fragments of **LP**, $\mathbf{K_3}$, $\mathbf{G_3}$, and $\mathbf{DG_3}$. In Sect. 3, we introduce some natural deduction systems for three-valued logics as well as the method of correspondence analysis and illustrate how it works for **LP** and $\mathbf{K_3}$. In Sect. 4, we generalize correspondence analysis for the negative fragments of **LP**, $\mathbf{K_3}$, $\mathbf{G_3}$, and $\mathbf{DG_3}$. Sect. 5 is devoted to soundness and completeness proofs. In Sect. 6, we illustrate the power of generalized correspondence analysis with examples of **LP** itself as well as Béziau and Franceschetto's [9,10] logics **L3A** and **L3B**. Sect. 7 contains concluding remarks.

## 2. Some Examples of Three-Valued Logics

To begin with, let us recall some basic notions regarding many-valued logic. A logical matrix $\mathfrak{M}$ is a triple $\langle \mathfrak{T}, \mathfrak{F}, \mathfrak{D} \rangle$, where $\mathfrak{T}$ is a non-empty set of truth values, $\mathfrak{F}$ is a non-empty set of functions over $\mathfrak{T}$, and $\mathfrak{D} \subset \mathfrak{T}$ is a non-empty set of *designated* values. For each logical matrix $\mathfrak{M} = \langle \mathfrak{T}, f_{F_1}, \ldots, f_{F_n}, \mathfrak{D} \rangle$, there is a propositional language $\mathfrak{L}$ with an alphabet $\langle \mathcal{P}, F_1, \ldots, F_n, (,) \rangle$, where $\mathcal{P}$ is a set $\{p, q, r, s, p_n, q_n, r_n, s_n \mid n \in \mathbb{N}\}$ of propositional variables and $F_1, \ldots, F_n$ are propositional connectives of the same arity as $f_{F_1}, \ldots, f_{F_n}$. The notion of an $\mathfrak{L}$-formula is defined in a standard way. The notion of a valuation $v$ of a formula $\alpha$ in a logical matrix $\mathfrak{M} = \langle \mathfrak{T}, f_{F_1}, \ldots, f_{F_n}, \mathfrak{D} \rangle$ is defined as follows:

- if $\alpha = \pi$, where $\pi \in \mathcal{P}$, then $v(\alpha) \in \mathfrak{T}$;
- if $\alpha = F_i(\pi_1, \ldots, \pi_k)$, where $1 \leqslant i \leqslant n$ and $\pi_1, \ldots, \pi_k \in \mathcal{P}$, then, for each $\tau_1, \ldots, \tau_k \in \mathfrak{T}$, it holds that $v(F_i(\pi_1, \ldots, \pi_k)) = f_{F_i}(\tau_1, \ldots, \tau_k)$.

The notion of an entailment relation is defined as follows, for each set of formulas $\Gamma$ and each formula $\alpha$: $\Gamma \models \alpha$ iff for each valuation $v$, it holds that $v(\gamma) \in \mathfrak{D}$ (for each $\gamma \in \Gamma$) implies $v(\alpha) \in \mathfrak{D}$.

Since in this paper we consider three-valued logics, $\mathfrak{T} = \{1, 1/2, 0\}$ and either $\mathfrak{D} = 1$ or $\mathfrak{D} = \{1, 1/2\}$. Besides, we restrict $\mathfrak{F}$ to the case of unary and binary functions, since the most popular and important three-valued connectives are unary or binary ones.

For this paper the following three connectives are of huge importance:[1]

[1] We use the same notation for these negations. However, we hope it will not cause a confusion in the further exposition.