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Multichannel Phase-Equivalent Transformation and Supersymmetry*

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Abstract—A phase-equivalent transformation of local interaction is generalized to the multichannel case in the direct-scattering problem. Generally, the transformation does not change the number of bound states in the system and their energies. For a special choice of the parameters involved, however, the transformation removes one of the bound states and is equivalent to the multichannel supersymmetry transformation recently proposed by J.M. Sparenberg and D. Baye (1997). With the aid of the transformation, it is also possible to add a bound state to the discrete spectrum of the system at a given energy $E < 0$ if the angular momentum $l \geq 2$ in at least one of the coupled channels. © *2000 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Nucleon–nucleon, nucleon–cluster, and cluster–cluster potentials are an input for various microscopic calculations of nuclear structure and reactions. Unfortunately, the exact form of the potentials describing these interactions is unknown. It is conventionally supposed that the available scattering data and bound state properties can be fitted with approximately the same accuracy by different local potentials. For example, there are a lot of so-called realistic *NN* potentials on the market describing *NN* scattering and deuteron properties with high accuracy. Moreover, a description of phenomenological data can be achieved with potentials very different in structure. In particular, meson-exchange *NN* potentials of the Nijmegen kind [1] are known to have a short-range repulsive core in a triplet *s* wave. The same high-quality description of the nucleon–nucleon data is provided by the latest versions of the Moscow potential [2, 3] that does not have a repulsive core but instead is deeply attractive in the triple *s* wave at short distances and supports an additional forbidden state. The possibility of alternative descriptions of various cluster–cluster and nucleon–cluster interactions by means of repulsive-core and deeply attractive potentials with forbidden states is also well known (see, e.g., the discussion in [2] and references therein).

Principally, it is possible to distinguish experimentally between alternative potentials by studying their off-shell properties in interactions with an additional particle. The simplest probe is the photon, and as it was shown in [4–6], the proton–proton bremsstrahlung reaction $pp \longrightarrow pp\gamma$ in the energy range of 350–400 MeV can be used to discriminate between various *NN* potentials. However, the $pp \longrightarrow pp\gamma$ reaction has not been examined experimentally in this energy range.

Another possibility is to study the properties of three- and four-body systems bound by two-body potentials of interest. From this point of view, it looks like we do not have satisfactory nucleon–nucleon, cluster-nucleon, and cluster–cluster potentials at present. It is well known that none of the realistic *NN* potentials provides proper binding of tritium or ^{3}He. There have been successful attempts in generating phenomenological three-nucleon interactions tuned to fit the properties of light nuclei [7] (see also [8] and references therein). However, as it was shown in the detailed study of Picklesimer *et al.* [9], the effect of three-nucleon forces consistent with realistic two-body ones on the binding energy of the triton is canceled by the effects of virtual excitations of Δ isobars, etc. Hence, the trinucleon cannot be satisfactorily described using known realistic two-body potentials supplemented by three-body potentials consistent with them. All calculations within three-body cluster models also fail to reproduce the correct binding energy of three-cluster nuclear systems with known local cluster–cluster and cluster–nucleon potentials fitted to the corresponding scattering data.

To design a potential consistent with two-body phenomenological data and providing the correct binding of few-body systems, it seems promising to make use of phase-equivalent transformations depending on a continuous parameter(s). Some attempts in this direction have been performed using nonlocal phase-equivalent transformations. The results of these attempts are encouraging: in [10], an oversimplified *NN* potential providing a satisfactory description of *s*-wave *NN* scattering data was fitted to exactly reproduce the triton binding energy, while in [11], realistic $n\alpha$ potentials were tuned to reproduce various ^{6}He properties, including the binding energy within the $\alpha + n + n$ cluster

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model. The interactions suggested in [10, 11] are nonlocal ones. Various applications (see, e.g., [12, 13]) of local phase-equivalent transformations to few-body problems were restricted to the supersymmetry transformation [14–16] that removes one of the bound states in a two-body system. The supersymmetry transformation does not contain parameters and cannot be used for fine tuning of the interaction of interest.

