

Schrödinger plasmon–solitons in Kerr nonlinear heterostructures with magnetic manipulation

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We investigate surface plasmon–soliton (SPS) propagation in transverse magnetic field in heterostructures with Kerr nonlinearity. The nonlinear Schrödinger equation in the framework of perturbation theory has been derived for three cases: a single-interface metal/nonlinear-dielectric structure and double-interface structures of nonlinear-dielectric/metal/dielectric with either ferromagnetic or nonmagnetic dielectric. The effect of the magneto-optical nonreciprocity in the Schrödinger equation is found. The estimations show that the effect is the strongest for the double-interface structure with a magnetic substrate in the vicinity of the resonant plasmonic frequency. We have also shown that the external magnetic field modifies SPS amplitude and width. © 2015 Optical Society of America

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Surface plasmon-polaritons (SPPs) have a broad range of applications for development of nano-optical circuits and other nanoscale devices [1] due to their subwavelength confinement. At the same time, nonlinear plasmonics brings many opportunities for nanoscale photonic devices [2,3], in particular, for the novel nonlinear plasmonic metamaterials [4–7]. It has been shown, theoretically, that in the presence of Kerr-type nonlinearity, temporal [8] and spatial [9–11] surface plasmon–soliton propagation is allowed. Temporal surface plasmon–solitons are wave packets with slowly varying pulse envelope (when a group velocity dispersion is compensated for by self-phase modulation), possibly observed in plasmonic heterostructures with one of the media possessing Kerr-type nonlinearity [8,12]; when refractive index depends on the intensity of the propagating wave $n = n_0 + n_2 I$, n_2 is the nonlinear refractive index. Nonlinearity leads to the self-phase modulation, resulting in spectral narrowing of the temporal profile of the pulse. Temporal plasmon–soliton propagation [2,13], as well as its

fundamental aspects, including self-trapping and frequency conversion are currently of great interest [14,15].

Today, SPPs control is a quite challenging and topical problem. The field of nonlinear plasmonic pulses, and the excitation and control of the pulses, is being intensively developed [15–17]. Among different approaches for SPP control, an external magnetic field application that gives rise to magneto-optical phenomena seems to be of prime importance. The magnetic control has been demonstrated in the case of continuous SPP waves. In particular, a transversal magnetic field provides linear variation of the SPP wavenumber in respect to the magnetization in the case of continuous SPP waves [18–24]. To the best of our knowledge, magneto-optical properties of the plasmonic solitons have not been considered yet.

In this Letter, we study three different types of the heterostructures, shown in Fig. 1. First, we consider a single-interface structure of metal/nonlinear-dielectric. Being placed in an in-plane magnetic field, the metal part acquires magneto-optical properties that open the possibility for the magnetic control of the plasmonic solitons. In double-interface structures of nonlinear-dielectric/metal/linear-dielectric, we consider the following two options: either the metal or the linear dielectric has magneto-optical properties. The substrate must not necessarily be dielectric; structures with semiconductor substrate can be

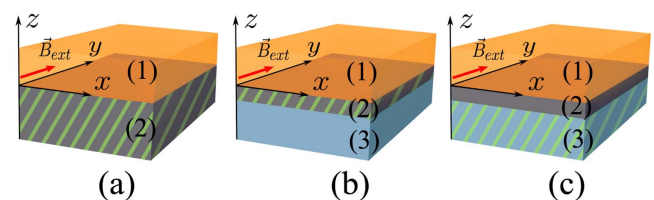


Fig. 1. Considered nonlinear plasmonic structures. (a) Metal/nonlinear-dielectric structure and (b),(c) the nonlinear-dielectric/metal/linear-dielectric structure. The media exhibiting the magneto-optical properties (shown by hatching) are the metal [(a),(b)] or the magnetic dielectric (c). (1), a dielectric with Kerr nonlinearity; (2), a metal; (3), a linear dielectric. Red arrow denotes the external magnetic field direction.

considered as well. We have found that a significant nonreciprocity effect can be present in such structures, and the parameters of the SPSs can be strongly altered using the external magnetic field.

