Parton-medium cross section and average QGP viscosity in lead-lead collisions at LHC

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Abstract. The recent experimental CMS and E-by-E ATLAS data on the elliptic anisotropy of particles in $Pb-Pb$ collisions are used to extract parton medium cross-section and the quark-gluon plasma viscosity-to-entropy ratio within the incomplete equilibration medium model. Our method assumes extrapolation of the measured pseudorapidity spectrum of charged particles to low-$p_T$ range. Then the rapidity distribution of both charged and neutral hadrons is restored. The extracted value of the parton-medium cross section is found to be $\sigma = (3.1\pm0.2)$mb. It yields $\eta/s = 0.17\pm0.02$ for the ratio of shear viscosity to entropy density in QGP phase. The last result is in a good agreement with the estimates obtained by calculations within the viscous hydrodynamics.

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1. Introduction

Observations at the RHIC and LHC suggest that an extremely dense partonic medium, called quark-gluon plasma (QGP), with nearly perfect fluid properties has been formed at the extreme temperatures and energy densities produced in relativistic heavy ion collisions. For instance, ideal hydrodynamics with zero transport coefficients was able to describe first experimental measurements of hadronic radial and elliptic flow. Application of advanced methods for the extraction of the components of particle collective flow, however, indicated clearly that the description of the data on relativistic heavy ion collisions required some degree of QGP viscosity (for review see, e.g., [1]).

Theoretical works on strongly coupled quantum field theories, which utilize results from superstring theory, have established a lower limit of $1/4\pi$ for the specific shear viscosity $\eta/s$ of the QGP [2]. Here $\eta$ is the shear viscosity and $s$ is the entropy density in the system. QGP shear viscosity is an important transport coefficient characterizing the properties of the QGP medium. Furthermore, it is expected that the certain irregularities in the $\eta/s$ ratio as a function of temperature, such as kinks and jumps, should help us to pin down the critical temperature of the quark-hadron phase transition (or crossover) at which the specific shear viscosity is the smallest [3]. The measurement of the specific shear viscosity cannot be done directly in the experiment. To obtain this information out of the data one has to rely on several approaches and assumptions.

One of the possible ways to extract the QGP viscosity is to confront the data with the macroscopic model calculations. Here the shear viscosity, or rather $\eta/s$ ratio, is inserted as a free parameter in the hydrodynamic part of the code. Its value is fixed after providing the best agreement with the data. The most successful approaches, however, combine a viscous fluid dynamic description of the QGP phase with a microscopic Boltzmann simulation of the hadronic phase [4, 5]. Recall that (3+1)-dimensional hydrodynamics can be reduced to (2+1)-d case by assumption of the boost-invariant velocity profile in the longitudinal direction, and further to (1+1)-d case for the longitudinally boost-invariant and the azimuthally symmetric flow. The next distinction between the viscous hydrodynamic models is related to the order of employed hydrodynamic approximation, such as first-order Navier-Stokes theory, second-order Israel-Stewart theory [6], third-order viscous hydrodynamics, and so forth. For instance, the VISHNU model [4] matches the (2+1)-d viscous hydrodynamic algorithm VISH2+1 [7, 8] for the solution of the second-order Israel-Stewart equations to the well-known UrQMD cascade model [9] as an afterburner. The similar approach, MUSIC+UrQMD [5], also employs the Israel-Stewart formalism, but the evolution here is fully (3+1)-dimensional. Calculations within this model, performed with the IP-Glasma initial conditions [10] together with an average value $\eta/s = 0.2$ for $Pb-Pb$ collisions at the LHC and a somewhat smaller value $\eta/s = 0.12$ for $Au-Au$ collisions at top RHIC energies, provide a good description of all presently available data for charged hadron flow harmonics, both $v_n$ and $v_n(p_T)$ from $n = 2$ to $n = 5$ [11], including their centrality...
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and $p_T$-dependencies. Usually this approach is called the viscous hydrodynamic hybrid model.