However, a local phase-equivalent transformation which preserves the number of bound states and depends on a continuous parameter exists and is well known in the inverse scattering theory [17]. To the best of our knowledge, so far nobody has used it in few-body calculations. This transformation was developed for a single-channel case only and cannot be applied without some approximations to realistic *NN* interactions that mix triplet *s* and *d* partial waves. Another drawback of the transformation is that it involves a bound-state wave function and, thus, cannot be used to modify *nn* and *pp* interactions and the *np* interaction in all "nondeuteron" partial waves.

Recently, Sparenberg and Baye [18] suggested a multichannel supersymmetry transformation. We use some ideas of [18] to derive a multichannel phase-equivalent transformation which depends on continuous parameters. The transformation can be treated as a generalization of both the single-channel phase-equivalent transformation [17] and the multichannel supersymmetry transformation of [18]. Generally, the transformation does not change the number of bound states in the system and their energies. However, with a special choice of the parameters, the transformation removes one of the bound states and becomes equivalent to the multichannel supersymmetry transformation suggested in [18]. If the angular momenta in all coupled channels are less than two, a parameter-dependent family of local interactions phase-equivalent to the given initial one can be constructed by means of the transformation, even in the case when the system does not have a bound state. If the angular momentum is $l \geq 2$ in at least one of the coupled channels, the transformation can be used to add a bound state to the discrete spectrum of the system at a given energy $E < 0$. Having a bound state, one can construct a family of phase-equivalent potentials and afterwards remove the bound state by the supersymmetry version of the transformation. Thus, the suggested transformation can be used in a multichannel case to produce phase-equivalent interactions without any restriction on the structure of the discrete spectrum of the system. In particular, the transformation can be applied to the realistic *NN* interaction in all partial waves.

2. GENERAL FORM OF LOCAL MULTICHANNEL PHASE-EQUIVALENT TRANSFORMATION

Multichannel scattering and bound states are described by the Schrödinger equation

$$\sum_j (H_{ij} - E\delta_{ij})\varphi_j(E, r) = 0, \qquad (1)$$

where indices i and j label the channels, E is the energy,

$$H_{ij} = \frac{\hbar^2}{2m}\left[-\frac{d^2}{dr^2} + \frac{l_i(l_i+1)}{r^2}\right]\delta_{ij} + V_{ij}(r) \qquad (2)$$

is the Hamiltonian, m is the reduced mass, and l_i stands for the angular momentum in the channel i. We suppose that the potential $V_{ij}(r)$ (i) is Hermitian and (ii) tends asymptotically at large distances to a diagonal constant matrix,

$$V_{ij}(r) \xrightarrow[r \to \infty]{} \epsilon_i \delta_{ij}, \qquad (3)$$

where ϵ_i is a threshold energy in the channel i. We suppose that $\epsilon_1 = 0$ and $\epsilon_i \geq \epsilon_j$ if $i > j$.

The boundary conditions for the wave functions are

$$\varphi_i(E, 0) = 0, \qquad (4)$$

$$\varphi_i(E, \infty) < \infty. \qquad (5)$$

Except for the discussion in Section 3.3, we suppose that there is at least one bound state in the system at the energy E_0. The corresponding wave function $\varphi_i(E_0, r)$ is supposed to be normalized:

$$\sum_i \int_0^\infty \varphi_i^*(E_0, s)\varphi_i(E_0, s)ds = 1, \qquad (6)$$

where "*" denotes complex conjugation. Of course, $\varphi_i(E_0, r)$ fits a more severe boundary condition for $r \longrightarrow \infty$ than (5):

$$\varphi_i(E_0, \infty) = 0. \qquad (7)$$

We define the transformed potential $\tilde{V}_{ij}(r)$ as

$$\tilde{V}_{ij}(r) = V_{ij}(r) + v_{ij}(r), \qquad (8)$$

where

$$v_{ij}(r) = -2C\frac{\hbar^2}{2m}\frac{d}{dr}\frac{\varphi_i(E_0, r)\varphi_j^*(E_0, r)}{A + C\sum_k \int_a^r |\varphi_k(E_0, s)|^2 ds} \qquad (9)$$

and A, C, and a are arbitrary real parameters.

