Stress Fields in Two-Dimensional Mantle Convection Model with Non-Newtonian Rheology

A. Bobrov[1*], A. Baranov[1,2] and N. Bobrova[1]
[1] Schmidt Institute of Physics of the Earth, Moscow, 123995, Russian Federation.
[2] Institute of Earthquake Prediction Theory and Mathematical Geophysics, Moscow, 117997, Russian Federation.
*a_m_bobrov@yahoo.com

1 Introduction

Plenty of studies on mantle convection with different rheologies were carried out during the past years. The models with different effective Rayleigh numbers, non-Newtonian viscosity, different values for the activation energy and volume which determine the *P*,*T* dependence, different modes of heating, different aspect ratios and dimensions (two-dimensional, three-dimensional, three-dimensional spherical: 2D, 3D, 3S) were considered.

These works mainly studied the temperature and velocity fields of the mantle flows. However, only relatively few works address the stress fields. In the study [1], the stress fields were calculated in the lithosphere for different depth profiles of viscosity for spherical model with plates on the surface. It was found that, depending on the boundary conditions, the stresses in the lithosphere varied between ±50 MPa and ±140 MPa. In our previous works [2], [3] we studied (with different distributions of viscosity) 2D stresses in the presence of floating continental plates. The stresses in the upper part of plates reach 40 MPa.

A more complex rheology was investigated in [4]. For the generation of 2D plate boundary, authors introduced a history-dependent rheology with the yield strength which is determined by past fractures. The results show horizontal stresses varying inside the interval ±100 MPa. Yoshida [5] analyzed spherical models with a heat production rate $H = 10$ and a simpler *P*,*T*- dependent mantle viscosity with a highly viscous area simulating a supercontinent. The maximum deviatoric tensional stress generated in the area of immobile supercontinent is (30÷90) MPa.

Thus, in these and another studies, the authors focus mainly on the lithospheric stress fields. In our opinion, insufficient attention has been paid to the mantle stress fields.

In the present work, we study how non-Newtonian viscosity affects the convection structure and stress fields in the mantle and lithosphere. In our model, the convection is solely driven by the temperature anomalies in the mantle. The subduction process develops in a self-consistent manner after introducing the strain rate dependence of the viscosity in addition to its P,T dependence. Of course, the real mantle has a more complex rheology than the considered model.

2 The model and equations

We use a 2D Cartesian model in order to minimize numerical calculations and to provide high resolution.

The mantle is assumed to be heated from below (from the core) and internally heated by the decay of radiogenic elements, which are uniformly spatially distributed.

With these assumptions, thermal convection is governed by the common equations for conservation of mass, momentum, and energy. In Boussinesq approximation, 2D fluid convection equations for coordinates x and z (with the z axis directed upwards) have the following dimensionless form [6]:

the momentum transfer equations

$$-\partial p/\partial x + \partial \tau_{xx}/\partial x + \partial \tau_{xz}/\partial z = 0, \quad (1)$$

$$-\partial p/\partial z+\partial \tau_{xz}/\partial x+\partial \tau_{zz}/\partial z+RaT = 0, \quad (2)$$

the heat transfer equation

$$\partial T/\partial t+V_x\partial T/\partial x+V_z\partial T/\partial z = \partial^2 T/\partial x^2+\partial^2 T/\partial z^2+H, \quad (3)$$

and the continuity equation

$$\partial V_x/\partial x + \partial V_z/\partial z = 0. \quad (4)$$

Here,

$$Ra = (\alpha\, g\, \Delta T\, D^3)/(\kappa \nu_0)$$

is the Rayleigh number determined in terms of the reference kinematic viscosity ν_0. The unknown variables are the velocity components, V_x and V_z; the overlithostatic pressure, p (i.e., the perturbation in the lithostatic pressure P caused by convection); the superadiabatic (potential) temperature, T; and the deviatoric viscous stress tensor, τ_{ij}.