To investigate the magnetoplasmonic solitons, following previous studies of the nonmagnetic case [8,25], we consider a slowly varying wave packet envelope $\psi(x, t)$. An equation determining its behavior is derived in the framework of the first-order perturbation theory. In particular, we assume that the nonlinearity is relatively weak and, hence, the magnetic field of the SPS has only a transverse component $\mathbf{H} = (0, H_y, 0)$, and the electric field component \mathcal{E}_y is negligibly small. Then we assume that the magnetic field of the SPS in the nonlinear layer can be represented in the form of a wave packet [25,26] with $H_y(x, z, t) = \psi(x, t)\mathcal{H}_y(x, z, t)$, where $\mathcal{H}_y|_{z=0} = \exp[i(\omega t - kx)]$, $z = 0$ corresponds to the surface of the nonlinear medium, ω is the carrier frequency, and k is the corresponding wavenumber. First, we solve Maxwell's equations for each layer of the structure, using a perturbation theory for the nonlinear layer of the structure and using the following dielectric permittivity tensor of a medium placed in a magnetic field:

$$\varepsilon = \varepsilon(\omega) \begin{pmatrix} 1 & 0 & iQ(\omega) \\ 0 & 1 & 0 \\ -iQ(\omega) & 0 & 1 \end{pmatrix}, \quad (1)$$

where Q is the magneto-optical (MO) parameter [27]. We assume that the MO parameter is proportional to the magnitude of the external magnetic field, and we neglect its dispersion in simulations to preserve uniformity of the results (this assumption is valid at the off-resonant frequency range [27]), as the results discussed in our Letter are primarily qualitative. However, our approach is not restricted to these assumptions and, for a specific media, the dispersion of the MO parameter can be quite complex. The nonmagnetic part of the permittivity for metals is defined by $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - i\Gamma\omega}$ [28], ω_p is the plasma frequency, and Γ is the relaxation coefficient. The external magnetic field is applied along the y -axis, and the SPS propagates along the x -axis, while interface normals are along the z -axis (Fig. 1). Substituting the solutions into the boundary conditions, we obtain the equation that defines the behavior of the wave packet envelope $G_{nl}^{(1)}(\omega, k|\psi)\psi(x, t) = 0$, where index (1) defines the order of perturbation theory that has been used for the nonlinear layer. The function $G_{nl}^{(1)}$ depends on $|\psi(x, t)|^2$ as the dielectric permittivity $\varepsilon = \varepsilon_0 + \alpha|\mathbf{E}|^2$; $\alpha = \frac{\varepsilon_0 n_2 c}{4\pi}$ (in esu units) depends on it. Here, ε_0 is the linear part of the dielectric constant, and c is the vacuum speed of light.

After that, the function $G_{nl}^{(1)}(\omega, k|\psi)$ is expanded into a series following [25]. Therefore, we obtain

$$i \frac{\partial \psi(x, t)}{\partial t} + i v_g \frac{\partial \psi(x, t)}{\partial x} + \frac{1}{2} \beta \frac{\partial^2 \psi(x, t)}{\partial x^2} - \gamma |\psi(x, t)|^2 \psi(x, t) = 0, \quad (2)$$

where the coefficients are defined as

$$v_g = \frac{\partial \omega}{\partial k}; \beta = \frac{\partial^2 \omega}{\partial k^2}; \gamma = \frac{\partial \omega}{\partial |\psi(x, t)|^2}, \quad (3)$$

where all the derivatives are calculated at the point $(\omega, k, |\psi|^2) = (\omega_0, k_0, 0)$, where (ω_0, k_0) is the root of the

dispersion relation in the linear case. Equation (2) is a nonlinear parabolic equation for $\psi(x, t)$ or the nonlinear Schrödinger equation. The coefficients [Eq. (3)] can be calculated solely from the dispersion relation. The magnetic action is assumed to be contained in the latter as the magneto-optical contribution to the dielectric permittivity [Eq. (1)]. Consequently, the coefficients of the nonlinear Schrödinger equation depend on the external magnetic field.

There are two types of fundamental solutions of Eq. (2): bright (pulses of certain amplitude) and dark (inverse pulses on constant bright background) solitons. The latter have little practical interest. The condition for the existence of bright SPS is $\beta\gamma < 0$ and, in this case, the solution of Eq. (2) with pulse duration τ follows the expression

$$\psi(x, t) = \frac{1}{v_g \tau} \sqrt{-\frac{\beta}{\gamma}} \exp\left(it \frac{\beta}{2v_g^2 \tau^2}\right) \frac{1}{\cosh\left(\frac{t-x/v_g}{\tau}\right)}. \quad (4)$$

When dispersion is normal, the medium must necessarily be defocusing (with $n_2 < 0$) so that the condition for the existence of the bright SPS is fulfilled. This consequence was first reported in [8] and, since then, the range of nonlinear materials was broadened. For instance, a variety of conjugated polymers, as well as nonlinear glasses, which exhibit large values of the negative nonlinear refractive index, was studied [29,30]. It is expected that losses in these media can be further reduced. In our study, we consider the 4-BCMU as the nonlinear medium with $n_2 = -15 \times 10^{-14} \text{ cm}^2/\text{W}$ [29] and $n_0 \approx 1.56$.