The main result of this paper can be formulated as the following statement.

The wave function

$$\tilde{\varphi}_i(E, r) = \varphi_i(E, r) - C\varphi_i(E_0, r)\frac{\sum_k \int_a^r \varphi_k^*(E_0, s)\varphi_k(E, s)ds}{A + C\sum_k \int_a^r |\varphi_k(E_0, s)|^2 ds} \qquad (10)$$

fits the nonhomogeneous multichannel Schrödinger equation

$$\sum_j (\tilde{H}_{ij} - E\delta_{ij})\tilde{\varphi}_j(E, r) = C\frac{\hbar^2}{2m}\frac{\varphi_i(E_0, r)}{A + C\sum_k \int_a^r |\varphi_k(E_0, s)|^2 ds}\mathcal{W}(E_0, E; a), \quad (11)$$

where the Hamiltonian

$$\tilde{H}_{ij} = \delta_{ij}\frac{\hbar^2}{2m}\left[-\frac{d^2}{dr^2} + \frac{l_i(l_i+1)}{r^2}\right] + \tilde{V}_{ij}(r) \quad (12)$$

and the quasi-Wronskian

$$\mathcal{W}(E_0, E; a) \equiv \sum_k [\varphi_k^*(E_0, a)\varphi_k'(E, a) - \varphi_k^{*\prime}(E_0, a)\varphi_k(E, a)]. \quad (13)$$

We use the prime to denote derivatives: $f' \equiv df/dr$.

To prove the statement, one can verify (11) by the direct calculation of $\sum_j (\tilde{H}_{ij} - E\delta_{ij})\tilde{\varphi}_j(E, r)$ using definitions (8)–(10) and (12) and other formulas given above, as well as the fact that the interaction $V_{ij}(r)$ is Hermitian, $V_{ij}^*(r) = V_{ji}(r)$. The calculation is lengthy but straightforward.

It is clear from (10) and (7) that the suggested transformation is phase-equivalent at any energy $E > 0$; all the bound states supported by the initial potential V_{ij} are preserved by the transformation, since the wave functions $\tilde{\varphi}_i(E_b, r)$ for the corresponding energies $E_b < 0$ (including E_0) fit both boundary conditions (4) and (7). However, the denominator in the last term in (10) should be nonzero at any distance r; therefore, one should be accurate in assigning values to arbitrary parameters A, C, and a. This requirement can be easily satisfied in a wide and continuous range of parameter values.

3. PARTICULAR CASES OF THE PHASE-EQUIVALENT TRANSFORMATION

3.1. Homogeneous Schrödinger Equation

Of course, we are mostly interested in phase-equivalent transformations that result in the homogeneous Schrödinger equation

$$\sum_j (\tilde{H}_{ij} - E\delta_{ij})\tilde{\varphi}_j(E, r) = 0 \quad (14)$$

instead of the nonhomogeneous Schrödinger equations (11). To derive the transformation leading to (14), we can search for the parameters A, C, and a providing zero values of the right-hand side of (11). The choice $C = 0$ brings us to the equivalent (contrary to phase-equivalent) transformation that is of no interest. Thus, we should search for the parameters that fit the equation

$$\mathcal{W}(E_0, E; a) = 0. \quad (15)$$

Two obvious solutions of (15) are $a = 0$ and $a = \infty$. Various other solutions of (15) can be found for particular potentials $V_{ij}(r)$. However, the nonzero finite solutions a of (15) are energy-dependent. With the solutions $a(E)$ of (15), we can obtain energy-dependent potentials $\tilde{V}_{ij}(E; r)$ phase-equivalent to the initial energy-independent potential $V_{ij}(r)$. It may be interesting for some applications, but we shall not discuss the energy-dependent transformation and shall concentrate our attention on the solutions $a = 0$ and $a = \infty$.