Other methods link the $\eta/s$ ratio to the system parameters, such as temperature and partonic cross section, which can be obtained from the analysis of particle spectra. The eccentricity scaled integrated elliptic flow $v_2/\varepsilon$ versus transverse charged-particle density $n_T = (1/S)(dN_{ch}/dy)$ is a "universal" function, since this function describes energy and centrality dependencies of the flow in a wide range of collision energies. There are many examples of application of this "universal" function for data analysis, see, e.g., [4, 7, 12, 13, 14, 15]. One can also note a good agreement with the numerous experimental data on $v_2/\varepsilon$ as a function of $1/S \ dN/\eta$ from 7.7 AGeV to 2.76 ATeV for the hydrodynamic model with the Monte Carlo Glauber (MC-GI) initial conditions in contrast to the Kharzeev-Levin-Nardi (MC-KLN) ones [7]. Here the main reason for the discrepancy was traced to the different centrality dependencies of the overlap areas in the two models.

In [12] the authors suggest a simple equation for the "universal" function as a function of transverse particle density within the incomplete equilibration medium (IEM) approach. It is worth noting that within the ideal fluid hydrodynamics the magnitude of the elliptic flow $v_2$ is determined solely by the spatial eccentricity $\varepsilon$, whereas its dependence on the transverse size of the system is absent. Under condition of incomplete equilibration, however, $v_2$ does depend on the size of the system via the number of collisions per particle. The inverse of the number of collisions is just the Knudsen number $K$, which is very small, $K \ll 1$, in case of local equilibrium. The proposed in Ref. [12] semi-phenomenological formula thus naturally explains the linear growth of the elliptic flow with the increasing value of $K^{-1}$ and the flow saturation in the hydrodynamic limit $K^{-1} \rightarrow \infty$. This equation was directly used in [13, 14] to extract the IEM parameters from the RHIC data at 200 AGeV. To our best knowledge, no similar analysis of the LHC data has been done yet.

Partonic and later on hadronic cascades in a system of colliding nuclei lead to development of a collective flow, which has both isotropic and anisotropic components. The strength of the collective anisotropic flow is measured by means of harmonics of the Fourier expansion of the charged hadron azimuthal distributions with respect to the event plane, defined by the maximum particles in direction of the harmonic studied. The azimuthal dependence of the particle yield can be written in terms of the harmonic expansion [16]

$$E \frac{d^3N}{dp^3} = \frac{d^2N}{2\pi p_T dp_T d\eta} \times \left\{1 + 2 \sum_{n=1}^{\infty} v_n(p_T, \eta) \cos [n(\phi - \Psi_n)] \right\},$$

where $\phi$ is the azimuthal angle with respect to the event plane $\Psi_n$, and $v_n$ is the Fourier coefficient. In this paper a parameter $v_2$, dubbed elliptic flow, is considered. Elliptic flow was studied extensively at both LHC [17, 18, 19] and RHIC [20, 21, 22, 23] energies. Its description is amenable to hydrodynamic calculations [24, 25, 26, 27]. Comparisons
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to theoretical results indicate that at the LHC energies most of the flow is primarily
generated during an early stage of the system evolution before the hadronization, and
the relative impact of the late hadronic stage is weak. An integrated elliptic flow can
be expressed as

$$
\left( p_T, y \right) = \frac{\int_{\Delta y} dy \int_{\Delta p_T} p_T d^2N}{\int_{\Delta y} dy \int_{\Delta p_T} p_T d^2N} \left( p_T, y \right) v_2(p_T, y)
$$

(2)

Several methods have been developed to extract the elliptic flow from the experimental
data. In present paper we utilize the approach described in the CMS publication [19].

