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How quantum is the “quantum vampire” effect?: testing with thermal light: supplementary material

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This document provides supplementary information to "How quantum is the “quantum vampire” effect?: testing with thermal light," https://doi.org/10.1364/OPTICA.5.000723. An exact mathematical description of the photon annihilation and beam splitter action on the single-mode thermal state of light in terms of photon number distributions and generating functions is given.

Generating functions. All quantum states presented in the work can be described with diagonal density matrixes, which are completely defined by photon number distributions $P(n)$. Any particular distribution corresponds to its generating function $G(z)$, which can be defined by the following equations:

$$G(z)=\sum_n P(n)z^n, \qquad P(n)=\frac{G^{(n)}(0)}{n!}, \tag{S1}$$

where $G^{(n)}$ is an nth-order derivative.
Normalization condition leads to $G(1)=1$.

For example, a thermal state with a mean photon number μ_0 is described by the Bose-Einstein photon number distribution $P_{BE}(n\,|\,\mu_0)$ (3), which corresponds to the generating function $G_{BE}(z\,|\,\mu_0)=[1+\mu_0(1-z)]^{-1}$.

Photon subtraction. It was shown [1,2], that photon subtraction leads to the differentiation and renormalizing of the generating function:

$$G_{-1}(z)=\frac{G^{(1)}(z)}{G^{(1)}(0)} \tag{S2}$$

where $G_{-1}(z)$ is the generating function, which corresponds to the photon subtracted state.

Therefore, k-photon subtracted thermal state can be described with the generating function

$$G_{-k}(z\,|\,\mu_0)=\frac{G_{BE}^{(k)}(z\,|\,\mu_0)}{G_{BE}^{(k)}(1\,|\,\mu_0)}=\left[1+\mu_0(1-z)\right]^{-k-1}, \tag{S3}$$

which corresponds to the compound-Poisson distribution $P_{cP}(n\,|\,\mu_0,k)$ (4).

Beam splitting. Consider the beam splitter BS (Fig. 1b), which splits a quantum state in the mode “1” into the modes “A” and “B”. The two-mode photon number distribution $P(n_A,n_B)$ corresponds to the generating function $G(z_A,z_B)$ as follows:

$$\begin{gathered} G(z_A,z_B)=\sum_{n_A,n_B} P(n_A,n_B)\,z_A^{n_A}z_B^{n_B}, \\ P(n_A,n_B)=\frac{1}{n_A!\,n_B!}\frac{\partial^{n_A}\partial^{n_B}}{\partial z_B^{n_B}}G(0,0). \end{gathered} \tag{S4}$$

It can be shown, that

$$G(z_A,z_B)=G_{in}\left(|t|^2 z_A+|r|^2 z_B\,|\,\mu_0\right), \tag{S5}$$

where $G_{in}(z)$ is a generating function of the initial photon number distribution at the input mode “1”.

Marginal distributions $P(n_A)$ and $P(n_B)$ can be calculated with use of their generating functions:

$$G(z_A) = G(z_A, z_B = 1), \quad G(z_B) = G(z_A = 1, z_B). \qquad \textbf{(S6)}$$

If we have a *k*-photon subtracted thermal state $\hat{\rho}_{th-}(\mu_0, k)$ in the mode "1", we need to substitute (S3) and (S5) into (S6) and therefore we get

$$\begin{aligned} G(z_A) &= \left[1+\mu_0 |t|^2 (1-z_A)\right]^{-k-1} = G_{th-}\left(z_A \mid \mu_0 |t|^2, k\right)_A, \\ G(z_B) &= \left[1+\mu_0 |r|^2 (1-z_B)\right]^{-k-1} = G_{th-}\left(z_B \mid \mu_0 |r|^2, k\right)_B. \end{aligned} \qquad \textbf{(S7)}$$

Therefore, the beam splitter doesn't change parameter *k*, but change μ_0 in accordance with its transmission and reflection coefficients.

Quantum vampire description in terms of generating functions. All considered states in the modes "1", "*A*" and "*B*" are defined by the two-mode photon number distribution (S4). Photon subtraction in the mode "*A*", "*B*" or "1" leads to its differentiation with respect to z_A, z_A, or $z_1 = |t|^2 z_A + |r|^2 z_B$, and appropriate renormalization. One can directly check, that

$$\frac{\partial G(z_A, z_B)/\partial z_A}{\partial G(1,1)/\partial z_A} = \frac{\partial G(z_A, z_B)/\partial z_B}{\partial G(1,1)/\partial z_B} = \frac{\partial G_1\left(z_1 = |t|^2 z_A + |r|^2 z_B\right)/\partial z_1}{\partial G_1(1)/\partial z_1}, \qquad \textbf{(S8)}$$

therefore, conditional photon subtracted states are invariant under the mode where photon subtraction takes place.

References

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