Mathematical Model for Three-Phase Compressible Dual Porosity Streamline Simulation

Abstract

Flow simulation of fractured reservoirs usually is performed using a dual porosity model. The dual porosity system is modeled by using two coupled grids: one for matrix and one for fracture. These two continua communicate by transfer functions. Until now there were no mathematical models of dual porosity 3-phase compressible flow for streamline simulators. To realize this model it was necessary to reformulate the matrix and fracture pressure equations. We have incorporated conventional transfer function as a source/sink term not only in the streamline saturation equations (as it was in incompressible case) but also in the pressure equation.

The dual porosity model has been implemented into a streamline simulator. This tool has its main application in the geological modeling domain for analyzing uncertainty, model ranking and screening of geologically detailed models including fractures.

In this paper we describe the mathematical model for 3-phase compressible dual porosity model for a streamline simulator and compare the results and run times of the streamline-based approach with a conventional dual porosity grid-based commercial simulator. The results from streamline simulator for dual porosity show good agreement with those produced by a commercial finite difference simulator with order of magnitude improvement in simulation time.

Streamline methods as a reservoir simulation tool has found much interest in petroleum engineering because of its ability to calculate fluid flow in multi-million cell geological models with reasonable CPU times. However, important physical properties of geo-scale fluid flow models are still not properly modeled by streamline methods. Enhancing the range of physical properties that can be simulated accurately in a timely manner will enable improved workflows in the geological modeling domain.

Introduction

Streamline methods have been used for fluid flow simulation since the nineteenth century (Helmoltz [10] and later Muskat [19]). Improvements of the method as a reservoir simulator were published by a number of authors (LeBlanc [16], Higgins [11]) but still it was not a general reservoir simulation technology. In 1960-90's the reservoir simulation community mainly focused on the finite difference/finite volume schemes. Still it has several defects: there are many complicated geological multiwell models that require a large number of grid cells. Time step length have to satisfy CFL condition for the explicit case and the convergence of the non-linear equations for the implicit approach may also require short time step length. Simulation of geological scale reservoir models with finite difference/finite volume methods may be impractical due to constraints in available CPU time. Also numerical smearing and grid orientation effects can be observed for this technique.

The streamline method reduces these problems and has been getting increased attention over the past decade (Blunt, Thiele, Batycky, Bratvedt, King, Datta-Gupta and others) [1, 4, 5, 6, 7, 8, 9, 12, 13, 20, 21, 27]. With this method the 3D domain is transferred to many 1D domains for the flow calculations - streamlines, and the fluid flow calculations are done along them. Streamline simulation is considered as an alternative to finite difference/volume simulation. However, there are many tools in the grid-based finite difference reservoir simulators that are not available in streamline based reservoir simulators. One of them is the modeling of fractured reservoirs using a dual porosity model.

Simulation of fractured reservoirs is an interesting and essential task. Flow of fluids through this kind of reservoirs is usually performed as flow through the high-permeability, low-porosity fractures surrounding...
individual high-porosity low-permeability matrix blocks. Those blocks contain the majority of the reservoir pore volume and act as a source or sink terms to the fractures. In other words: dual porosity model represent the reservoir split into two continua’s (matrix and fracture), which only interact by transfer functions. The study of naturally fractured reservoirs started in the middle of 20-th century by Barenblatt, Zheltov and Kochina [2]. Dual porosity model is a subject for numerous papers: [3, 8, 9, 12, 18, 20, 21, 22, 24, 25, 26]. The form of transfer function is not unified and is the subject for discussion (Kazemi [14,15]). It is an empirical parameter. In our work we use a conventional form. Its value depends directly on pressure difference between matrix and fracture continua.

This work is devoted to simulation of dual porosity model within the streamline concept. All previous work considers incompressible two phase flow along the streamlines. We extend the existing models to an immiscible 3-phase compressible model for streamlines. We did not consider vaporized oil and dissolved gas in this work. Several experiments on dual porosity have been done to compare regular grid-based and streamline simulator. A good agreement has been observed. cured reservoirs is an interesting and essential task. Flow of fluids through this kind of reservoirs is usually performed as flow through the high-permeability, low-porosity fractures surrounding individual high-porosity low-permeability matrix blocks. Those blocks contain the majority of the reservoir pore volume and act as a source or sink terms to the fractures. In other words: dual porosity model represent the reservoir split into two continua’s (matrix and fracture), which only interact by transfer functions.