It should be noted that similar methods have been used to study SPSs earlier [8,14] with small differences. However, to the best of our knowledge, the magnetic impact on the SPSs has never been taken into account, and this problem is considered in detail in this Letter. For the single-interface problem, we obtained

$$G_{nl}^{(1)}(\omega, k|\psi)\psi(x, t) = \left(\frac{\chi_1}{\varepsilon_1^0 + \frac{\alpha}{4} \left(\frac{\varepsilon \chi_1}{\omega \varepsilon_1^0}\right)^2 \left(1 + \frac{k^2}{\chi_1^2}\right)^2 |\psi(x, t)|^2} + \frac{\chi_2 + kQ(\omega)}{\varepsilon(\omega)(1 - Q(\omega)^2)} \right) \psi(x, t) = 0, \quad (5)$$

where $\chi_i = \sqrt{k^2 - \varepsilon_i \frac{\omega^2}{c^2}}$; the dielectric permittivity of the nonlinear medium is $\varepsilon_1 = \varepsilon_1^0 + \alpha|\mathbf{E}|^2$. In the case of $\alpha \rightarrow 0$, $G(\omega, k|\psi)$ defines the dispersion relation for the linear problem. Similar asymptotics is valid for the double-interface cases, but the corresponding relations are omitted here due to their bulkiness. The general procedure remains the same for all considered structures with the only difference in the coefficients in Eq. (2). The magnetic field impact is provided by the occurrence of the $Q(\omega)$ in Eq. (5) and, subsequently, in Eq. (3).

The nonlinear parameter of the SPS, γ , defines purely nonlinear properties of a pulse propagation. The external magnetic field alters this coefficient, as it is seen in Fig. 2, where we present the relative difference of the SPS nonlinear parameter for forward (F) and backward (B) SPS propagation at a fixed B -field $\Delta\gamma = 2(\gamma_F - \gamma_B)/(\gamma_F + \gamma_B)$ [the γ_F is equivalent to $\gamma(+\mathbf{B})$ and γ_B is equivalent to $\gamma(-\mathbf{B})$]. The latter defines the magneto-optical nonreciprocity effect for γ . In fact, the nonreciprocity effect is of higher order for the wavevector than for γ . However, since only the (ω_0, k_0) point is substituted in Eqs. (2) and (3), which is the root of the linear dispersion

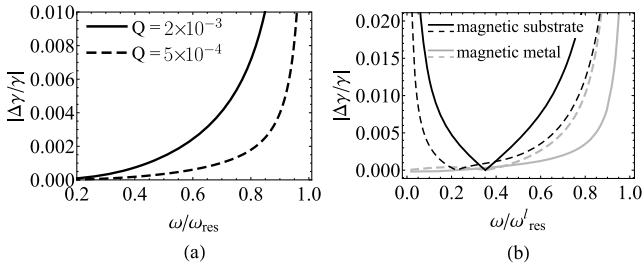


Fig. 2. Absolute value of the magnetic field induced relative change of γ vs SPS central frequency for (a) the single-interface structure [Fig. 1(a)] and (b) the double-interface structure with the gold film of 40 nm (solid lines) or 9 nm (dashed lines) thickness [Figs. 1(b) and 1(c)]. Black lines correspond to the structure with the ferromagnetic layer of bismuth iron garnet ($\epsilon = 6.3$), while gray lines represent data for the structure with the nonmagnetic substrate of $\epsilon = 6.3$ and the magnetization in the gold film. For both structures, $\omega_{\text{res}}^{\text{HIM}} = 4.35 \times 10^{15} \text{ s}^{-1}$, $Q = 2 \times 10^{-3}$.

relation, the problem can be reduced to the linear SPP case (see, for example, [31]). This effect plays a significant role in all of our results, since we used the wavevector value in the calculations.