Recent theoretical studies of elliptic flow were also focused on the quantifying of
the ratio of the shear viscosity to the entropy density of the produced medium in the
framework of viscous hydrodynamics [24, 28] with a variety of possible initial conditions.
In the theory of strongly interacting systems [24, 29] for the case of isotropic cross section
and massless particles the QGP shear viscosity $\eta$ is related to the scattering cross-section $\sigma$ via the equation

$$
\eta = 1.267 \frac{T}{\sigma}
$$

(3)
in natural units system, where $\hbar = 1$ and $c = 1$. There are also higher order calculations
extended up to the 16th order that do not change the obtained result [30]. For the gluonic
QGP which obeys the Boltzmann distribution, the entropy density is given by $s = 4n$, where
$n = \frac{g}{\pi^2} T^3$ is the QGP medium density and $g = 16$ is the gluon degeneracy. Thus,
the ratio of shear viscosity to entropy density may be written as

$$
\eta = 0.194 \frac{1}{T^2} \frac{1}{\sigma}
$$

(4)

We assume that the anisotropic flow is formed until the freeze-out of the fireball and
it is not changed further by the final-state interactions of hadrons. Therefore, we need
to know $T = T_{freeze}$, which can be obtained from the experimental data. We used the
standard Statistical Hadronization Model (SHM) with the modification factors from the
UrQMD [31] to fit the yields of identified charged hadrons produced in $Pb-Pb$ collisions
at $\sqrt{s_{NN}} = 2.76$ TeV. Data are taken from the ALICE experiment at the LHC [17].
The resulting value of the temperature of chemical freeze-out, $T_{freeze} = 166 \pm 3$ MeV,
is compatible with that at RHIC energies and expectations from the lattice QCD
calculations [32, 33].

The paper is organized as follows. The features of analysis of the azimuthal
harmonics on the Event-by-Event (E-by-E) basis are sketched in Sec. 2. The model
of incomplete equilibration medium (IEM) is considered in Sec. 3. Section 4 presents
details of our analysis of experimental data and main results obtained for parton-medium
cross section and share viscosity-to-entropy ratio. Conclusions are drawn in Sec. 5.

2. Event-by-Event analysis

The Event-by-Event analysis was implemented by ATLAS Collaboration to study the
azimuthal harmonics in $Pb-Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV, see [18, 52, 53, 54]. Here
the density distribution $P(v_n)$ was unfolded by the standard Bayesian procedure. It was shown that the true parameters density distribution did not depend on experimental extraction methods such as event plane, cumulant or two particle correlation with the triggering particle. The E-by-E analysis also excludes nonflow contributions.

If $v_2$ is understood as the hydrodynamic response to the initial anisotropy, $\varepsilon$, of the initial density profile, then $v_n = k_n \times \varepsilon_n$, where $\varepsilon_n$ and $k_n$ is the initial anisotropy of the $n$-th harmonic and the response coefficient of the same harmonic, respectively. In [55] the authors suggested a simple functional Elliptic Power form of the initial state without assuming any particular model of the initial conditions, which fitted eccentricity density distribution better than Gaussian or Bessel-Gaussian distribution.

The Elliptic Power distribution of eccentricity reads [55]

$$p(\varepsilon_n) = \frac{2\alpha \varepsilon_n (1 - \varepsilon_n^2)^{\alpha - 0.5}}{\pi} \int_0^\pi \frac{(1 - \varepsilon_n^2)^{\alpha - 1} d\varphi}{(1 - \varepsilon_n \varepsilon_n \cos \varphi)^{2\alpha + 1}}$$

Eq. (5) is reduced to the Power distribution

$$p(\varepsilon_n) = 2\alpha \varepsilon_n (1 - \varepsilon_n^2)^{\alpha - 1}$$

The probability distribution of anisotropic flow, $P(v_n)$, is related to the distribution of the initial anisotropy $p(\varepsilon_n)$ via

$$P(v_n) = \frac{d\varepsilon_n}{dv_n} p(\varepsilon_n) = \frac{1}{k_n} p\left(\frac{\varepsilon_n}{k_n}\right)$$

Below we will use the notation $k_n = k_n^{EbyE}$. Equation (7) was utilized in [55] to get the eccentricity scaled elliptic flow out of the ATLAS event-by-event data. It is very tempting, therefore, to compare the distributions of $v_2/\varepsilon_2$ as functions of centrality extracted by different methods from the data of two different experiments, ATLAS and CMS. This comparison is done in Sec. 4.