We use here the dimensionless variables assuming the following scaling factors: the thickness of the mantle D for length; κ/D for velocity; D^2/κ for time; $\Delta T=T_2-T_1$ for temperature; η_0 for dynamic viscosity; $\eta_0\kappa/D^2$ for pressure and stresses and $\kappa\Delta T/D^2$ for thermometric heat source density H. The components of the deviatoric viscous stress tensor are

$$\tau_{xx} = 2\, \eta\partial V_x/\partial x, \quad (5)$$

$$\tau_{zz} = 2\, \eta\partial V_z/\partial z, \quad (6)$$

$$\tau_{xz} = \eta\, (\partial V_x/\partial z +\partial V_z/\partial x), \quad (7)$$

where η is the dimensionless dynamical viscosity at a given point.

The deviatoric stresses τ_{ij} are associated with the total viscous stresses σ_{ij}. In particular, the total normal horizontal stresses and the total normal vertical stresses are

$$\sigma_{xx} = p - 2\, \eta\partial V_x/\partial x, \quad (8)$$

$$\sigma_{zz} = p - 2\, \eta\partial V_z/\partial z. \quad (9)$$

In terms of this definition, the compressive stresses are assumed positive. Here, the stress sign is consistent with the definition accepted in geophysics and engineering and is opposite to that adopted in physics.

The average values of the parameters that can be used to describe the overall mantle of the Earth [6] give the Rayleigh number $Ra = (\alpha\, g\, \Delta T\, D^3)/(\kappa\nu_0) = 2 \times 10^7$, as it was used in the work. The dimensionless stress of 1×10^4 in the given units of σ_0 corresponds to the dimensional value of 2.35 MPa.

The temperature on the upper and lower boundaries of the calculation domain is constant, and equal to 0 and 1 dimensionless units, respectively: $T_1 = 0$ and $T_2 = 1$. The boundaries of the domain are assumed to be free-slip and impermeable (the shear stresses and normal velocity components are zero). The side boundaries are assumed to be insulated, that is, the normal derivative of temperature is zero: $\partial T/\partial x = 0$.

We consider a model of the mantle with *P,T*- and strain-dependent viscosity. Generally, the viscosity can be described as

$$\eta = A^{-1/n} \, [\acute{\varepsilon}]^{(1-n)/n} \exp[(E + PV)/(nRT)], \quad (10)$$

n – nonlinearity index, $\acute{\varepsilon}$ is the second invariant of the strain rate tensor, E is activation energy, V is the activation volume. The member $[\acute{\varepsilon}]^{(1-n)/n}$ in this equation describes the plastic deformation. The numerical simulations for the parameters of the real Earth require a lot of computation time. So we use the simplified dependence of viscosity on temperature and lithostatic pressure [7]:

$$\eta(P,T) = 200 \times \exp[-9.2T + 2.3(1-z)], \quad (11)$$

Here, the depth z is connected with the dimensional lithostatic pressure P by the formula $P = \rho g D(1 - z)$. Simultaneously, the viscosity depends on the strain invariant $\acute{\varepsilon}$ as

$$\eta = \min(\eta(P,T), \tau/\acute{\varepsilon}), \quad (12)$$

where $\acute{\varepsilon}$ is the second invariant of the strain rate tensor, τ is an effective yield stress [8]. The latter relation describes, by means of the drop in the effective viscosity η, the effect of plasticity of the oceanic lithospheric plate in the areas of high strain rate (when the slab bends into the mantle; when sinking slab separates from the oceanic lithospheric plate). We assume the yield stress to be 25 MPa, which falls in the interval of “mobile lid regime” [9]. The symbol “min[*a*,*b*]” indicates that a function assumes a value of a and b, whichever is smaller.

In our simulation, we use well benchmarked 2D CITCOM code [10]. The calculations were carried out in the rectangular box with an aspect ratio of $L : D = 5 : 1$ with uniform 401 × 201 grid, i.e. with a horizontal resolution of 36 km and vertical resolution of 15 km. In the process of calculating, the model evolutionarily enters the typical mode (i.e., systematic trend of the solution disappears).