The case $a = 0$ presents a generalization of the single-channel phase-equivalent transformation from [17]. For the bound state at the energy E_0, the wave function obtained by means of the transformation is of the form

$$\tilde{\varphi}_i(E_0, r) = \frac{A\varphi_i(E_0, r)}{A + C\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds}. \quad (16)$$

The wave function (16) is not normalized. The normalization constant can be easily calculated. The normalized bound-state wave function is

$$\sqrt{\frac{A+C}{A}}\tilde{\varphi}_i(E_0, r) = \frac{\sqrt{A(A+C)}\varphi_i(E_0, r)}{A + C\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds}. \quad (17)$$

It is interesting that the components of the bound-state wave function in all channels are modified by the transformation synchronously: all the components $\varphi_i(E_0, r)$ are multiplied by the same factor $\sqrt{A(A+C)}(A + C\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds)^{-1}$. Nevertheless, the relative weight of the components $\varphi_i(E_0, r)$ in the norm of the total multichannel wave function can be changed by the transformation.

Let us now discuss the case of $a = \infty$. The transformed wave function in this case is of the form

$$\tilde{\varphi}_i(E, r) = \varphi_i(E, r) - \frac{C\varphi_i(E_0, r)\sum_j \int_\infty^r \varphi_j^*(E_0, s)\varphi_j(E, s)ds}{A + C\sum_j \int_\infty^r |\varphi_j(E_0, s)|^2 ds}. \quad (18)$$

If $E \neq E_0$, the functions $\varphi_i(E, r)$ and $\varphi_i(E_0, r)$ are orthogonal:

$$\int_0^\infty \varphi_i^*(E_0, s)\varphi_i(E, s)ds = 0. \qquad (19)$$

With the help of (19) and (6), we can rewrite (18) as

$$\tilde{\varphi}_i(E, r) = \varphi_i(E, r) - \frac{C\varphi_i(E_0, r)\sum_j \int_0^r \varphi_j^*(E_0, s)\varphi_j(E, s)ds}{A - C + C\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds}. \qquad (20)$$

It is seen from (20) that the case of $a = \infty$ is identical (up to the redefinition of the parameter $A \longrightarrow A + C$) to the case of $a = 0$ if $E \neq E_0$. It is clear, however, that after the redefinition of the parameter $A \longrightarrow A + C$, the potential $v_{ij}(r)$ obtained with $a = \infty$ becomes equivalent to the potential $v_{ij}(r)$ corresponding to the case of $a = 0$. Hence, the case of $a = \infty$ appears to be equivalent to the case of $a = 0$ at any energy E, including $E = E_0$. To demonstrate this explicitly, let us examine the wave function $\tilde{\varphi}_i(E_0, r)$ in the case of $a = \infty$. Replacing E by E_0 in (18), we obtain

$$\tilde{\varphi}_i(E_0, r) = \frac{A\varphi_i(E_0, r)}{A + C\sum_j \int_\infty^r |\varphi_j(E_0, s)|^2 ds} \qquad (21)$$

or, equivalently,

$$\tilde{\varphi}_i(E_0, r) = \frac{A\varphi_i(E_0, r)}{A - C + C\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds}. \qquad (22)$$

Replacing A by $A + C$ and normalizing the wave function (22), we obtain expression (17).