3. Incomplete equilibration medium model

Deviation from the local equilibrium can lead to an indicative dependence of the eccentricity scaled elliptic flow on charged particle multiplicity. Local equilibrium is not a necessary condition for the elliptic flow, but it is commonly accepted that deviation from the equilibrium can only reduce the magnitude of the effect.

Degree of thermalization in the fluid, produced in heavy ion collisions, can be characterized by the dimensionless parameter, Knudsen number $K$ [34]. The Knudsen number $K = \lambda/R_{tr}$ is defined by evaluating the mean free path $\lambda$ at average QGP life time $\tau = R_{tr}/c_s$, with the transverse medium size, $R_{tr}$, and the speed of sound in ultrarelativistic medium, $c_s = 1/\sqrt{3}$ [35]. By definition, $K^{-1}$ is the mean number of collisions per particle (also known as “opacity”); thus the ideal hydrodynamic limit corresponds to $K \rightarrow 0$. Hence, $\lambda = 1/(\sigma \rho)$, where $\sigma$ is the effective parton-medium
(mostly gluon-gluon) cross section and \( \rho \) is the (time-dependent) density of the medium for a typical time \( \tau \).

The inverse Knudsen number \( K^{-1} \) can be determined from the experimental data [12]. It is proportional to transverse particle density at midrapidity \( n_T = \frac{1}{S} \frac{dN}{dy} \bigg|_{y=0} \), where \( \frac{dN}{dy} \bigg|_{y=0} \) is the multiplicity density, an indicator of the number of collisions per particle at the time when elliptic flow is formed, and \( S \) is a square of the transverse area of the collision zone. Therefore, the inverse Knudsen number reads

\[
K^{-1}(n_T) = c_s \sigma n_T = c_s \sigma \frac{1}{S} \frac{dN}{dy} \bigg|_{y=0}.
\] (8)

In the incomplete equilibration medium (IEM) model the measured ratio \( \frac{v_2}{\varepsilon} \) is related to the Knudsen number by a simple equation suggested in [12, 36]

\[
\frac{v_2}{\varepsilon}(n_T) = \left( \frac{v_2}{\varepsilon} \right)_{\text{max}} \frac{K^{-1}}{K^{-1} + K_0^{-1}}
= \left( \frac{v_2}{\varepsilon} \right)_{\text{max}} \frac{1}{1 + K/K_0}
= \left( \frac{v_2}{\varepsilon} \right)_{\text{max}} \frac{n_T}{n_T + 1/(c_s \sigma K_0)},
\] (9)

where \( \left( \frac{v_2}{\varepsilon} \right)_{\text{max}} \) represents the hydrodynamic limit. Local thermal equilibrium is achieved if \( K^{-1} \gg 1 \). In this case the scaled elliptic flow coincides with that predicted by the hydrodynamic model, whereas for systems far from the equilibrium \( (K^{-1} \sim 1) \) it rapidly drops below the hydrodynamic limit. The value of parameter \( K_0 = 0.70 \pm 0.03 \) is estimated from the Monte-Carlo simulation of transport equations [37]. As seen in Eq.(9), using the known values of \( c_s \) and \( K_0 \), we can extract from the experimental data two parameters \( \left( \frac{v_2}{\varepsilon} \right)_{\text{max}} \) and \( \sigma \), or, alternatively, the combined parameter \( (c_s \sigma K_0) \). The success of IEM model, which gives the universal function, is caused perhaps by the fact that the physical uncertainties of three individual parameters \( c_s \), \( \sigma \), and \( K_0 \) are translated into the uncertainties of one parameter, \( \sigma \) [13].