1. Governing equations for 3 phase compressible flow

Let us consider three phase compressible dual porosity flow with conventional transfer functions:

\[
\frac{\partial \phi_i \rho_{a,i} s_{a,i}}{\partial t} + \frac{\partial \rho_{a,i} w_{a,i}}{\partial x} = (-1)^{i+1} \rho_{a} FK_m \lambda_{a,fm} \left( p_{a,m} - p_{a,f} \right), \quad w_{a,i} = -K_i \frac{\partial p_{a,i}}{\partial x} \quad (1.1)
\]

where subscripts \( \alpha = O,G,W \) denotes oil, gas or water phase, \( i = f,m \) denotes fracture and matrix parameters, \( \rho_{a,i} \) and \( p_{a,i} \) are the densities and pressures of each phase within the reservoir, \( F \) is fracture shape factor, \( \lambda_{a,fm} \) - upstream mobilities. Let us introduce the formation volume factors \( B_{a,i} \) of phase by relations

\[
\rho_{a,i} = \frac{\rho_{a}^{stc}}{B_{a,i}}, \quad B_{a,i} = B(p_{a,i})
\]

where \( \rho_{a}^{stc} \) are the densities of each phase at the standard conditions. Let us also assume, that the matrix and fracture porosities depends on the effective average pressure \( \phi = \phi(\bar{p}) \), where \( \bar{p} = \sum s_{a,i} p_{a,i} \) and the capillary pressures defined as

\[
p_{CG,i} = p_{G,i} - p_{O,i} = p_{G,i} - p_i, \quad p_{CW,i} = p_{O,i} - p_{W,i} = p_i - p_{W,i}
\]

The capillary pressures are the given functions on saturation; the phase and rock compressibility

\[
c_{a,i} = -\frac{1}{B_{a,i}} \frac{dB_{a,i}}{dp_{a,i}}, \quad c_{R,i} = \frac{d\phi}{d\bar{p}}
\]

are the given functions of pressure.

Let us restrict ourselves by the consideration of a single permeability case, that is, we assume the phase velocities to be zero in the matrix. The equations (1.1) can be written in the form:
\[ \frac{\partial p_m}{\partial t} = -\frac{1}{\Delta} \left( \Psi_G \Gamma_G + \Gamma_o + \Psi_w \Gamma_w \right) \]  

(1.2)

\[ \frac{\partial s_{G,m}}{\partial t} = G_G \Gamma_G + G_w \Gamma_w - \frac{\delta_G}{\Delta} \left( \Psi_G \Gamma_G + \Gamma_o + \Psi_w \Gamma_w \right) \]  

(1.3)

\[ \frac{\partial s_{w,m}}{\partial t} = W_w \Gamma_G + W_G \Gamma_G - \frac{\delta_w}{\Delta} \left( \Psi_G \Gamma_G + \Gamma_o + \Psi_w \Gamma_w \right) \]  

(1.4)

where \( \Gamma_a = F K_m \lambda_{a,m} \left( p_{a,m} - p_{a,f} \right) \). The system (1.2)-(1.4) is more suitable for the streamline technology since the pressure and saturation equations solution should be solved there separately. The expressions for the coefficients in (1.3) are quite cumbersome in the general case and we will not show them here. In particular case of incompressible matrix they have a simple form:

\[ \Psi_G = \Psi_G^{(0)} \left( 1 + c_{G,m} s_{G,m} \frac{dp_{CG,m}}{ds_{G,m}} \right)^{-1}, \quad \Psi_w = \Psi_w^{(0)} \left( 1 - c_{w,m} s_{W,m} \frac{dp_{CW,m}}{ds_{W,m}} \right)^{-1} \]

(1.5)

