Wavelet-analysis of series of observations of relative sunspot numbers. The dependence of the periods of cyclic activity on the time at different time scales

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Abstract. We applied the method of continuous wavelet-transform to high-quality time-frequency analysis to the sets of observations of relative sunspot numbers. Wavelet analysis of these data reveals the following pattern: at the same time there are several activity cycles whose periods vary widely from the quasi biennial up to the centennial period. These relatively low-frequency periodic variations of the solar activity gradually change the values of periods of different cycles in time. This phenomenon can be observed in every cycle of activity.

Key words. Solar cycle-observations-solar activity indices.

1 Introduction

It's known that the various activity indices which characterized the different aspects of the solar magnetic activity correlate quite well with the most popular solar index such as the relative sunspot numbers (SSN) and with others indices over long time scales.

The relative sunspot numbers is a very popular, widely used solar activity index: the series of relative sunspot numbers direct observations continue almost two hundred years.

First we have studied yearly averaged values of SSN during activity cycles 1 - 23, the tree-hundred years data set we demonstrate on Figure 1. We use

the data from NGDC web site available at http://www.ngdc.noaa.gov/stp and combined observational data from National Geophysical Data Center Solar and Terrestrial Physics, Solar-Geophysical Data Bulletin (2012) and Reports of National Geophysical Data Center Solar and Terrestrial Physics (2012).

On Figure 2 we illustrated with help of wavelet - analysis the fact that the long time series of observations give us the very useful information for study of the problem of solar flux cyclicity on long time scales. The result of wavelet - analysis of series of observations of average annual SSN is presented in form of many isolines. For the isoline of the value of the wavelet-coefficients are of the same. The maximum values of isolines specify the maximum values of wavelet-coefficients, which corresponds to the most likely value of the period of the cycle. On Figure 2 there are three well-defined cycles of activity: - the main cycle of activity is approximately equal to a 10 - 11 years, 40-50- year cyclicity and 100 to 120-year (ancient) cyclicity.

Then we have studied monthly averaged values of SSN during activity cycles 21, 22 and 23. We also use the data set from NGDC web site available at http://www.ngdc.noaa.gov/stp.

Floyd *et al.* (2005) showed that the mutual relation between sunspot numbers and three solar UV/EUV indices, and also with the $F_{10,7}$ flux and the Mg II core-to-wing ratio remained stable for 25 years of satellite EUVobservations. Other solar activity indices are also closely correlated to the SSN and radio flux $F_{10,7}$, see Bruevich *et al.* (2014a). Some physics-based models have been developed with using the combined proxies describing sunspot darkening (sunspot number or areas) and facular brightening (facular areas, CaII or MgII indices), see Viereck et al. (2001), Krivova et al. (2003), Viereck et al. (2004), Skupin et al. (2005).

The SSN index has an advantage over other indices of activity because data on annual variation available from the 1700's.

The magnetic activity of the Sun is called the complex of electromagnetic and hydrodynamic processes in the solar atmosphere and in the underphotospheric convective zone, see Rozgacheva and Bruevich (2002), Bruevich et al. (2001). The analysis of active regions (plages and spots in the photosphere, flocculae in the chromosphere and prominences in the corona) requires to study the magnetic field of the Sun and the physics of magnetic activity. This task is of fundamental importance for astrophysics of the Sun and the stars. Its applied meaning is connected with the influence of solar active processes on the Earth's magnetic field.

2 Wavelet-analysis of series of observations of SSN

The study indices of solar activity are very important not only for analysis of solar radiation which comes from different altitudes of solar atmosphere. The most important for solar-terrestrial physics is the study of solar radiation influence on the different layers of terrestrial atmosphere (mainly the solar radiation in EUV/UV spectral range which effectively heats the thermosphere of the Earth and so affects to our climate).

The relative sunspot number is an index of the activity of the entire visible disk of the Sun. The SSN is a commonly used index of solar activity, Vitinsky et al. (1986). Sunspots are temporary phenomena on the photosphere of the Sun that appear visibly as dark spots compared to surrounding regions. They are caused by intense magnetic activity, which inhibits convection by an effect comparable to the eddy current brake, forming areas of reduced surface temperature. Although they are at temperatures of roughly 3000-4500K (2700 - 4200°C), the contrast with the surrounding material at about 5,780 K (5,500°C) leaves them clearly visible as dark spots. Manifesting intense magnetic activity, sunspots host secondary phenomena such as coronal loops(prominences) and reconnection events. Most solar flares and coronal mass ejections originate in magnetically active regions around visible sunspot groupings. Similar phenomena indirectly observed on stars are commonly called stars pots and both light and dark spots have been measured.

We have to point out that close interconnection between radiation fluxes characterized the energy release from different atmosphere's layers is the widespread phenomenon among the stars of late-type spectral classes, see Bruevich (2015a). It was confirmed that there exists the close interconnection between photospheric and coronal fluxes variations for sun-like stars of F, G, K and M spectral classes with widely varying activity of their atmospheres, see Bruevich and Alekseev (2007), Bruevich et al. (2001). It was also shown that the summary areas of spots and values of X-ray fluxes increase gradually from the sun and sun-like HK project stars with the low spotted discs to the highly spotted K and M-stars. The main characteristic describing the photospheric radiation is the spottiness of the stars. Thus, the study of the relative sunspot numbers is very important to explain the observations of sun-like stars.



Figure 1: Yearly averaged relative sunspot numbers 1700 - 2015.

In Fig.1 we can see that the duration of the 11 yr cycle of solar activity ranged from 7 to 17 years. The results become more accurate with the beginning of the of direct solar observations (1850-2015).



