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Asymptotic Behaviour of Hadron Electromagnetic Form Factors in the Dynamical Model of Factorizing Quarks

Within the dynamical model of factorizing quarks we have derived formulae for the asymptotic behaviour of the hadron and deuteron electromagnetic form factors which depend upon the number of particle-constituent quarks. The model predictions well agree with the experimental data on the pion and proton electromagnetic elastic form factors.

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It is known that the small-angle hadron scattering is well described in the framework of the additive quark amplitude model \(^1\). The large-angle scattering seems to be controlled by another mechanism \(^2\). A hypothesis of factorizability of the quark scattering amplitudes \(^3\) successfully describes the relations between cross sections of different large-angle scattering processes \(^4\). In this model the colliding quarks are thought of as to produce an effective force field \(V_{ij}\), and the scattering process is considered as the process of independent quark scattering on this field. For independent events the total probability is equal to the product of individual probabilities. Thus the scattering amplitude of the whole hadron at the angle \(\Theta\) is the product of the amplitudes of the quark scattering at \(\Theta\) on the effective potential.

In \(^5\) the model of factorizing quarks (amplitudes) was enlarged by the dynamical assumption that the size of the interaction region, that determines the quark large-angle scattering in the kinematical region \(-\xi, \xi \rightarrow \infty \xi / /\) fixed, is of an order of the quark Compton wave length \(1/M_q\). To this picture there corresponds the following quark scattering amplitude \(^5\):

\[
g_q(\Theta) \sim \frac{y_q}{x h q^2} = \frac{2M_q^2 \ln(1-t_q/M_q^2) + \frac{\xi}{2M_q^2}}{\sqrt{t_q(t_q-4M_q^2)}},
\]

where \(y_q = \text{Arch} \left(1-t_q/M_q^2 \right)\) is the rapidity.
corresponding to the transfer momentum per one quark
\[ \mathcal{E}_q = \mathcal{E}/m^2 = \left( (r-K)/N \right)^2 \]

\((N)\) is the number of the valence quarks in a hadron and \(m_q\) is the effective quark mass, the parameter of our model. Really, the size of the interaction region for this amplitude is \(1/\mathcal{E}^2\)

\[ \langle \mathcal{E}^2 \rangle = \mathcal{E} \frac{\partial}{\partial \mathcal{E}} \left( \frac{1}{\mathcal{E}^2} \right) \bigg|_{\mathcal{E} = 0} = \frac{\mathcal{E}}{m_q^2}. \]

Thus, in the dynamical model of the factorizing quarks the cross section for the elastic fixed-angle scattering is defined by the formula

\[ \frac{d\sigma}{dt} (q\bar{e} \to q\bar{e}) \sim \frac{x^2}{s^{\alpha}} \left[ \prod_{q_i} \frac{Y_{q_i}}{s_{i}h_i} \prod_{\bar{q}_i} \frac{Y_{\bar{q}_i}}{s_{\bar{q}_i}h_{\bar{q}_i}} \right]^2, \]

where \(n\) and \(m\) are the numbers of valence quarks in hadrons \(q\) and \(\bar{e}\) resp.

Consider now the electron-hadron elastic scattering \(e\bar{h} \to e\bar{h}\). Following the Wu-Yang idea on the analogy of the hadron electromagnetic structure to the distribution of the "strong-interacting" matter, we adopt that at large transfer momenta the photon-hadron block in a one-photon exchange diagram corresponds to the excitation of the same self-consistent field \(V_{\text{eff}}\). Then the differential cross section is

\[ \frac{d\sigma}{dt} (e\bar{h} \to e\bar{h}) \sim \frac{x^2}{t^{\alpha}} \left[ \prod_{q_i} \frac{Y_{q_i}}{s_{i}h_i} \prod_{\bar{q}_i} \frac{Y_{\bar{q}_i}}{s_{\bar{q}_i}h_{\bar{q}_i}} \right]^2. \]
Comparing the latter with the Rosenbluth formula for the asymptotic cross section \( \frac{d^2 \sigma}{dt} (e^- + e^- \rightarrow h) \sim \frac{A}{t^2} \cdot C_h^2(t) \), we obtain the electromagnetic form factor \( C_h(t) \) in the form

\[
C_h(t) \sim \prod \frac{y_{q_h}}{s_h y_{\gamma^*}}.
\]

(3)

At large transfer momenta \( lt_0/M_q^2 \), \( y_{q_h} \), \( s_h y_{\gamma^*} \) \( \sim \frac{e_0(1/t_0/M_q^2)}{1/t_0/M_q^2} \).

For the form factors of pion, proton and deuteron formula (3) gives

\[
C_\pi(t) \sim \left( \frac{y_{\pi^*}}{s_{\pi^*} y_{\gamma^*}} \right)^2 \times \left( \frac{e_\pi(1/t_0/4M_q^2)}{1/t_0/18M_q^2} \right)^2,
\]

(4)

\[
C_p(t) \sim \left( \frac{y_p}{s_h y_{\gamma^*}} \right)^3 \times \left( \frac{e_p(1/t_0/4M_q^2)}{1/t_0/18M_q^2} \right)^3,
\]

(5)

\[
C_d(t) \sim \left( \frac{y_d}{s_h y_{\gamma^*}} \right)^6 \times \left( \frac{e_d(1/t_0/36M_q^2)}{1/t_0/72M_q^2} \right)^6.
\]

(6)

Formulae (4-6) can be rewritten in a form of the conventional power law \( C_h(t) \sim (1/t_0/n^2 M_q^2)^{-N_{\text{eff}}} \), where \( n \) is the number of quarks in particle \( h \) and \( N_{\text{eff}} \) depends on the momentum transfer as follows

\[
N_{\text{eff}} = n - n \frac{e_n e_h (1/t_0/n^2 M_q^2)}{e_n (1/t_0/n^2 M_q^2)}.
\]

(7)
With increasing $\xi$, the effective power $N_{eff}$. (7) smoothly increases and reaches the asymptotic limit equal to the number of valence quarks in particle.

The figure represents the results of comparison of predictions by formulae (4) and (5) with experimental data on the form factor of pion $^8$ and proton $^9$. It is seen from the figure, that the experimental data are really grouping near the straight line with the integer values of the slope that is due to (2) equal to the number of the valence quarks. In fitting the data by formulae (4,5) we have obtained the following values of $\Lambda^2$ per one degree of freedom: for proton $\chi^2_{df.} = 13.3/(14-2)$, for pion $\chi^2_{df.} = 20.6/(16-2)$. It should be noted that the found effective quark mass $M_q = 0.16$ GeV, defining the pion form factor asymptotics coincides, within the experimental error, with the value of $M_q$ found in fitting the data on the proton form factor and is consistent also with the value obtained in analysis of the $p\bar{p}$-scattering $^5$.

Formulae (4–6) contain manifestly the scale parameter the quark mass value $M_q$, thus the asymptotic behaviour of form factors depends on the ratio $t_q/M_q^2$. Since the transfer momentum per one quark $t_q$ is reciprocal to the squared number of valence quarks in particles, the asymptotic limit for the form factor of deuteron sets is later than for pion and proton.
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