Finite difference modeling of elastic wave propagation on curvilinear grid: a generalized rotated operator approach

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Summary

Modeling of wave propagation in a realistic geological environment needs a numerical scheme able to handle complex shapes and geometries. Finite difference scheme based on a generalization of the rotated staggered grid method can be used for modeling of elastic waves on curvilinear grid. This scheme has been validate with classical analytical solutions and used to simulate elastic wave propagation in complex geometries. The proposed method is simple and computationally performing.

Introduction

Numerical modeling of wave propagation through irregular interfaces between layers, especially for the case of the sea floor having complex shapes is a fundamental problem in seismology. The methods which allow modeling correctly the boundary effects (e.g. Rayleigh waves on the free surface with complex topography, scattering of elastic waves on fractures in rocks, etc) are of great interest. For such complex geological structures, finite or, more recently, spectral element methods (FEM/SEM) proved to be well adapted [Komatitsch and Vilotte, 1998; Seriani, 1998]; However, these methods are more complex and computationally more expensive than classical staggered finite difference methods. Moreover; in the case of FEM/SEM methods special care should be taken when dealing with fluid-solid interfaces as spurious modes are generated in fluid regions [Komatitsch et al., 2000].

In this article we describe an alternative method for the wave propagation problem. It combines the simplicity of finite difference methods and the flexibility of FEM/SEM to model complex geometries without the need of making any special treatments for the fluid-solid interfaces. The method can be seen as a generalization of the rotated operators finite difference method [Saenger et al., 2000]- well known in the geophysical literature; or, it can be referred as the HEMP method well known in the rock mechanic literature [Wilkins, 1999]. Our purpose is to demonstrate that our finite difference scheme on a curvilinear grid correctly models the propagation of Rayleigh waves and is capable to model the wave propagation through the surface between the liquid environment and an elastic body. For comparison purposes with the SEM method, we reproduced the synthetic tests with the same geometry and the same physical parameters described in the papers [Komatitsch and Vilotte, 1998; Komatitsch et al., 2000] where the authors demonstrated the efficiency of the SEM method.

Method

To construct the numerical scheme, we start from the standard staggered finite difference scheme:

$$(\sigma_{xx})_{i,j}^{n+1} = (\sigma_{xx})_{i,j}^{n} + \tau(\lambda+2\mu)_{i,j}(u_x)_{i,j}^{n} + \tau\lambda_{i.j}(v_y)_{i,j}^{n}$$

$$(\sigma_{yy})_{i,j}^{n+1} = (\sigma_{yy})_{i,j}^{n} + \tau\lambda_{i,j}(u_x)_{i,j}^{n} + \tau(\lambda+2\mu)_{i.j}(v_y)_{i,j}^{n}$$

$$(\sigma_{xy})_{i,j}^{n+1} = (\sigma_{xy})_{i,j}^{n} + \tau\mu_{i,j}(u_y+v_x)_{i,j}^{n}$$

$$u_{i,j}^{n+1} = u_{i,j}^{n} + \frac{\tau}{\rho_{i,j}}(\sigma_{xx,x}+\sigma_{xy,y})_{i,j}^{n+1},$$

$$v_{i,j}^{n+1} = v_{i,j}^{n} + \frac{\tau}{\rho_{i,j}}(\sigma_{xy,x}+\sigma_{yy,y})_{i,j}^{n+1}$$

To define the spatial Cartesian x and y derivatives in the case of curvilinear mesh system, let's consider the new coordinate system (ξ,η) being the direction of diagonals of a convex quadrangular grid cell as shown in (Figure 1).

