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ECG Averaging Based on Hausdorff Metric

Leonid S Fainzilberg
International Research and Training Center of Information Technologies and Systems, Kiev, Ukraine

Correspondence: L.S.Fainzilberg, IRTC ITS, Prospect Academica Glushkova 40, Kiev-680, Ukraine, 03680.
E-mail: fainzilberg@svitonline.com, phone +380 44 4116904, fax +380 44 2661570

Abstract. A new method for estimation of ECG's averaged cycle is proposed. The method consist of following steps: transformation from time domain signal to the phase space, estimation of reference trajectory in the phase space using Hausdorff metric, estimation of average trajectory and its inverse transformation to the time domain. Proposed method is more suitable for processing signal with nonlinear perturbation like ECG in comparison with traditional methods.

Keywords: ECG Stochastic Model; Phase Space

1. Introduction

When the problems of computer processing and analysis of ECG are solving the traditional representation of ECG in the time domain $y = y(t)$ leads to some errors. The situation is caused by nonlinear distortions of P -wave, QRS - complex and $ST-T$ segment from one cycle to other. Moreover, it is known that boundaries of fragments of real ECG usually are fuzzy. Hence, alternative approaches to the problem have to be studied. One of them based on transformation of time domain signal to a specific image in the phase space was considered in [Fainzilberg, 1998]. Now we present further study results of this method to a problem of ECG averaging.

2. Basic Results

Let's assume that observed ECG signal $y(t) = \Phi[\hat{y}(t), \zeta(t)]$ be a result of distortions of some periodic process $\hat{y}(t)$ by random perturbation $\zeta(t)$, where $\Phi(\cdot)$ - unknown function.

Let $y_0(t)$ is a part of unobserved function $\hat{y}(t)$ on one period T_0 and $y_0(t)$ have to be estimated by ECG processing. We assume that $y_0(t)$ is the function consisting of K fragments

$$y_0(t) = \begin{cases} y_0^{(1)}(t) \text{ при } 0 \le t < t_0^{(1)}, \\ \cdots \\ y_0^{(K)}(t) \text{ при } t_0^{(K-1)} \le t < t_0^{(K)} = T_0 \end{cases} \quad (1)$$

We suppose also that any i -th fragment ($i = 1,..., K$) on the m -th ECG cycle is a result of operator transformation to corresponding fragments of $y_0(t)$:

$$y_m^{(i)}(t) = a_m y_0^{(i)}\left(\frac{t - \tau_m^{(i)}}{b_m^{(i)}}\right) \quad (2)$$

where $a_m, b_m^{(i)}$ are random parameters of perturbation (by amplitude and time) and $\tau_m^{(i)}$ is the parameter of time shift.

In this case, the nonlinear stochastic model to simulate real ECG signal may be obtained:

$$y_m^{(i)}(t) = (1 + \xi_m) y_0^{(i)}(\theta), \quad (3)$$

where

$$\theta = \frac{t-(m-1)T_0+t_0^{(i-1)}-\sum_{j=1}^{m-1}\sum_{i=1}^{K}(t_0^{(i)}-t_0^{(i-1)})\delta_j^{(i)}-\sum_{l=1}^{i-1}(t_0^{(l)}-t_0^{(l-1)})(1+\delta_m^{(l)})}{1+\delta_m^{(i)}} \qquad (4)$$

and ξ_m , $\delta_m^{(i)}$ are sequences of the limited on a level random variables with zero average (see Fig. 1).

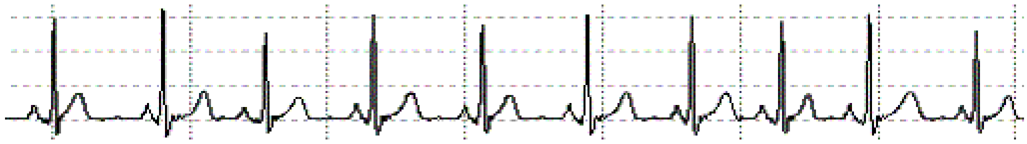

Figure 1. *Result* of *ECG simulation according to stochastic model (3).*

The nonlinear stochastic model (3) may be easy generalized to simulate ECG signal with broken morphology of beats (for example, extra systoles) by using $G > 1$ etalons $y_{01}(t),\dots, y_{0G}(t)$ which generate m -th ECG cycle according to probabilities P_g, $\sum_{g=1}^{G} P_g = 1$.

Despite of nonlinear distortions of etalons it may be show that diagnostic features of distorted etalons have close phase coordinates. This gives following method for estimation of ECGs averaged cycle.

Let we have set $\Omega_M = \{Q_1,\dots Q_M\}$ of vectors $z = (y,\dot{y})$ corresponding to M cycles of observed ECG in normalized phase space. Then we may define reference cycle Q_0 as trajectories having minimum sum of Hausdorff distances to other trajectories:

$$Q_o = \underset{\substack{1\le j\le M \\ j\ne i}}{\operatorname{argmin}} \sum_{i=1}^{M} R_H(Q_i,Q_j) \qquad (5)$$

where

$$R_H(Q_i,Q_j) = \max\left\{\max_{z_i\in Q_i}\ \min_{z_j\in Q_j} \rho(z_i,z_j), \max_{z_j\in Q_j}\ \min_{z_i\in Q_i} \rho(z_i,z_j)\right\} \qquad (6)$$

and $\rho(z_i,z_j) = \|z_i - z_j\| = \sqrt{(y_i - y_j)^2 + (\dot{y}_i - \dot{y}_j)^2}$ - Euclid distance.

The average trajectory may be easy estimated by points placed near corresponding point of Q_0 . Its projection gives good estimate of etalon cycle $y_0(t)$ in the time domain (see Fig. 2).

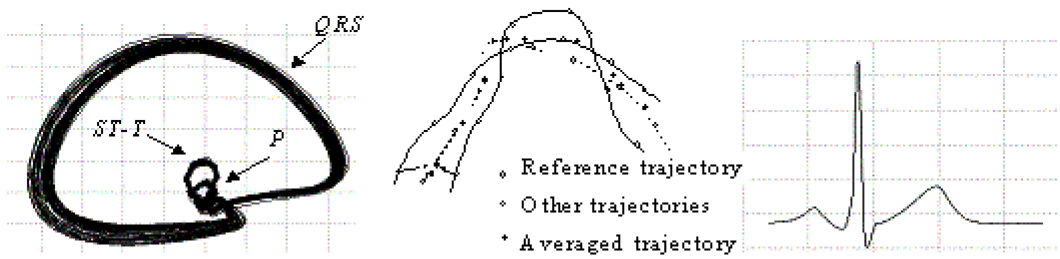

Figure 2. *ECG in the phase space (left), its fragment (middle) and averaged cycle in the time domain (right).*

3. Discussion and Conclusion

We use Hausdorff metric to construct the average trajectory of observed ECG in the phase space. In comparison with traditional this method is more suitable for processing signal with nonlinear perturbation like real ECG. The projection of constructed average trajectory gives good presentation of ECG average cycle in the time domain and may be used for patients' diagnoses.

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