3.2. Supersymmetry

Let us discuss a particular choice of parameters: $C = 1$, $a = \infty$, and $A = 1$. The wave function in this case is

$$\tilde{\varphi}_i(E, r) = \varphi_i(E, r) + \frac{\varphi_i(E_0, r)\sum_j \int_r^\infty \varphi_j^*(E_0, s)\varphi_j(E, s)ds}{\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds} \qquad (23)$$

or

$$\tilde{\varphi}_i(E, r) = \varphi_i(E, r) - \frac{\varphi_i(E_0, r)\sum_j \int_0^r \varphi_j^*(E_0, s)\varphi_j(E, s)ds}{\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds}. \qquad (24)$$

Equation (23) can be used at any energy E, while (24) is applicable only if $E \neq E_0$. In the case of $E = E_0$, the wave function can be rewritten in a simpler form as

$$\tilde{\varphi}_i(E_0, r) = \frac{\varphi_i(E_0, r)}{\sum_j \int_0^r |\varphi_j(E_0, s)|^2 ds}. \qquad (25)$$

Equation (23) is just Eq. (4) of [18]. In [18], Sparenberg and Baye suggested a multichannel supersymmetry transformation. Thus, Eqs. (24) and (25) describe the multichannel supersymmetry transformation, or, in other words, the multichannel supersymmetry transformation is a particular case of the phase-equivalent multichannel transformation discussed in this paper that corresponds to the particular choice of the parameters. Let us discuss how it works.

It is clear from (25) that $|\tilde{\varphi}_i(E_0, r)| \longrightarrow \infty$ as $r \longrightarrow 0$. Hence, at the energy E_0, the wave function $\tilde{\varphi}_i(E_0, r)$ does not match the required boundary condition (4) at $r = 0$. At the same time, $\tilde{\varphi}_i(E_0, r)$ fits the boundary condition (7) at $r = \infty$. Therefore, it is impossible to construct another solution of the Schrödinger equation (14) consistent with both boundary conditions at the energy $E = E_0$. As a result, the phase-equivalent transformation removes the bound state at $E = E_0$. At the same time, it is clear from (24) that for all energies $E \neq E_0$, the zero in the denominator arising in the limit $r \longrightarrow 0$ is canceled by the zero in the numerator and the wave function (24) matches the boundary conditions at the origin and at infinity both at once. Thus, the transformation in this case removes the bound state at $E = E_0$ but none of the other bound states, while the S-matrix at any energy $E > 0$ is unchanged.

Of course, the supersymmetry transformation can also be formulated in the case of $a = 0$. It is interesting that the bound state in this case is removed by a different mechanism. Suppose that $A = 0$ and that C is arbitrary. The wave function at any energy E in this case may be written as (24). However, it is seen that, at $E = E_0$, the wave function $\tilde{\varphi}_i(E_0, r) \equiv 0$.

We used the boundary condition (4) to construct the supersymmetry transformation: the bound state is removed because, for some particular parameter values, the wave function $\tilde{\varphi}_i(E_0, r)$ diverges at the origin and appears to be inconsistent with (4). One can suppose that it is also possible to use the boundary condition at $r = \infty$ to remove the bound state and to construct another supersymmetry transformation. This is not so. Let us discuss the case of $a = \infty$, $A = 0$, and arbitrary C. As is seen from (18), $\tilde{\varphi}_i(E_0, r) \equiv 0$ in this case; thus, the bound state is removed. However, the transformation is no longer phase-equivalent. Indeed, at energies $E > E_0$, the last term in (18) does not vanish when $r \longrightarrow \infty$ and provides an additional phase shift, or, in other words, it modifies the S matrix.

3.3. Inverse Supersymmetry

We shall refer to a transformation that adds a bound state to the discrete spectrum of the system and leaves unchanged the S matrix and the energies of all bound states supported by the initial Hamiltonian as the inverse supersymmetry transformation.

Let us suppose that there is no bound state at the energy $E_0 < 0$. By $\varphi_i(E_0, r)$, we now denote the wave function at energy E_0 that matches the boundary condition (7) at infinity but diverges at the origin as r^{-l_i} (see, e.g.,[2)] [19]), where l_i is the angular momentum in the channel i.