For the single-interface structure, $\Delta\gamma/\gamma$ becomes rather large only in the vicinity of the plasmonic resonant frequency of the structure $\omega_{\text{res}} = \omega_p/\sqrt{1 + \epsilon}$, where ϵ denotes the linear part of the dielectric permittivity of the dielectric medium. At the same time, for the double-interface structures, there is a zero point in $\Delta\gamma/\gamma$ vs ω dependence, and $\Delta\gamma/\gamma$ is rather large for both low and high frequencies. The position of the zero point shifts to the higher frequencies with the increase of the gold thickness.

Note that for the double-interface structures there are two SPP modes: the low-index mode (LIM) and the high-index mode (HIM). Figure 2(b) shows the dependency only for the HIM, since for the LIM the propagation length is too small (tenths of μm) in considered structures, especially compared to the length of the SPS formation. For Q of gold of 5×10^{-4} and 2×10^{-3} estimation of the magnetic field which must be applied gives 9 T and 35 T, respectively, according to the free electron model at $\omega = 3.5 \times 10^{15} \text{ s}^{-1}$. For the structure with magnetic dielectric [Fig. 2(b), gray lines], $Q = 2 \times 10^{-3}$ of the dielectric might be already achieved at the magnetic field of about 2 mT.

Apart from the alteration of γ , the external magnetic field modifies the SPS amplitude, as seen in Fig. 3, when comparing $Q = 0$ to $Q = 0.15$. Specifically, with the increase of $|Q|$, the amplitude of the SPS propagating backward, opposite the x -axis, increases for $Q > 0$ and decreases for $Q < 0$. Note that Fig. 3 can be viewed as a change from $Q = -0.15$ to $Q = 0$ for the SPS propagating along the x -axis. Here, we also face the phenomenon of the MO nonreciprocity, but this time in terms of the SPS amplitude. The influence of the magnetic field on the SPS width, w [Fig. 3(c)], is determined similarly to the magnetic impact on γ .

Further analysis shows that the magnetic field impact depends primarily on the SPS localization in a structure. The increase of the localization of the SPS field in the magnetic medium explains the nonreciprocity growth with the increase of frequency, in all three types of the considered structures. For

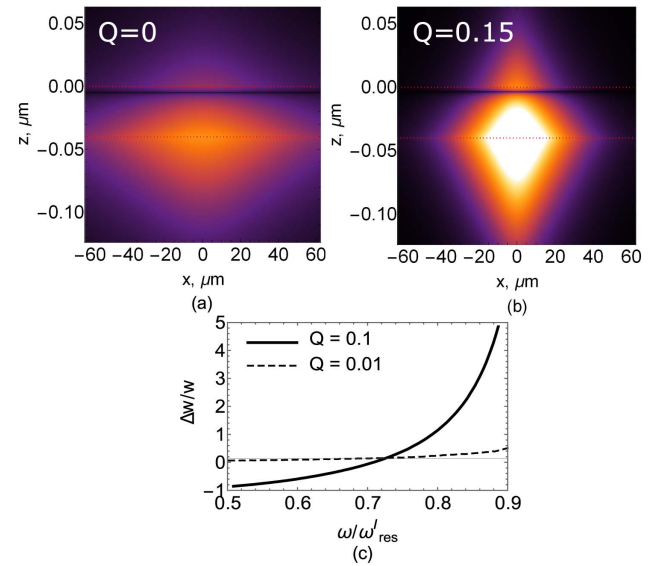


Fig. 3. (a), (b) Contour plots (not scaled) for profiles of the absolute value of the SPS magnetic field envelope in the double-interface structure with 40 nm thick gold film, the nonlinear-dielectric, and the ferromagnetic dielectric at $\omega = 0.78\omega_{\text{res}}^{\text{HIM}}$. (c) Spectrum of the SPS width relative change due to the transversal magnetic field, for the same structure. Parameters of all layers are the same as in Fig. 2.

example, in Fig. 2(b), the nonreciprocity is larger when the MO response is due to the dielectric and smaller when it is due to the metal film. At the same time, the nonlinear properties depend on the level of localization of the SPS field in the nonlinear medium. Selection of the opto-geometric parameters of the structure should be done paying attention to balancing the field localization between nonlinear and magnetic medium, and depends on the losses in the media.

