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Remarks on Quark Stars.

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Some time ago we have drawn attention to the necessity of admitting the existence of a quarkian core in the interior of hypothetical superstars of very great masses (1). Similar considerations were proposed also by PACINI and developed by DE SABBATA *et al.* (2) and by BURBIDGE. We propose now to investigate some properties of a hypothetical quarkian substance. The fundamental role is played by the reaction

$$\mathrm{B} \leftrightarrows \mathrm{q}_1 + \mathrm{q}_2 + \mathrm{q}_3 \,. \tag{1}$$

Among baryons and quarks a deep potential well is needed to secure a great mass defect in baryon building; we draw attention to our previous evaluation of the interaction constants between quarks obtained in the nonlinear quark theory of mesons (3), *i.e.*

$$\begin{cases} (q_1 \pi q_1) = (q_2 \pi q_2) = 2g \,, & (q_3 \pi q_3) = 0 \,, \\ (q_1 \eta q_1) = (2/\sqrt{3})\, g \,, & (q_1 K^* q_2) = 2\sqrt{2}\, g, \text{ etc.} \end{cases} \tag{2}$$

For definiteness we admit for the quark masses

$$\begin{cases} x_1 \approx x_2 \approx 2.5\, M_\mathrm{B} \,, \quad x_3 = \alpha_3 M_\mathrm{B} \,, \quad \alpha_3 \approx 10 \,, \\ \alpha = \alpha_1 + \alpha_2 + \alpha_3 \approx 15 \,, \end{cases} \tag{3}$$

and for magnetic moments, also taking the values from our previous investigation (3),

$$\left| \begin{aligned} \mu_{\mathrm{q}_1} &= \left\{ \frac{2}{3} \frac{M_\mathrm{B}}{x_1} + 1.63 \right\} \mu_\mathrm{B} \,, \\ \mu_{\mathrm{q}_3} &= -\left\{ \frac{1}{3} \frac{M_\mathrm{B}}{x_3} - 0.64 \right\} \mu_\mathrm{B} \,. \end{aligned} \right. \tag{4}$$

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(1) D. IVANENKO and D. E. KURDGELAIDZE: *Astrophysics* (Russ.) vol. 1, No. 6 (1965).
(2) D. BOCCALETTI, V. DE SABBATA and C. GUALDI: *Nuovo Cimento*, 45 A, 513 (1966).
(3) D. KURDGELAIDZE and A. BASSIUNI: *Journ. Nucl. Phys.*, 8, 154 (1968) and subsequent papers.

It is not excluded that the quarkian substance can exist as a primordial state from which afterwards baryons and nuclei were formed; on the other hand a quarkian substance can arise as a consequence of gravitational contraction of a stellarlike object. Developing the conception of a neutron star AMBARZUMIAN and SAAKIAN treated baryonic stars; at the particle density $n \approx 10^{40}\,\text{cm}^{-3}$ the main component is represented by Ξ-hyperons. For definiteness we admit for a baryonic star the average masses of particles

$$M'_B \approx \gamma M_B\,, \qquad (\gamma \sim 2 \div 3)\,. \tag{5}$$

In the baryon-quarkian star all particles are ultrarelativistic. The critical density of the transition of a B-star to a Q-star would be

$$n_c \geqslant 2(\gamma\alpha)^3 \cdot 10^{38}\ \text{cm}^{-3} \geqslant 6 \cdot 10^{42}\ \text{cm}^{-3} \tag{6}$$

and the corresponding Fermi temperature for the degenerate B-gas is

$$T_F \approx 2 \cdot 10^{12}\ {}^\circ\text{K}\,. \tag{7}$$

The equilibrium density is given by relation (ε_F = Fermi energy)

$$\varepsilon_F(\text{B}) = 2\varepsilon_F(\text{q}_1) + \varepsilon_F(\text{q}_3)\,. \tag{8}$$

We have

$$\varepsilon_F(\text{B}) \approx 3\varepsilon_F(\text{q}_1)\,, \qquad \varepsilon_B(\text{q}) \approx cp_F = c(3\pi\ h^3 n_q)^{\frac{1}{3}} \tag{9}$$

and the Fermi temperature

$$T_F(\text{B}) \approx 3T_F(\text{q}_1)\,. \tag{10}$$

So at $n \geqslant 10^{43}\,\text{cm}^{-3}$ a B-star always will contain quark admixture. Taking, for simplicity, the baryons as neutral we have a system similar to a ionized gas where neutral baryons play the part of atoms in ordinary plasma and positive and negative quarks the role of electrons and ions. As the two quarks have identical mass but different charges $(2e/3, -e/3)$ one can introduce an effective quasi-particle

$$q' = q_1 + q_2\,, \qquad x' = x_1 + x_2 \approx 5\,M_B\,, \qquad Q' = +e/3\,. \tag{11}$$