Elliptic flow develops gradually in the system as the system evolves. Measured value of \( v_2 \) is formed at the final stage of hydro evolution at the temperature of freeze-out. The quantities that we shall extract from the IEM model should be interpreted as the averages over the transverse area \( S \), and over some time interval around \( \tau = R_{tr}/c_s \), which is the typical time scale for the build-up of \( v_2 \) in hydrodynamics.

There are several works, e.g. [13, 14, 15], where Eq. (9) is applied for the parameter extraction. First analysis of the experimental data for \( Au-Au \) and \( Cu-Cu \) collisions at \( \sqrt{s_{NN}} = 200 \) GeV within the IEM model was made in [13]. The authors determined parameters \( v_2^{\text{max}} \) and \( \sigma \) for different initial conditions, such as MC-Gl and MC-KLN ones. Later on, in [14] the Eq. (8) was generalized by including the ratio of the shear viscosity to entropy density:

\[
K^{-1}(n_T) = c_s \sigma n_T \left[ 1 + \frac{2}{3 \tau_i T_i} \left( \frac{\eta}{s} \right) \right]^{-3}.
\] (10)
However, the parameters $\eta/s$ and $\sigma$ are strongly correlated, as can be easily seen from the relation

$$\sigma = \text{const} \cdot \left[1 + \frac{2}{3\tau_i T_i} \left(\frac{\eta}{s}\right)^3\right].$$

It means that any fit will provide us the same $\chi^2/NDF$ for two different pairs of parameters, $\eta/s$ and $\sigma$. Therefore, this model is not used in our analysis.

4. Eccentricity-scaled elliptic flow as a function of transverse particle density

For the analysis we use the experimental data on the azimuthal anisotropy in $Pb-Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV obtained with the Compact Muon Solenoid (CMS) detector at the LHC [19]. The $v_2$ coefficient is determined as a function of charged particle transverse momentum $p_T$ and overlap of the colliding nuclei (centrality) in the pseudorapidity region of $|\eta| < 2.4$, where $\eta = -\ln(\tan(\theta/2))$, with $\theta$ being the polar angle relative to the counterclockwise beam direction. We analyze the integrated elliptic flow in the measured transverse momentum range of $0.3 \leq p_T \leq 3.0$ GeV/$c$ and extrapolate it to the region $0 \leq p_T \leq 0.3$ GeV/$c$.

The collective motion of the system and, therefore, the anisotropy parameter depends on the initial shape of the nucleus-nucleus collision area and the fluctuations in the positions of the interacting nucleons. By dividing $v_2$ to the participant eccentricity, one may potentially remove this dependence across centralities, colliding species, and center-of-mass energies, thus enabling a comparison of results in terms of the underlying physics driving the flow. Two initial state definitions are used in MC-Gl and MC-KLN models. Initial state in the MC-KLN model [38, 39, 40, 41, 42, 43] is based on the color glass condensate (CGC) concept [44] accounting for the fact that at very high energies or small values of Bjorken $x$ the gluon density becomes saturated.

The CGC model predicts eccentricities that exceed the Glauber-model ones by a factor of about 1.2, with some deviations from this value in the most central and the most peripheral collisions [41, 43], and a bit different behavior of the initial overlap area $S$ [7].

