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3D Reconnection of Random Magnetic Fields

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ABSTRACT

The problem of three-dimensional magnetic reconnection in random fields is of importance both in astrophysics and fusion science. As is known, a crucial ingredient of reconnection are the null points of the magnetic field, where the condition of the magnetic-flux freezing becomes broken. In the 2D approximation, the null points have a universal topology of X-type, while much more diverse configurations are possible in the 3D situation. The case that was most studied before is the axially-symmetric fan-like structure (which is called also the "proper radial null"), i.e. "collision" of two oppositely-directed magnetic fluxes with subsequent outflow in the equatorial plane. On the other hand, the configurations with a finite number of the fan "vanes" (or the "improper radial nulls") were usually assumed to be the specific particular cases. However, the probability of occurrence of the various configurations was never calculated in a systematic way.

The aim of this report is to present a self-consistent evaluation of the above-mentioned probabilities in the potential field approximation. The basic results can be formulated as follows:

(i) The most likely case of the 3D reconnection (i.e., occurring with the dominant probability) is the six-tail structure, in which the magnetic field lines come out of the null point in 6 asymptotic mutually-orthogonal directions.
(ii) The axially-symmetric fan-type structure (with infinite number of vanes) is admissible but emerges with a very small probability.
(iii) The generic six-tail configuration possesses 4 "dominant" and 2 "recessive" asymptotic directions and, thereby, is approximately reduced at the sufficiently large scales to the well-known 2D structure of X-type. So, the specific 3D effects should manifest themselves, first of all, in the small-scale reconnection events.

Introduction

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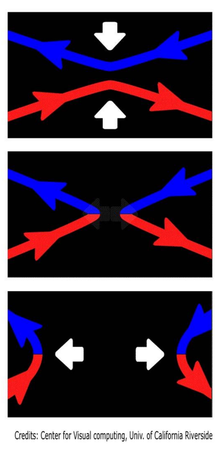

Magnetic reconnection is the process of changing topology of the magnetic field lines.

As a result of reconnection, the magnetic field usually tends to the state with less energy; while the excessive energy is released near the spot of reconnection for heating the plasma, acceleration of the charged particles, *etc*.

A necessary prerequisite of the magnetic reconnection is presence of the **null point** (*i.e.*, the point where all components of the magnetic field vanish), because it is necessary to break down the theorem of uniqueness of the magnetic field lines.

Magnetic reconnection was usually studied in the 2D approximation.

As will be shown later, 3D reconnection possesses some specific features, which are important, first of all, at the sufficiently small scales.

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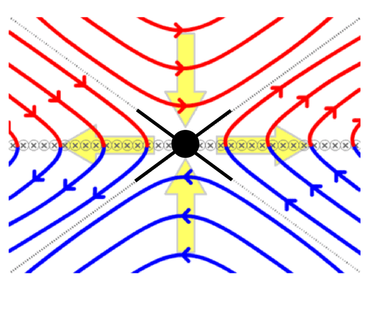
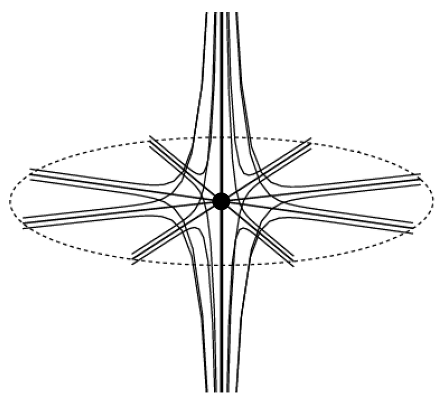
- A key ingredient in the development of the magnetic reconnection process are null points of the magnetic field.

