Dynamic multi-criteria problem: method to solving

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We study a dynamic model that describes the process of transition of a controlled object from the initial state $x_0 \in \mathbb{R}^n$ at the time $t_0$ in the terminal state $x(t_1) = x_*^1$ at $t_1$. For example, a group of people united by a common goal to implement a joint project can act as a object of control. The dynamics of the process is described by a controlled system of linear equations

$$\frac{d}{dt} x(t) = D(t)x(t) + B(t)u(t), \quad t_0 \leq t \leq t_1, \quad x(t_0) = x_0,$$

$$x(t_1) = x_*^1 \in X_1 \subseteq \mathbb{R}^n, \quad u(\cdot) \in U,$$

where $x_*^1$ is a component of the solution to the problem of multicriteria equilibrium:

$$\langle \lambda^*, f(x_1^*) \rangle \in \text{Min}\{\langle \lambda^*, f(x_1) \rangle \mid x_1 \in X_1\},$$

$$\langle \lambda - \lambda^*, f(x_1^*) - \lambda^* \rangle \leq 0, \quad \lambda \geq 0.$$

Here $f(x_1) = (f_1(x_1), f_2(x_1), \ldots, f_m(x_1))$ is a vector criterion; $f_i(x_1), \ i = 1, 2, \ldots, m,$ are convex scalar functions. The boundary value problem (3), (4) is a two-person game with Nash equilibrium. The solution of (1)-(4) is the set $(\lambda^*; x_*^1, x^*(\cdot), u^*(\cdot))$. Specifically, we are looking for a control $u^*(\cdot) \in U$ such that the right end of the trajectory $x^*(\cdot)$ coincides with the component $x_*^1$ of boundary value problem solution.

For the problem (1)-(4) we introduce a function analogous to the Lagrange function in convex programming problems:

$$\mathcal{L}(\lambda, \psi(t); x_1, x(t), u(t)) =$$

$$= \langle \lambda, f(x_1) - \frac{1}{2} \lambda \rangle + \int_{t_0}^{t_1} \langle \psi(t), D(t)x(t) + B(t)u(t) - \frac{d}{dt} x(t) \rangle dt,$$

$$1$$
which is defined for all \((\lambda, \psi(\cdot)) \in \mathbb{R}_+^n \times \Psi_2^2[t_0, t_1], (x_1, x(\cdot), u(\cdot)) \in X_1 \times AC^n[t_0, t_1] \times U\). Related approaches were considered in [1-2]. It is shown that the saddle point of the function (5) is a solution of (1)-(4).

To solve the problem, we use the dual extraproximal method [3]:

\[
\bar{\lambda}^k = \text{argmin} \left\{ \frac{1}{2} \lambda - \lambda^k | \lambda, f(x_1^k) - \frac{1}{2} \lambda \right\},
\]

\[
\bar{\psi}^k(t) = \psi^k(t) + \alpha \left( D(t)x^k(t) + B(t)u^k(t) - \frac{d}{dt}x^k(t) \right),
\]

\[
(x_1^{k+1}, x^{k+1}(\cdot), u^{k+1}(\cdot)) = \text{argmin} \left\{ \frac{1}{2} |x_1 - x_1^k|^2 + \alpha \langle \bar{\lambda}^k, f(x_1) - \frac{1}{2} \bar{\lambda}^k \rangle 
\]

\[
+ \frac{1}{2} \|x(t) - x^k(t)\|^2 + \frac{1}{2} \|u(t) - u^k(t)\|^2
\]

\[
+ \alpha \int_{t_0}^{t_1} \langle \bar{\psi}^k(t), D(t)x(t) + B(t)u(t) - \frac{d}{dt}x(t) \rangle dt \right\},
\]

\[
\lambda^{k+1} = \text{argmin} \left\{ \frac{1}{2} \lambda - \lambda^k | \lambda, f(x_1^{k+1}) - \frac{1}{2} \lambda \right\},
\]

\[
\psi^{k+1}(t) = \psi^k(t) + \alpha \left( D(t)x^{k+1}(t) + B(t)u^{k+1}(t) - \frac{d}{dt}x^{k+1}(t) \right), \alpha > 0.
\]

The theorem on the convergence of the saddle-point method to the solution of the problem was proved.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-01-06045-a), and the Program for Support of Leading Scientific Schools (project no. NSh-4640.2014.1.)

REFERENCES