With $\varphi_i(E_0, r)$, we can use our transformation to obtain the homogeneous Schrödinger equation (14) in the case of $a = \infty$. The transformed wave function $\tilde{\varphi}_i(E, r)$ is given by (18). It is seen from (18) that $\tilde{\varphi}_i(E, r)$ does not diverge at the origin and matches the boundary conditions, both at the origin and at infinity, at any energy $E \neq E_0$. For $E = E_0$, the transformed wave function $\tilde{\varphi}_i(E_0, r)$ is given by (21). It is clear that $\tilde{\varphi}_i(E_0, r)$ at the origin is proportional to $r^{2L - l_i - 1}$, where $L = \max\{l_i\}$. Hence, $\tilde{\varphi}_i(E_0, r)$ matches the boundary condition (4) if $L \geq 2$ and is not consistent with (4) if $L \leq 1$. Therefore, our transformation with $\varphi_i(E_0, r)$ irregular at the origin is the inverse supersymmetry transformation in the case of $L \geq 2$. In the case of $L \leq 1$, the transformation appears to be a phase-equivalent transformation that does not make use of the bound state and can be applied to a system that does not support a bound state. If the transformation is applied to the free Hamiltonian with $V_{ij}(r) \equiv 0$ in the s or p partial wave, it produces a nonzero "transparent" potential $\tilde{V}_{ij}(r)$ that provides the phase shift $\delta = 0$ at any energy E. The multichannel version of the transformation couples s and p partial waves to produce a two-channel "transparent" interaction that provides the S matrix of the form $S_{ij} = \delta_{ij}$.

[2)] The r^{-l} divergence of the wave functions at the origin is derived in [19] for the single-channel case only. However, the derivation of the r^{-l} rule from [19] can be easily generalized to the multichannel case, at least for the potentials that do not diverge at the origin.

It is interesting that the inverse supersymmetry transformation is not unique: we have three parameters E_0, A, and C that provide a family of inverse supersymmetry partner potentials. Contrary to it, the supersymmetry transformation is unique; however, it can be used in combination with the phase-equivalent transformation to construct a family of potentials phase-equivalent to the initial one but not supporting one of the bound states.

4. CONCLUSION

We derived a multichannel phase-equivalent transformation that can be used without restrictions on the structure of the discrete spectrum of the system in various scattering problems like NN scattering, nucleon–cluster, or cluster–cluster scattering. The multichannel supersymmetry and inverse supersymmetry transformations appear to be particular cases of the suggested general phase-equivalent transformation corresponding to particular choices of the parameter values. The inverse supersymmetry transformation is possible if only the orbital angular momentum $l_i \geq 2$ in at least one of the coupled channels. It is interesting to note that, from the point of view of the NN system, this means that a deep attractive NN potential, supporting an additional forbidden state like the Moscow NN potential, can be constructed through the inverse supersymmetry transformation of the realistic meson-exchange potential with a repulsive core only due to the d-wave admixture in the deuteron wave function.

By using the suggested transformation, one can construct a family of phase-equivalent potentials depending on continuous parameters. Such families may be very useful for fine tuning of the interaction aimed to fit not only two-body observables but also three- and few-body ones. If the system has at least one bound state, the phase-equivalent potential family is constructed using formulas (8) and (9) directly. One can construct phase-equivalent single- or multichannel potential families also in the case when there are no bound states in the system: if all channel orbital angular momenta $l_i \leq 1$, one can directly apply the transformation with the irregular function $\varphi_i(E_0, r)$; if at least one of the channel orbital angular momenta $l_i \geq 2$, one can produce a bound state using inverse supersymmetry at the first stage and remove the bound state at the final stage by using the supersymmetry version of the transformation. Thus, one can, for example, construct a family of phase-equivalent potentials for any combination of coupled partial waves in the NN system.

It should be noted that our method allows to one construct the family of phase-equivalent potentials with given properties of the spectrum in the multichannel case without use of Gelfand–Levitan–Marchenko procedure applied in the inverse scattering problem (for example, see [20]).

We hope that the suggested transformation will be useful in various few-body applications.

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Note added in proof. Some of the problems discussed here were also considered in [21]. In particular, it was found there that it is impossible to add a bound state in the case of $L < 2$.

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