For the fracture continuum we accept another simplifying assumption, that is the capillary forces are equal to zero there, so that \( p_{G,f} = p_{o,f} = p_{o,f} = p_f \) and we can write the fracture equations in the form:

\[ \frac{\partial p_f}{\partial t} - \frac{1}{\Delta_f} \frac{\partial}{\partial x} \left( \Theta_f \frac{\partial p_f}{\partial x} \right) + p_f \frac{\partial}{\partial x} \left( \Omega_f \frac{\partial p_f}{\partial x} \right) = \frac{1}{\Delta_f} \sum \Gamma_a \]  

(1.6)

\[ \phi_f \frac{\partial s_{a,f}}{\partial t} + \frac{\delta_{a,f}}{\Delta_f} \frac{\partial}{\partial x} \left( \Theta_f \frac{\partial p_f}{\partial x} \right) - \frac{\partial}{\partial x} \left( \Theta_a \frac{\partial p_f}{\partial x} \right) - \frac{\delta_{a,f}}{\Delta_f} p_f \frac{\partial}{\partial x} \left( \Omega_f \frac{\partial p_f}{\partial x} \right) + p_f \frac{\partial}{\partial x} \left( \Omega_a \frac{\partial p_f}{\partial x} \right) \]  

(1.7)

where the notations are introduced:

\[ \Delta_f = (\phi_f C_{T,f} s_{G,f} + c_{R,f}), \quad \delta_{a,f} = (\phi_f c_{a,f} + c_{R,f}) s_{a,f} \]

\[ \Omega_T = K \sum \lambda_a c_{a,f}, \quad \Omega_a = K \lambda_a c_{a,f} \]  

(1.8)

\[ \lambda_T = K \sum \lambda_a, \quad \lambda_a = K \frac{k_a}{\mu_a} \]

\[ \Theta_T = \lambda_T + \Omega_T p_f, \quad \Theta_a = \lambda_a + \Omega_a p_f \]

and \( C_{T,i} = \sum c_{a,i} s_{a,i} \).
2. Computational model

Since we consider the model for the streamline simulator, we present here the computational model for the 1D displacement problem only. In the finite difference form the equation (1.2) can be written in the form:

\[
\left( p_n \right)_i^{n+1} = \left( p_n \right)_i^n + \tau \left( \Lambda_{fn} \right)_i^n \left( p_f \right)_i^{n+1} - \Phi_i^n \left( 1 + \tau \left( \Lambda_{fn} \right)_i^n \right)
\]

(1.9)

where

\[
\Phi = FK_m \Delta_m \frac{\Psi G \lambda_{G,fn} p_{CG,fn} - \Psi W \lambda_{W,fn} p_{CW,fn}}{\Delta_m}, \quad \Lambda_{fn} = \frac{FK_m \left( \Psi G \lambda_{G,fn} + \lambda_{O,fn} + \Psi W \lambda_{W,fn} \right)}{\Delta_m}
\]

The meaning of the superscript \( p \) will be clarified below, the subscript \( i \) will be omitted from now. Taking into account (1.9), the fracture pressure equation (1.6) can now be written in a three diagonal form:

\[
a_i \left( p_f \right)_{i-1}^n + b_i \left( p_f \right)_i^n + c_i \left( p_f \right)_{i+1}^n = d_i
\]

(1.10)

where

\[
a_i = -\frac{\tau}{\Delta_f h^2} \left( \left( \Theta \right)_i^{n+1} - p_f \left( \Omega \right)_i^{n+1} \right)
\]

\[
b_i = 1 + \frac{\tau \Lambda_{ij}}{\left( 1 + \tau \Lambda_{ij} \right)} + a_i + c_i
\]

\[
d_i = p_f^n + \frac{\tau \Lambda_{ij} p_m^n}{\left( 1 + \tau \Lambda_{ij} \right)} + \tau \Lambda_{ij} p^n - \tau \Lambda_{ij} p^{n+1} - \frac{\tau^2 \Lambda_{ij}^2 \Phi_i^n}{\left( 1 + \tau \Lambda_{ij} \right)}
\]

And

\[
\Lambda_{ij} = \frac{FK_m \left( \Psi G \lambda_{G,fn} + \lambda_{O,fn} + \Psi W \lambda_{W,fn} \right)}{\Delta_f}, \quad \Lambda_{ij} = \frac{FK_m \lambda_{az,fn}}{\Delta_f}
\]

After new values of both pressures are obtained, the exchange terms can be recalculated and the saturations can be updated. The values with half-integer subscripts in the above equations as well as in the finite difference analogues of the fracture saturation equation were evaluated in the upwind technique. Finally, the value of superscript \( p \) was typically taken \( p = n \). We also carried out a series of simulation where the nonlinearities were treated iteratively.