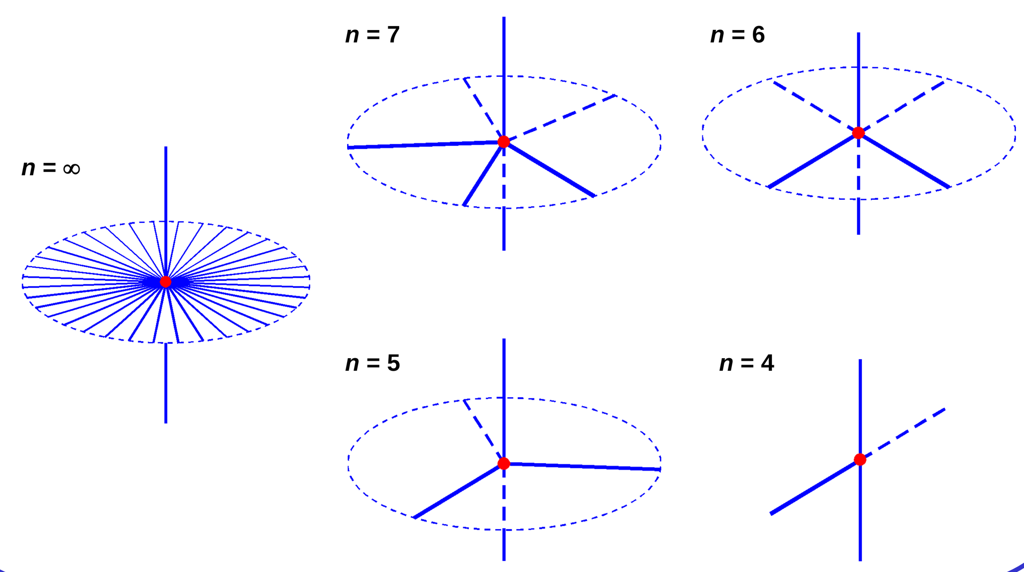




- In **2D geometry**, there is only one kind of the null points responsible for the magnetic reconnection; these are the null points of **X-type**.
- In **3D geometry**, there may be much more diverse types of the null points, *e.g.*, the widely-discussed **fan-type** null points.

3

- **What are other admissible kinds of the 3D null points?**
- **Which of them are realized most frequently in a random magnetic field?**
(In particular, are there null points with a finite number of asymptotic tails?)



Method of analysis

4

(in the potential field approximation)

$$\mathbf{B} = -\mathrm{grad}\,\psi$$

$$\psi(r,\theta,\varphi) = \sum_{j=0}^{\infty}\sum_{m=0}^{j} r^j\, P_j^m(\cos\theta)\left[a_{jm}\cos(m\varphi) + b_{jm}\sin(m\varphi)\right]$$

- If a_{jm} and b_{jm} are the random quantities, then this formula represents a random magnetic field.
- For the sake of simplicity, we shall restrict our consideration by the first N terms of the expansion. Therefore, the random field will be realized in the N-dimensional parametric space.
- The magnetic field intensity is expressed through the potential as

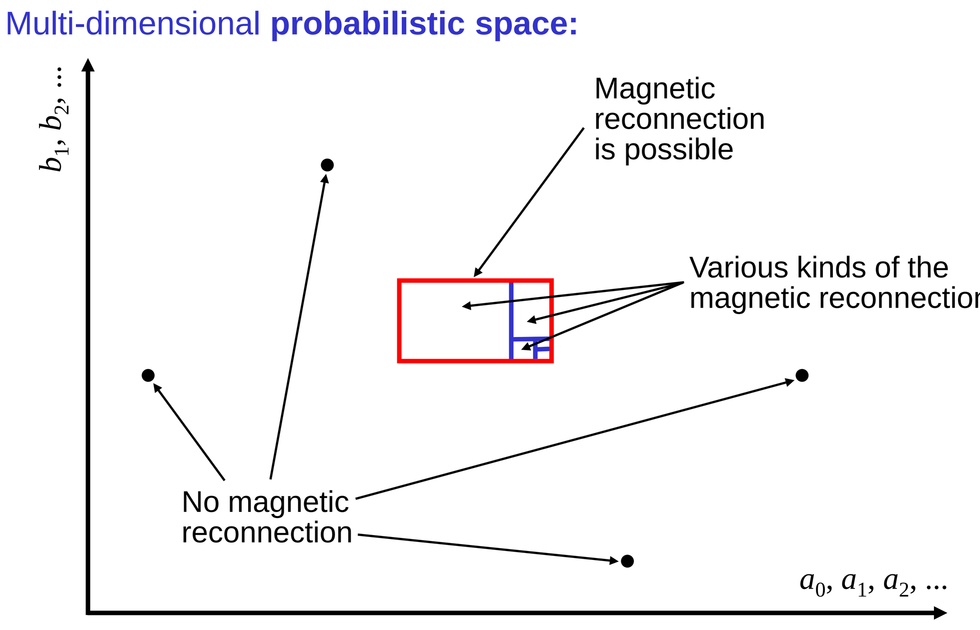
$$B_r = -\frac{\partial\psi}{\partial r}\,, \quad B_\theta = -\frac{1}{r}\frac{\partial\psi}{\partial\theta}\,, \quad B_\phi = -\frac{1}{r\,\sin\theta}\frac{\partial\psi}{\partial\varphi}$$

- The coefficient a_{00} is, of course, arbitrary.