Figs.1 - 2 show two typical stages of the examined convection.

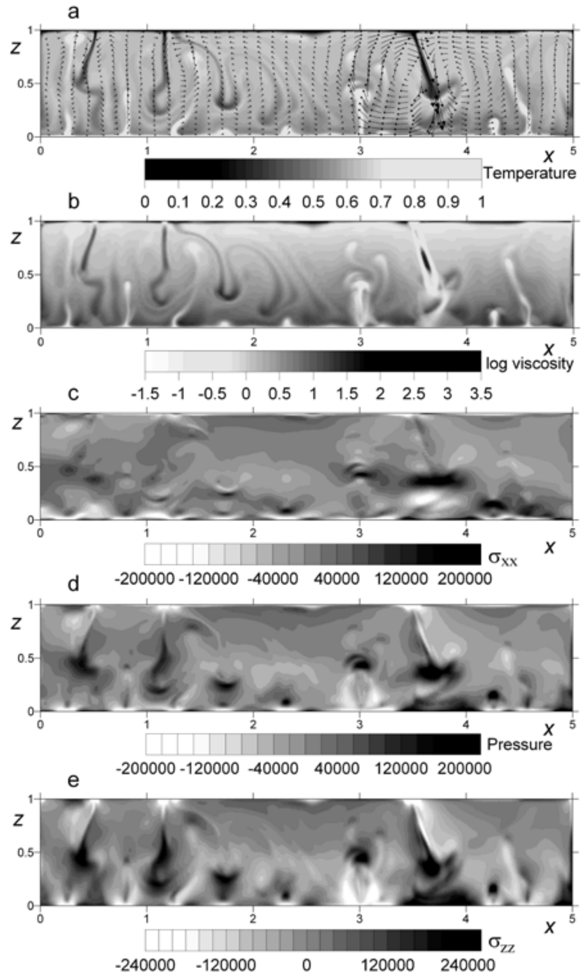

Figure 1. The non-Newtonian viscosity model at *Ra* = 2 × 10^7 and *H* = 15, for the first time moment (*t* = 0). Panels (*a*) – (*e*) from top to bottom: (*a*) The spatial distributions of dimensionless temperature and flow velocity. The dimensionless velocities are shown by the arrows with a maximum value of 15 cm/year. (*b*) The spatial distribution of the logarithm of dimensionless viscosity. (*c*) The spatial distribution of dimensionless σ_{xx}. Tones from white to light gray correspond to overlithostatic tensile stresses (negative values), dark gray tones correspond to compressional stresses. (*d*) The spatial distribution of dimensionless overlithostatic pressure. (*e*) The spatial distribution of dimensionless σ_{zz}. The dimensionless stress of 1 × 10^4 in the given units of σ_0 corresponds to the dimensional value of 2.35 MPa.