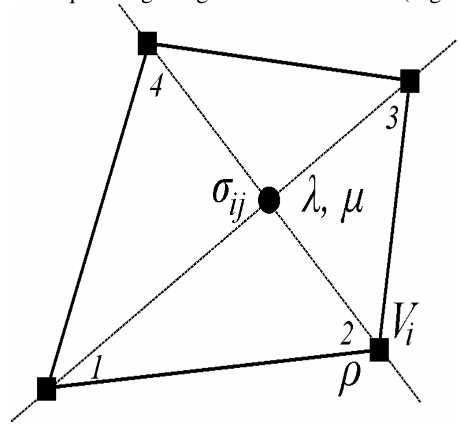


Figure 1 Quadrangular grid cell

Then, the operators of differentiation in the new system have the form $\partial_\xi = \Delta_{24}(x)\partial_x + \Delta_{24}(y)\partial_y$ and $\partial_\eta = \Delta_{13}(x)\partial_x + \Delta_{13}(y)\partial_y$, where

$\Delta_{mn}(f) \equiv (f_m - f_n)/r_{mn}$. r_{mn} is the distance between points m and n. If the velocities in the nodes of a cell are known, their ξ and η derivatives in the center of the cell can be defined as $(u,v)_\xi = \Delta_{24}(u,v)$ and $(u,v)_\eta = \Delta_{13}(u,v)$.

Combining these four equations into a linear system and having solved it relative to x and y derivatives, one obtains the relations:

$$(u,v)_x = \frac{\Delta_{13}(u,v)\Delta_{24}(y) - \Delta_{24}(u,v)\Delta_{13}(y)}{\Delta_{13}(x)\Delta_{24}(y) + \Delta_{13}(y)\Delta_{24}(x)},$$

$$(u,v)_y = \frac{\Delta_{24}(u,v)\Delta_{13}(x) - \Delta_{13}(u,v)\Delta_{24}(x)}{\Delta_{13}(x)\Delta_{24}(y) + \Delta_{13}(y)\Delta_{24}(x)}.$$

Analogously, the relations for the average stress spatial derivative can be derived. For the case of rectangular cells the described method of discretization degenerates to the rotated staggered grid method [Saenger et al., 2000]. At the same time, the scheme can be considered as a particular case of the HEMP scheme used by Wilkins [Wilkins, 1999] for modeling of 3D finite elastic body deformations.

Numerical tests

To check the accuracy of the solution obtained by means of our scheme, we performed three series of numerical tests.

Test #1. First, Lamb's classical problem was considered to check the scheme on an irregular grid. All physical and geometrical parameters of the computational domain for this test case are taken as in [Komatitsch and Vilotte, 1998]. The source represents a Ricker wavelet force, perpendicular to the free surface having a central frequency of 10 Hz. The grid for this problem has a convex-curved form (Figure 2a). We gradually refined the grid at the surface to have more than 80 grid points for the Rayleigh wavelength ($\lambda_R \approx 226$ meters in the case under consideration). Note that in the case of the rotated scheme and therefore in our generalized scheme more points per wavelength are needed than the classical Virieux scheme [Saenger et al., 2000]. The results presented in Figure 2 are obtained with a grid size of 3200×1600. Comparison with analytical solutions (Figure 3) demonstrates less than 1% error for this resolution.

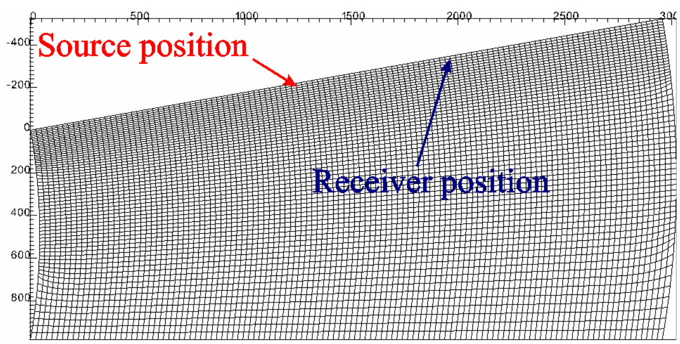
Tests #2 and #3. Second and third tests are performed for the problem of wave propagation through the water – solid interface. All physical parameters are taken the same as in [Komatitsch et al., 2000]. In the second test, two homogeneous half-spaces with an inclined contact interface having a slope of 10 degrees (Figure 4); in the third test the interface is sinusoidal (Figure 5). Both grids essentially thicken in the vicinity of the interface. The source in both problems also represents Ricker wavelet, but of compression type, with the central frequency of 10 Hz. Rigid wall conditions are implemented at the computational domain boundaries.