In [7] it was shown that only in the MC-Gl model the obtained universal curve was able to match the experimental data for the dependence of the eccentricity scaled elliptic flow on $1/S \, dN_{ch}/d\eta$ within the entire collision energy range between 7.7 AGeV and 2.76 ATeV. Therefore, we use the Glauber model, which is a multiple-collision model treating a nucleus-nucleus collision as an independent sequence of nucleon-nucleon collisions (see [45] and references therein), to calculate the participant eccentricity

$$\varepsilon_{\text{part}} = \sqrt{\frac{(\sigma_{x'}^2 - \sigma_{x''}^2)^2 + 4\sigma_{x'y'}^2}{\sigma_{x'}^2 + \sigma_{x''}^2}}$$

and its cumulant moments $\varepsilon\{2\} = \sqrt{\langle \varepsilon_{\text{part}}^2 \rangle}$, where $\sigma_{x'}^2$ and $\sigma_{x''}^2$ are the variances of the participant spatial distribution and $\sigma_{x'y'}^2$ is the covariance. The participant eccentricity
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is evaluated in a frame that maximizes its value. This frame, which defines the initial-state participant plane, may be shifted and rotated with respect to the frame defined by the impact parameter vector and the beam direction. From the Glauber model, we also calculate the transverse overlap area of the two nuclei $S \equiv \pi \sqrt{\sigma_x^2 \sigma_y^2 - \sigma_{x'y'}^2}$. The table of eccentricities $\varepsilon$ and overlap area $S$ for each centrality can be found in [19].

To restore the elliptic flow the Event Plane (EP) method is employed. Here the two-particle positive and negative event planes, $\Psi_{EP}^+$ and $\Psi_{EP}^-$, are determined by correlating the signals in the hadron calorimeter HF$^+$ ($3 < \eta < 5$) and HF$^-$ ($-5 < \eta < -3$), respectively. The elliptical anisotropies are measured by correlating particles from the negative (positive) $\eta$ range of the tracker, i.e., $-2.4 < \eta < 0$ ($0 < \eta < 2.4$) to $\Psi_{EP}^+$ ($\Psi_{EP}^-$). The factor $R$ corresponds to a resolution correction, which needs to be applied to the event plane. A gap of three units in $\eta$ is kept to reduce the contamination of non-flow effects.

In [19] the anisotropic parameter $v_2$ was measured at $|\eta| < 0.8$ in the region $p_T = 0.3 - 3.0$ GeV/$c$. Now we extrapolate the integrated value of $v_2$ for every centrality to the region $p_T = 0 - 0.3$ GeV/$c$, using the extrapolation of the normalized spectrum $dN/dp_T$ at each centrality by power-law dependence and polynomial of 5-th order fit to $v_2(p_T)$ in the region $p_T = 0.3 - 3.0$ GeV/$c$. Such procedure allows us to calculate the integral value of $v_2$ by using Eq. (2) in the region $0 < p_T < 3$ GeV/$c$. Note that the extrapolation of the integrated value of $v_2$ to the low-$p_T$ region reduces the elliptic flow by about 5-20%, being dependent on the normalized spectrum $dN/dp_T$ at each centrality, because maximum of the $dN/dp_T$ distribution is located around $p_T \approx 0.2$ GeV/$c$.

The eccentricity scaled integrated elliptic flow, $v_2/\varepsilon$, extracted by the two-particle cumulant method is nearly identical to the flow obtained by the event-plane method, provided the corresponding eccentricity fluctuations are taken into account. In the centrality range of 15 – 40%, the four-particle cumulant measurement of $v_2/\varepsilon$ is also in agreement with the other two methods. The main difference in the results obtained by different methods can be attributed to their sensitivity to non-flow contributions. Because of some irregularities of eccentricity $\varepsilon$ for very central and for peripheral collisions, we do not include the four-particle cumulant in the analysis of transverse particle density in present work.