Figure 2: Cyclic activity of relative sunspot numbers at different time scales.

In Fig.2 we can see that periods of cycles on different time scales are not constant.

The long-term behavior of the sunspot group numbers have been analyzed using wavelet technique by Frick et al. (1997) who plotted the changes of the Schwabe cycle (its period is about 11-yr) and studied the grand minima. The temporal evolution of the Gleissberg cycle (its period is about 100-yr)can also be seen in the time-frequency distribution of the solar data. According to Frick et al. (1997) the Gleissberg cycle is as variable as the Schwabe cycle. It has two higher amplitude occurrences: first around 1800 (during the Dalton minimum), and then around 1950. They found very interesting fact - the continuous decrease in the frequency (increase of period) of Gleissberg cycle. While near 1750 the cycle length was about 50 yr, it lengthened to approximately 130 yr by 1950.

In the late of XX century some of solar physicists began to examine with different methods the variations of relative sunspot numbers not only in high amplitude 11-yr Schwabe cycle but in low amplitude cycles approximately equal to half (5.5-yr) and fourth (quasi-biennial) parts of period of the main 11-yr cycle, see Vitinsky et al. (1986). The periods of the quasi-biennial cycles vary considerably within one 11-yr cycle, decreasing from 3.5 to 2 yrs, and this fact complicates the study of such periodicity using the method of periodogram estimates.

Using the methods of frequency analysis of signals the quasi-biennial cycles have been studied not only for the relative sunspot number, but also for 10.7 cm solar radio emission and for some other indices of solar activity, see Bruevich et al. (2014b); Bruevich and Yakunina (2015b). It was also shown that the cyclicity on the quasi-biennial time scale takes place often among the stars with 11-yr cyclicity, see Bruevich and Kononovich (2011).

For the wavelet analysis of relative sunspot numbers on the scales in 11 years and quasi-biennial scales we will use the monthly averaged values, see Fig.3.

In Fig.4 we can see that the periods of cycles on different time scales are not constant too.

Also as in the case of the learning of the Schwabe cycle we see that approximately during three cycles value of the periods decreases (for the Gleissberg cycle from the periods of 110 years to 70 years - Fig.2, for the Schwabe cycle from the periods of 12 years to 8 years - Fig. 4). Then during the next cycle there are two equal amplitude cycles (two Gleissberg cycles with periods which change from 130 to 60 years - Fig.2, the two Schwabe cycles with periods which change from 13 to 7 years Fig. 4). In the following activity cycle only the cycles with the greatest periods remain and then the



Figure 3: Monthly averaged relative sunspot numbers 1950 - 2015.



Figure 4: Cyclic activity of relative sunspot numbers in activity cycles 18 - 24.

value of the periods gradually decreases over the next three cycles.



Figure 5: Cyclic activity of relative sunspot numbers on the quasi biennial time scale.

In Fig.5 we can see that in the case of quasi-biennial cycles the behavior of these periods inside the 11-yr cycle is similar to the variation of cycle's periods of the Schwabe cycle and the Gleissberg cycle. The periods of quasi-biennial cycles change from 3.5 to 2 yr inside the 11-yr cycle.

For the solar-type F,G and K stars according to *Kepler* observations it was also found "shorter" chromosphere cycles with periods of about two years, see Metcalfe et al. (2010), Garcia et al. (2010). In Kollath and Olah (2009) the "shorter" cycle (like solar quasi-biennial) was determined for the star V CVn, it's duration is equal to 2.7 yr.

To describe this general trend we propose a formal representation of this process. The cyclic variations of fluxes of solar radiation (in particular, the SSN as the most frequently studied activity index) can be represented by a sinusoid with varying period and constant amplitude:

$$A(t) = \cos(2\pi \frac{t - t_0}{T})$$

Note that exactly this behavior we can see in different cycles of activity,

see Fig.2, Fig.4, Fig.5.

The smooth change of the cycle period can be represented as follows:

$$T(t) = T_0 - k(t) \cdot (t - t_0)$$

where t_0 is the peak time of the studied cycle, T_0 is the cycle's period at the time t_0 , t varies in the range $t_0 < t < t_0 + T_0$.

Cyclicity	Cycle's period	k(t)
Century cycle	100 yr	0.3
Half a century cycle	$50 \mathrm{yr}$	0.25
11-yr cycle	10 -11 yr	0.2
Quasi biennial cycle	2 - 3.5 yr	0.33

Table 1: Parameters of different solar cycles.

In Table 1. we presented the values of k(t) for different solar cycles. For each cycle (from the quasi biennial duration to 11-yr and 100-yr cycle's periods) the values of coefficient k(t) are different, see also Fig.2, Fig.3, Fig.4. We consider that it is necessary to take into account the temporal evolution of solar cycles for successful forecasts or the parameters of activity cycles.

3 Conclusion

The study of the evolution of solar cyclicity by example of the SSN variation using the wavelet analysis allows us to make more accurate predictions of indices of solar activity (and consequently the predictions of the parameters of the earth's atmosphere), and also to take a step towards a greater understanding of the nature of cyclicity of solar activity. The close interconnection between activity indices make possible new capabilities in the solar activity indices forecasts. For a long time the scientists were interested in the simulation of processes in the earth's ionosphere and upper atmosphere. For these purposes it is necessary the successful forecasts of maximum values and other parameters of future activity cycles and also it has been required to take into account the century component.

Wavelet analysis of these data reveals the following features: the period and phase of these relatively low frequency variations of the solar flux, previous to the studied time point, influence to the amplitudes and to the phase of studied solar flux which show the gradually changing of their values in time: as a result, the periods of variations are getting longer.

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