The main purpose of the test #2 was to make sure that no numerically reflected S-wave propagates in the water domain; as can be seen in Figure 4. Several P-waves of different origins (incident, reflected from the free surface and reflected from the interface) can be observed in water, Stoneley waves are seen at the interface, and transmitted P- and S-waves can be noticed in the solid media.

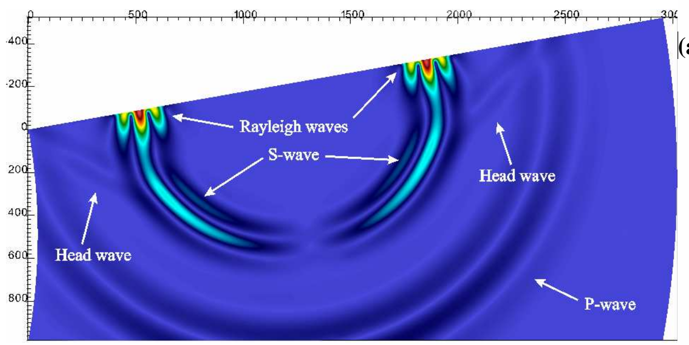
In the third test, the absence of spurious numerical diffraction is mainly demonstrated (Figure 5). Such diffraction is caused in general by the approximation of curvilinear surface by rectangular grids: this is a well known limitation for the classical finite-difference schemes for such cases. As can be seen in Figure 5b, the use of surface-oriented mesh admissible by our scheme allows to avoid this problem. Besides, as in the previous test case, we do not see any spurious S-waves propagating into the water domain.

Conclusions

We have shown that finite difference scheme based on simple generalization of rotated staggered grid method can be used for modeling of elastic/acoustic waves on curvilinear grid. We have validated this scheme with well know analytical solutions and used the same complex geometries as described in the literature to demonstrate that our scheme can be as flexible as finite or spectral element. The main advantage of this method with respect to finite element consists in it simplicity and its computational performance.

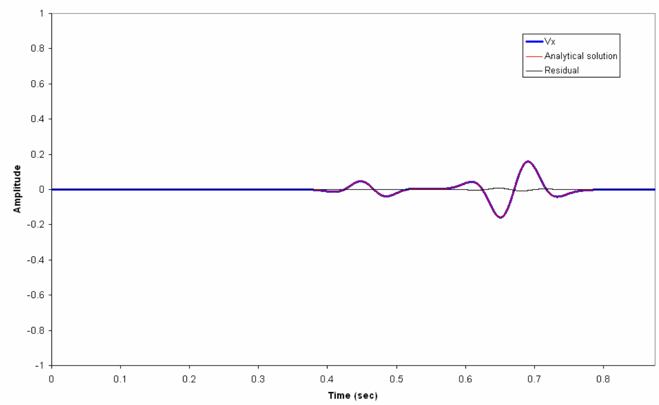


(a)

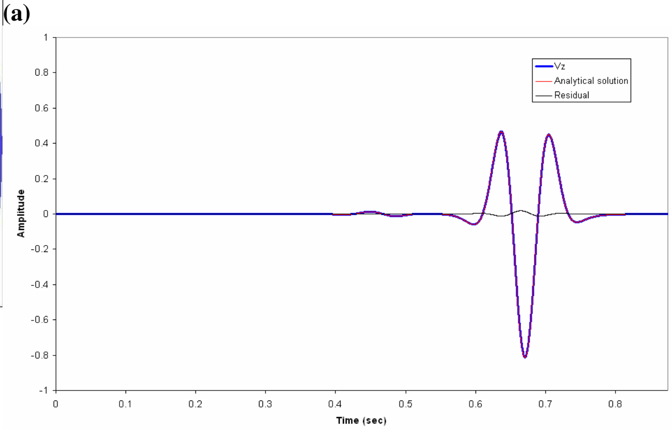


(b)

Figure 2 The grid used for the test #1 (every 10^{th} line is plotted) (a); snapshots of the wavefield (velocity modulus is plotted) at 0.6 sec (b).



(a)



(b)

Figure 3 Seismograms of the horizontal (a) and vertical (b) components of the velocity vector (the blue and red lines correspond to numerical and analytical solution respectively).