Basic idea of our statistical estimates

5

Multi-dimensional **probabilistic space:**



6

- In the 1st order with respect to r, we get:

$$B_r^{(1)} = -\left\{a_{10}\cos\theta - (1-\cos^2\theta)^{1/2}[a_{11}\cos\varphi + b_{11}\sin\varphi]\right\}$$

For a null point of the magnetic field to take place at $r = 0$, it is necessary that $a_{10} = a_{11} = b_{11} = 0$.

- Therefore, any null point can be realized only in the subset of coefficients with dimensionality not greater than $N-3$.
- In the 2nd order with respect to r, we get:

$$B_r^{(2)} = -2r\left\{\frac{1}{2}(3\cos^2\theta - 1) - \frac{3}{2}\sin(2\theta)\left[a_1\cos\varphi + b_1\sin\varphi\right] + 3\sin^2\theta\left[a_2\cos(2\varphi) + b_2\sin(2\varphi)\right]\right\}$$

$$B_\theta^{(2)} = -3r\left\{\sin(2\theta)\left[-\frac{1}{2} + a_2\cos(2\varphi) + b_2\sin(2\varphi)\right] - \cos(2\theta)\left[a_1\cos\varphi + b_1\sin\varphi\right]\right\}$$

$$B_\varphi^{(2)} = -3r\left\{2\sin\theta\left[-a_2\sin(2\varphi) + b_2\cos(2\varphi)\right] + \cos\theta\left[a_1\sin\varphi - b_1\cos\varphi\right]\right\}$$

For conciseness, we introduced the following designations:

$$a_1 = a_{21}/a_{20}\,, \quad b_1 = b_{21}/a_{20}\,, \quad a_2 = a_{22}/a_{20}\,, \quad b_3 = b_{22}/a_{20}$$

7

- In general, the magnetic-field-line equation takes the form:

$$\frac{dr}{B_r} = \frac{r\,d\theta}{B_\theta} = \frac{r\,\sin\theta\,d\varphi}{B_\varphi}$$

It can be also rewritten as

$$\begin{cases} r\,\dfrac{d\theta}{dr} = \dfrac{(B_\theta/r)}{(B_r/r)} \\ r\,\sin\theta\,\dfrac{d\varphi}{dr} = \dfrac{(B_\varphi/r)}{(B_r/r)} \end{cases}$$

- In particular, the field line passing immediately through the null point (*i.e.*, the origin of coordinates) should satisfy the following condition:

$$\begin{cases} (B_\theta^{(2)}/r) = 0 \\ (B_\varphi^{(2)}/r) = 0 \end{cases}$$

After the substitution of the magnetic field components, we get **a set of two algebraic equations**, defining the asymptotic directions.

8

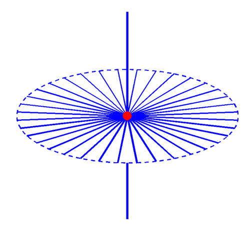
$$\begin{cases} \sin(2\theta^*)\left[-\frac{1}{2} + a_2\cos(2\varphi^*) + b_2\sin(2\varphi^*)\right] - \cos(2\theta^*)\left[a_1\cos\varphi^* + b_1\sin\varphi^*\right] = 0 \\ 2\sin\theta^*\left[-a_2\sin(2\varphi^*) + b_2\cos(2\varphi^*)\right] + \cos\theta^*\left[a_1\sin\varphi^* - b_1\cos\varphi^*\right] = 0 \end{cases}$$

- **The fan-type structure** (with infinite number of vanes) appears, in particular, when $a_1 = a_2 = b_1 = b_2 = 0$, since the above set of equations is reduced to the condition:

$$\sin(2\theta^*) = 0\,,$$

which has two solutions:

$$\begin{cases} \theta^* = 0,\ \pi: \text{ axis of the “fan”,} \\ \theta^* = \pi/2,\ \varphi^* \text{ is arbitrary: vanes of the “fan”.} \end{cases}$$