We have also recalculated $dN/dy$ from $dN/d\eta$ at $|\eta| < 0.8$. Under the assumption of region mid-rapidity, i.e., $\cosh^2 y \approx 1.0$, the charged particle spectrum $dN/dp_T$ is used for each centrality, and $dN/dy$ is calculated as

$$\frac{dN}{dy} = \int dp_T \frac{1}{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{dN}{d\eta dp_T} \approx \int dp_T \frac{m_T}{p_T} \left( \frac{dN}{dp_T} \Big|_{\eta=0} \right),$$

where $m_T^2 = m + p_T^2$. Recall, that the analysis in [19] is done for charged particles, where the spectrum is dominated by $\pi^\pm$'s. In present work we make two modifications. Namely, (i) we put $m = m_{\pi^0}$ and (ii) recalculate the total number of particles as
\[ \frac{dN}{dy} = \frac{3}{2} \frac{dN_{ch}}{dy}, \] thus taking into account neutral \( \pi \)-mesons. The systematic uncertainty caused by the replacement of all particles to pions is small, since pions heavily dominate the hadronic spectrum at \( 0 \leq p_T \leq 0.3 \text{ GeV}/c \).

In ideal hydrodynamics, the eccentricity-scaled elliptic flow is constant over a broad range of impact parameters. The deviations from this behavior are expected in peripheral collisions, in which the system freezes out before the elliptic flow fully builds up and saturates [46]. A weak centrality and beam-energy dependence is expected through variations in the equation-of-state. In addition, the system is also affected by viscosity, both in the sQGP and in hadronic stages [7, 27, 39, 47] of its evolution. Therefore, the centrality dependence of \( v_2/\varepsilon \) can be used to extract the ratio of the shear viscosity to the entropy density of the system.

It was previously observed [48, 49] that the \( v_2/\varepsilon \) values, obtained for different systems colliding at varying beam energies, scale with the charged-particle rapidity density per unit transverse overlap area, \((1/S)(dN_{ch}/dy)\), which is proportional to the initial entropy density. In addition, it was pointed out [40] that in this representation the sensitivity to the modeling of the initial conditions of heavy ion collisions is largely removed, thus enabling the extraction of the shear viscosity to the entropy density ratio from the data via the comparison with viscous hydrodynamic calculations.

In Fig. 1 we plot the CMS data on \( v_2\{EP\}/\varepsilon_{\text{part}} \) and \( v_2\{2\}/\varepsilon\{2\} \) measured in Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) [19]. Here the pseudorapidity spectrum of charged particles, \( dN_{ch}/d\eta \), was (i) extrapolated to the region \( 0 < p_T < 3 \text{ GeV}/c \), then (ii) converted to the rapidity distribution \( dN_{ch}/dy \), which finally was used (iii) to obtain the total spectrum, \( dN/dy = 3/2 dN_{ch}/dy \), of charged and neutral hadrons. Solid curve in Fig. 1 corresponds to the fit of \( v_2\{EP\}/\varepsilon_{\text{part}} \) distribution to Eq. (9) with two free parameters, \((v_2/\varepsilon)_{\text{max}}^\text{max} \) and \( \sigma \).