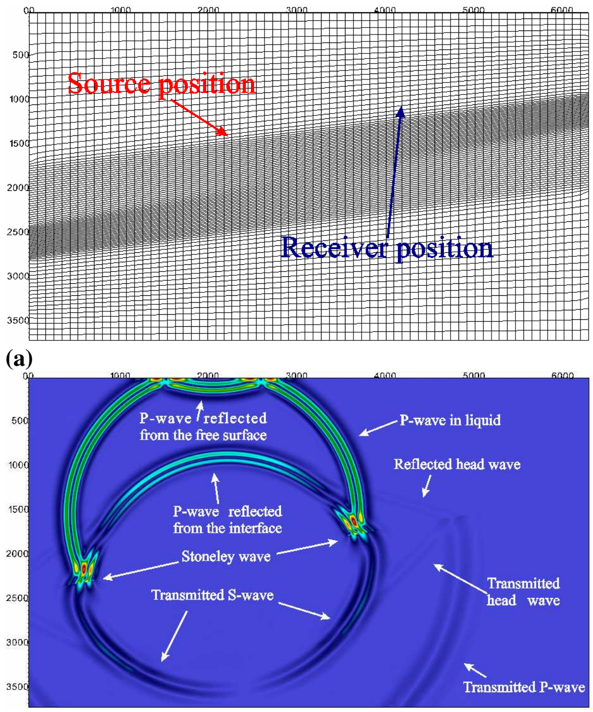


Figure 4: The grid used for the test #2 (every 10th line is plotted) (a); snapshots of the wave field (velocity modulus is plotted) at 1.3 sec (b)

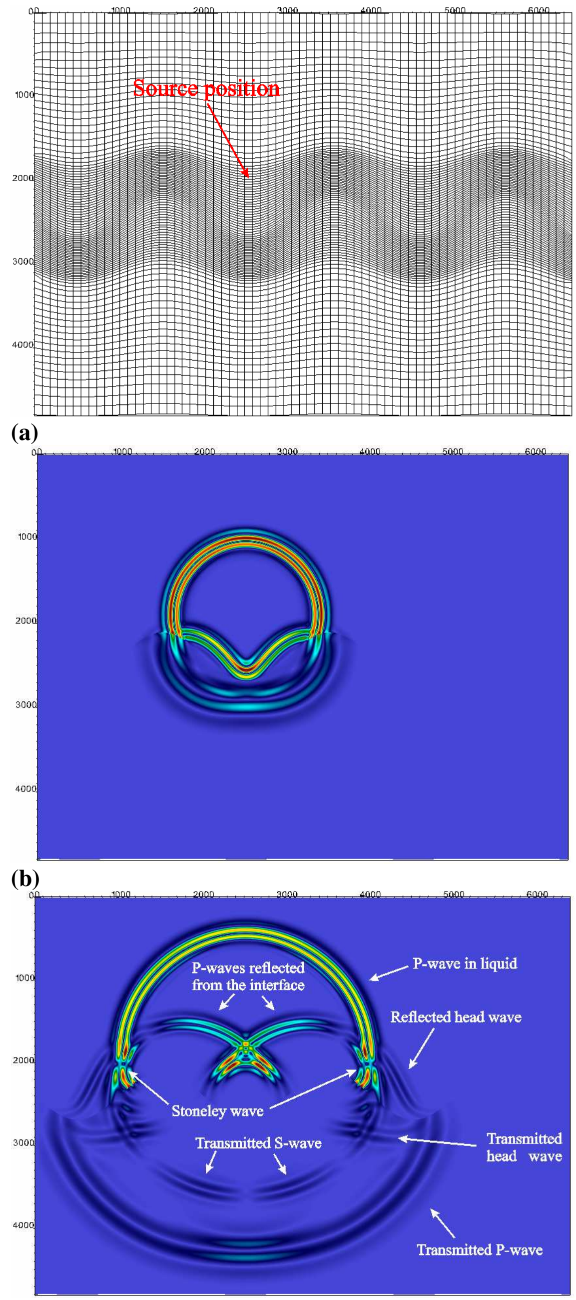


Figure 5: The grid used for the test #3 (every 10th line is plotted) (a); snapshots of the wave field (velocity modulus is plotted) at 0.8 sec (b) and 1.2 sec (c)

EDITED REFERENCES

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