- This geometric structure will be preserved also after rotation in space trough two Euler angles.
- Therefore, the **fan-type structure** is realized in the subspace of spherical harmonic coefficients with dimensionality

***N* – 5**

9

$$\begin{cases} \sin(2\theta^*)\left[-\frac{1}{2} + a_2\cos(2\varphi^*) + b_2\sin(2\varphi^*)\right] - \cos(2\theta^*)\left[a_1\cos\varphi^* + b_1\sin\varphi^*\right] = 0 \\ 2\sin\theta^*\left[-a_2\sin(2\varphi^*) + b_2\cos(2\varphi^*)\right] + \cos\theta^*\left[a_1\sin\varphi^* - b_1\cos\varphi^*\right] = 0 \end{cases}$$

- Yet another important property of this set of equations is its invariance under the transformation:

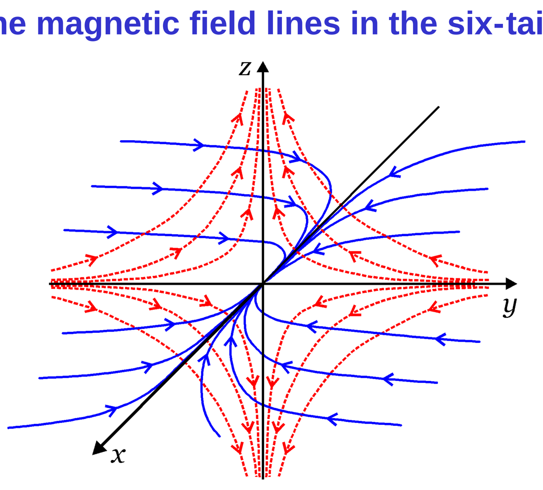
$$\begin{cases} \theta^* \rightarrow \pi - \theta^* \\ \varphi^* \rightarrow \varphi^* + \pi \end{cases}$$

- The magnetic field lines enter the null point (or leave it) as oppositely directed pairs. Therefore, the structures with an odd number of tails are prohibited.
- **In the most general case**, the system of two equations with two unknown variables should have a finite number of solutions (*i.e.*, a discrete set of magnetic field lines should pass through the null point in the real physical space). As follows from a more careful analysis, this number is equal to 6.
- Consequently, the most general type of topology of the 3D null point (in the probabilistic sense) is a **six-tail structure**. It is realized in the subspace of coefficients with dimensionality

***N* – 3**

Structure of the magnetic field lines in the six-tail configuration:

10



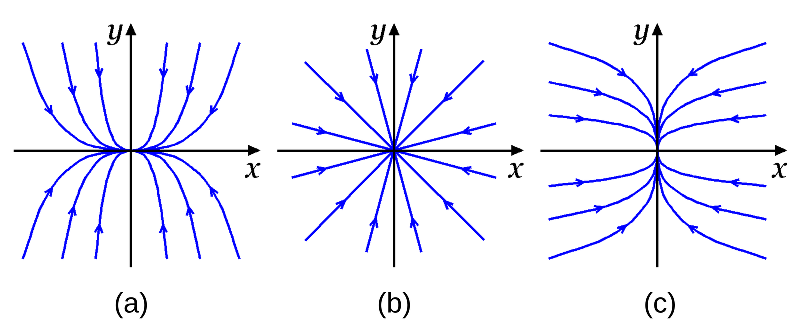
- Blue color shows field lines in the horizontal plane (xy); they have the **node-type structure.** Red color shows field lines in the vertical plane (yz); they have the **saddle-type structure.**
- There are two **dominant** directions (y and z) and one **recessive** direction (x).
- **If the “small-scale features” are not taken into account, the six-tail configuration is approximately reduced to the quasi-two-dimensional structure with the well-known topology of X-type.**

Pictorial illustration

11

why probability of occurrence of the six-tail structure should be much greater than for the fan-type structure

(configuration of the magnetic field lines in the vertical planes remains always of the same saddle type)



- There are infinitely many configurations of types (a) and (c), but only one “intermediate” configuration of type (b).

Summary:

12