The approximate intensity of the wave packet envelope that is needed to launch the SPS of certain pulse duration and central frequency, propagating along the x -axis, is given by

$$I_l = \langle S_x \rangle_{\text{max}} = \frac{c}{8\pi} \text{Re}[E_z H_y^*]_{\text{max}} \quad (\text{in esu units}). \quad (6)$$

Figure 4 presents the launch intensity dependence on the carrier frequency of the SPS in different structures. As seen in Fig. 4, at a certain frequency, the intensity is minimum, and its frequency depends on the gold film thickness and on the materials used in the structure. This leads to a large opportunity to control the launch intensity of the SPS by designing a structure with specific geometrical parameters. The values of intensity near the minima in all the plots are experimentally achievable.

Another important property of SPSs is their nonlinear length, defined as $L_{nl}^{(1)} \approx \left(\frac{\epsilon_d v_g |\psi|^2 |_{\text{max}}}{2c} \frac{\partial k}{\partial |\psi|^2} \Big|_{(\omega_0, k_0|0)} \right)^{-1}$; here, ϵ_d is the dielectric permittivity of the dielectric medium, in which the localization of the SPS field is higher. This length characterizes the distance at which nonlinear properties of the pulse are revealed; for example, at this distance, an arbitrary pulse forms a soliton in favorable conditions. To exploit experimentally nonlinear properties of the plasmonic pulses, the nonlinear length should be at least comparable with the propagation length. At the same time, the propagation length at higher frequencies,

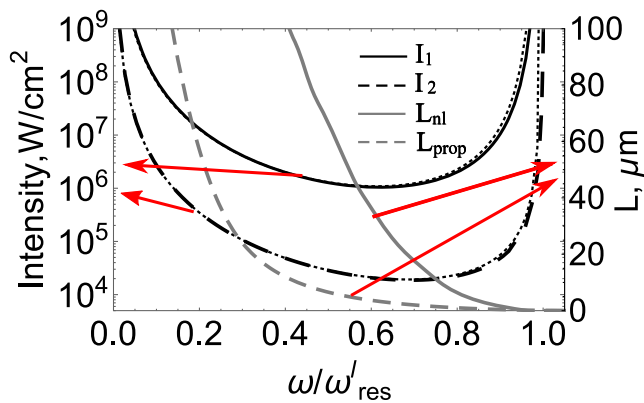


Fig. 4. Left axis, dependence of the launch intensity of SPSs; right axis, nonlinear (L_{nl}) and propagation (L_{prop}) lengths on frequency for the double-interface structure containing the same materials as in Fig. 2. Solid branch (I_1), 40 nm gold film; black dashed curve (I_2), 9 nm gold film; in each case, the linear dielectric film is magnetic with $Q = 0.01$. The thick lines denote values for the magnetic field \mathbf{B} , and the thin dotted lines are for the magnetic field $-\mathbf{B}$. The pulse duration is 1 ps.

where this condition is fulfilled, is too small for practical usage. The amplification of SPSs should be considered in the future to overcome this obstacle. There are two possible methods of SPS amplification: the optical [32] and the electric [33]. The scheme for the electric SPS pumping [33] lies in using Schottky contact between gold and the semiconductor. For example, involving InAs with Mn doping as a substrate may play a dual role: to provide the Schottky contact with gold and to exhibit the magneto-optical response. In addition, the SPSs of higher orders may be considered, which are the other solutions of Eq. (2) with periodically varying parameters, for which the $L_{nl}^{(N)} = L_{nl}^{(1)}/N^2$, where N is the order of soliton; however, their launch intensity grows as N^2 .

To sum up, we have investigated the possibility of magnetic control of temporal plasmon-solitons in the cases of the single-interface and the dielectric-metal-dielectric structures. The effect of the magneto-optical nonreciprocity for the SPS width and nonlinear parameter has been found. The effect is shown to be larger in a double-interface structure with a magnetic dielectric: nonreciprocity of SPS width reaches 400%, while variation of the nonlinear parameter is about 20%. At this, opto-geometric properties of the structure can be selected to diminish SPS launch intensity to 10^7 W/cm². In addition, the nonreciprocity effect of SPSs might be increased in periodic and microresonator plasmonic structures [34,35]. Our study of magnetic control of the SPSs opens a way for further investigations that, hopefully, will lead to practical implementations and use of the SPSs for various applications, such as plasmonic circuitry, sensing, and optical data processing.

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