The fitting procedure yields
\[
\sigma = (3.1 \pm 0.2)\text{mb}, \quad (v_2/\varepsilon)_{\text{max}}^\text{max} = 0.58 \pm 0.03 \quad (12)
\]
The indicated errors are only of statistical origin. And for the viscosity-to-entropy ratio we get
\[
\frac{\eta}{s} = 0.17 \pm 0.02 \quad \text{or} \quad 4\pi \left( \frac{\eta}{s} \right) = 2.14 \pm 0.25 \quad (13)
\]
Let us discuss the obtained results and compare it with the available estimates. Since the lattice QCD calculations become inapplicable for the nonequilibrium evolution, the common practice is to rely on the AdS/CFT correspondence for guidance to general properties of strongly coupled field theories at finite temperatures [50]. AdS/CFT imposes a lower boundary on viscosity to entropy ratio as \( 4\pi \eta/s \geq 1 \). The upper limit of this ratio, estimated both from pure theoretical considerations and from the analysis of heavy ion collisions at energies of RHIC and LHC, is about 0.6 (for review, see [51] and references therein). The hybrid hydrodynamic models provide \( \eta/s \approx 0.16 \) [4] or even \( \eta/s \approx 0.12 \) [11] at RHIC, and a bit higher values \( \eta/s \approx 0.22 \pm 0.02 \) [4] and \( \eta/s \approx 0.2 \) [11] at LHC.
Recently, in [55] the authors applied Eq. (7) to ATLAS E-by-E results [18] and got the eccentricity scaled elliptic flow \( k_2^{EbyE} = \varepsilon_{2true} / \varepsilon_2 \) as a function of centrality. We plot in Fig. 1 their distribution for \( k_2^{EbyE} \) (in ATLAS version with \( k' = 0.10 \)), taken from Fig. 2 of [55]. Compared to the CMS data, the ATLAS E-by-E points are located a bit higher because of absence of the \( v_2(p_T) \) extrapolation to the low-\( p_T \) region, \( 0 < p_T < 0.5 \) GeV/c, which decreases the values of the integrated flow. Nevertheless, two different methods of extraction the ratio \( v_2/\varepsilon \) give rather close dependencies of this ratio on transverse particle density. From the E-by-E analysis of the ATLAS data the value \( \eta/s \approx 0.19 \) was estimated [55]. It is remarkable that our result, obtained within the framework of incomplete equilibration model, agrees well with the hydrodynamic model estimates.

Other result is the combined parameter \( c_s\sigma K_0 \), which appears to be

\[
c_s\sigma K_0 = (1.25 \pm 0.14) \text{mb} \tag{14}
\]

It might be also useful for pinning down the transport properties of QGP.

5. Conclusions

The incomplete equilibration medium model is used to extract the medium modified gluon-medium cross section and to calculate the ratio of shear viscosity to entropy density. Both parameters represent the fundamental properties of QGP and play an important role in relativistic kinetic theory and hydrodynamics. Recent experimental CMS and ATLAS data on the elliptic anisotropy of charged particles in \( Pb-Pb \) collisions at \( \sqrt{s}_{NN} = 2.76 \) TeV are employed for the analysis. The integrated elliptic flow is estimated by extrapolation of measured differential flow and momentum spectrum of charged hadrons in the range \( 0.3 < p_T < 3 \) GeV/c to the range \( 0 < p_T < 3 \) GeV/c. In contrast to earlier applications of the IEM model, here the rapidity distribution of charged particles was recalculated from their pseudorapidity spectrum. The total hadron multiplicity distribution is then taken as \( dN/dy = 3/2dN_{ch}/dy \) to account for both charged and neutral particles. The values of obtained parameters are \( \sigma = (3.1 \pm 0.2) \text{mb} \) for the partonic cross section and \( \frac{\eta}{s} = 0.17 \pm 0.02 \) for the ratio of shear viscosity to the entropy density, respectively. The dependence of \( v_2/\varepsilon \) on particle transverse density obtained within the IEM model is very close to that obtained with the parameters from the E-by-E analysis of ATLAS data [55]. The extracted value of \( \eta/s \) also agrees well with that found in simulations of \( Pb-Pb \) collisions at LHC by the viscous hydrodynamic hybrid models, thus supporting the description of quark-gluon plasma as a nearly perfect fluid.

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Parton-medium cross section and average QGP viscosity in lead-lead collisions at LHC

Figure 1. (Color online.) Eccentricity-scaled $v_2\{EP\}/\varepsilon_{part}$ (full circles) and $v_2\{EP\}/\varepsilon\{2\}$ (open circles) from CMS [19] as a function of total particle transverse density $1/SdN/dy$ extended to $p_T=0$ region (see text for details). Full triangles denote the E-by-E results of ATLAS [18] with extraction response parameter $k_{EbyE}^2$ from [55]. The full black curve is a fit of $v_2\{EP\}/\varepsilon_{part}$ to Eq. (9). The dashed and dotted curves represent the bands of systematic uncertainties in the eccentricity determination.