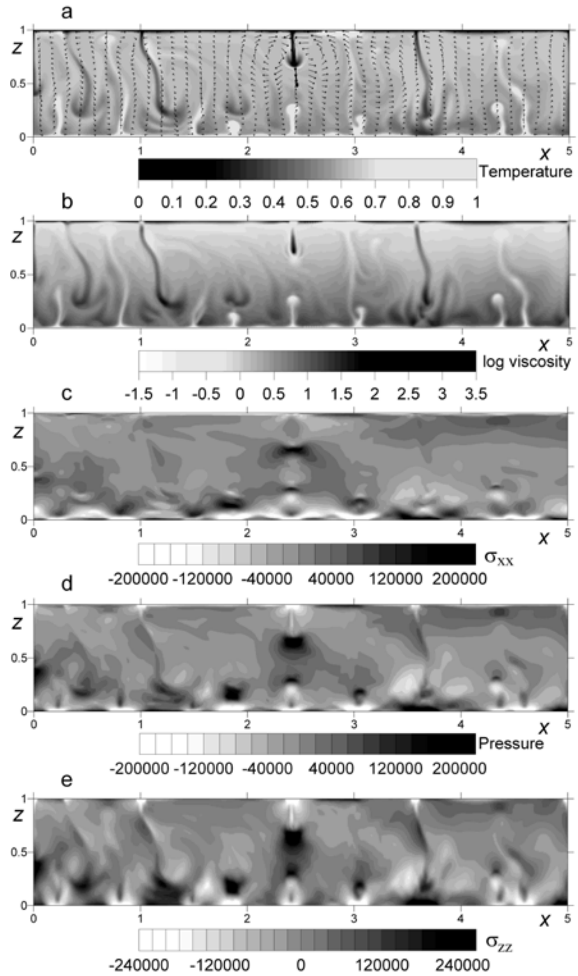

Figure 2. The non-Newtonian viscosity model for for the second time moment (*t* = 1.50 × 10^{-4}, in dimensional form 40 Ma).

3 Results

Compared to the models studied in our previous works [3], the present model exhibits significant distinctions. Our present model demonstrates jump-like migration of subduction zones and reveals large spatial and time heterogeneities of stresses and velocities in the area of slab subduction depending on the slab detachment stage. Due to more easy removal of lithospheric plates from the surface into the mantle, the model shows reasonable (not too low) values of the mantle velocities (with a maximum value of 5 cm/year at the surface and about 10 cm/year for rapidly sinking lithospheric slabs) and the surface heat flow. This rheology also provides plausible (not excessively large) stresses in the mantle and slabs. Thus, inclusion of the effect of plastic deformation essentially changes the behavior of the model, bringing it closer to the actual situation.

The model stresses in the parts of the mantle where vigorous subvertical flow is absent are small. Typical values of the horizontal stresses σ_{xx}, the overlithostatic pressure and the vertical stresses σ_{zz} in these areas vary within ± 6, ±8, ±10 MPa, respectively. However, the overlithostatic stresses exhibit strong concentrations in the areas of descending slabs, where the values are about an order of magnitude higher (±50 MPa). The stresses increase also in the surrounding area of the mantle, what is caused by viscous mantle interaction with rapidly sinking slab. These results give in terms of stresses the quantitative confirmation of current views on the oceanic downgoing slabs as the most important agent of the mantle convection. The ascending plumes play only a supplementary role (for our model, their contribution is about three times smaller).

We find significant differences between the σ_{xx}, σ_{zz}, and the pressure fields.The pressure field reveals both vertical and horizontal features of slabs and plumes, showing their long thermal conduits (areas of relatively low pressure) with broad, spherical or disk-like heads (areas of compression). Here, just as throughout the whole study, we consider the overlithostatic stresses. The σ_{xx} field is sensitive to subhorizontal features of the mantle flow. Conversely, the distribution of σ_{zz} mainly reflects vertical substructures (the conduits).

In the considered model, these fields differ in the mantle areas measuring in hundreds of kilometers where one of the fields shows the values almost an order (up to 50 MPa) greater than the typical, while the other field anomalies in this region are almost absent. Among these three fields, the structure of mantle flows is most clearly visible in the σ_{zz} field (the range of variations in σ_{zz} is wider than that of σ_{xx}).

The model predicts the presence of relatively cold remnants of lithospheric slabs at the bottom of the mantle just above the thermal boundary layer. The reason is more rapid detachment and subsequent immersion of the slabs as a result of the plastic properties of the material. The hot plumes penetrating through these remnants, which has relatively higher viscosity, as well as the descending slabs induce intense stress fields in the lower mantle, which are strongly nonuniform in space and time. By the same reasons, the upper surface of D" layer is an uneven.

The considered model is simplified in a number of respects. In the future, the use of the models with a more complex rheology would enable constructing more adequate model representations.

Acknowledgment

The work was supported by the Russian Foundation for Basic Research, grant no. 13-05-01123.

We are grateful to the authors of the CITCOM code (Louis Moresi, Shijie Zhong, Michael Gurnis, and others).

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Participant Notes