The discovered in these calculations practical coincidence of the iterative results with the simplest case \( p = n \) suggested us an idea to further significantly simplify the computational model. In fact, if we neglect the matrix compressibility or suppose that the matrix porosity depends on the oil pressure in matrix only, and, in addition, neglect the capillary pressure time derivatives, the set of governing equations (1.1) can be rewritten in the form:

\[
\frac{\partial}{\partial t} \left( \phi_m s_{az,m} \right) = \frac{\Gamma_{az}}{B_{az,m}}
\]

(1.12)
\[
\left( \phi_i C_{i,m} + c_{R,i} \right) \frac{\partial p_i}{\partial t} = -\sum \Gamma_{\alpha} \tag{1.13}
\]

\[
\frac{\partial}{\partial t} \left( \phi_i s_{\alpha,f} \right) + \frac{\partial}{\partial x} \frac{w_{\alpha,f}}{B_{\alpha,f}} = \frac{\Gamma_{\alpha}}{B_{\alpha,f}} \tag{1.14}
\]

\[
\left( \phi_i s_{\alpha,i} C_{i,f} + c_{R,f} \right) \frac{\partial p_f}{\partial t} + \sum B_{\alpha,f} \frac{\partial}{\partial x} \frac{w_{\alpha,f}}{B_{\alpha,f}} = \sum \Gamma_{\alpha} \tag{1.15}
\]

The set of pressure equations is solved in the described above way but with much more simple coefficients. However, instead of updating the saturation directly, we can update now the values of \( \phi_i s_{\alpha,i} / B_{\alpha,i} \), determine new values of porosity and the formation volume factor as the given functions of new pressures, and then to obtain the saturation. Although the basic assumptions of this procedure are not strictly physically proved, we found it to be acceptable since its accuracy is good enough.

Figure 1. Average reservoir pressure with time
Figure 2. Field gas production rate with time

Figure 3. Field oil production rate with time
Figure 4. Field water cut with time
3. Simulation Comparisons

To perform the simulation using the described above model one should specify the relative permeability and capillary pressure for oil and water in the matrix, the relative permeability for gas in the fracture, the gas relative permeability and capillary pressure for matrix and fracture, the PVT data for oil, water and gas and the compressibility of the phases and of the porosity. The initial fluid distribution and the production and injector well rates should be set as well. Since we goal here just to validate the results from the developed streamline simulator, we refer for all the specifications to the paper [16] and just note that the presented here results were obtained for the representative set of the listed above parameters. The validation was performed by comparison with a commercial finite difference simulator [30]. The red solid lines in Figures 1-4 are the results from the streamline implementation and the blue dotted line is the result from the commercial finite difference simulator.

Figure 1 shows exact agreement between the two simulators for the average pressure. Figure 2 shows very good agreement between the two simulators for the gas production rate with time. The difference between the simulators in total gas production at the end of the simulation is less than 0.3% of the total produced gas. Figure 3 show the oil production rate with time. Again the main features of the rate are in good agreement between the simulators, although the streamline simulator may exhibit slightly less dispersive behavior than the finite difference simulator for this coarse grid model. This may be observed in the two figures where the gas production from the initial gas saturation causes a drop at 20 days and at the onset of the water injection at 150 days. The figure 4 shows the water cut with time and there is good agreement between the simulators even though the streamline simulator has a slightly later water breakthrough and higher cut at the end.

Conclusions

A three-phase compressible dual porosity model has been developed for a streamline simulation technology and implemented into such a simulator. This model has been compared with the results produced by a commercial finite difference simulator and good comparisons have been demonstrated. It has also been demonstrated that the good scaling abilities associated with a streamline simulator also is valid for a three-phase compressible dual porosity simulation.

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