So we get the quark plasma built from two components:

$$Q^+ = \frac{e}{3}\,, \quad Q^- = -\frac{e}{3}\,, \quad \frac{x^+}{x^-} = \frac{x_1 + x_2}{x_3} \approx \left(\frac{5}{\alpha_3}\right) < 1\,, \qquad \left(\frac{x^+}{x} \approx \frac{1}{2}\right). \tag{12}$$

One can consider also the quarkian regions in stars, independently from gravitational contraction, as a prestellar state. One may be inclined to apply to this system magnetohydrodynamics and plasma theory. One has the characteristic plasma frequencies

$$\omega_+^2 = 4\pi ne\ /9x^+\,, \qquad \omega^- \simeq (3/2)\omega_{q^+}^2\,, \tag{13}$$

and the cyclic frequencies

$$\Omega_+ = \frac{eH_0}{3cx^+}, \qquad \Omega_- \approx \frac{1}{2}\Omega_+ \qquad (H_0 \text{ magnetic field}), \tag{14}$$

and the Alfvén velocity

$$u_A = \frac{\omega}{k} = \frac{H_0}{\sqrt{4\pi\varrho}} \qquad (\varrho = n(x^+ + x^-),\ \omega \ll \Omega). \tag{15}$$

In a more detailed treatment one must consider a three-component plasma. We have for the equations of motion and for the equation of magnetohydrodynamics of the cold plasma

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{i\varrho}[jH], \qquad \varrho = n\sum_1^3 x_\alpha. \tag{16}$$

Then the Maxwell equations give the Alfvén velocity. Introducing velocities and currents

$$j_\mu = en(z_1v_1 + z_3v_3), \qquad v_\mu^1 = \frac{x_1v_1 + x_3v_3}{x_1 + x_3}, \text{ etc.}, \tag{17}$$

we obtain

$$v_\mu = \tfrac{1}{2}(v_\mu^1 + v_\mu^2 + v_\mu^3), \qquad j_\mu = \tfrac{1}{2}(j_\mu^1 + j_\mu^2 + j_\mu^3). \tag{18}$$

The previous equations can be written in the form $(\alpha = 1, \alpha = 3)$

$$\frac{\mathrm{d}v_\mu^1}{\mathrm{d}t} = \frac{z_1 + z_3}{x_1 + x_3}ez + \frac{[j_\mu^1 H]}{en(x_1 + x_3)}, \qquad \Omega_1 = e\frac{Hz_1}{cx_1}, \text{ etc.} \tag{19}$$

Let us draw attention to the possibility of superconductivity in stellar matter as was pointed out by Kothari and Auluck (4) and recently considered by Trubnikov (5) for the special case of white dwarfs. Indeed one has in such objects as necessary conditions for the superconductivity state:

1) the presence of a degenerate Fermi gas of electrons,

2) the presence of a gas of phonons (crystal oscillation, plasma oscillation, etc.)'

3) a density satisfying certain conditions ($T_c > T$).

In white dwarfs ($T \sim 10^6$ °K, $n \approx 10^{30}$ cm^{-3}) one may expect the existence of superconductivity. Our proposal is to investigate the superconductivity of neutron, baryon and quark stellar objects. The essential point is the condition $T_c > T$ (T_c critical temperature of transition).

For preliminary rough considerations, the case of a quarkian star suggested by us and previously by Pacini seems to be rather favourable. For ultrarelativistic Fermi gas the temperature is proportional to $\sim n^{\frac{1}{3}}$, but the critical temperature for transi-

(4) D. S. Kothari and F. C. Auluck: *Proc. Nat. Inst. Sci. India*, **28** A, 228 (1962).
(5) B. A. Trubnikov: *Žurn. Èksp. Theor. Fiz.*, **55**, 1893 (1968).

tion to the superconductive state is proportional to $\sim n^{\frac{1}{3}}$, so we see that at increasing density one must expect with some certainty that

$$T_c \gg T_F .$$

On the other hand we have

$$T_c = (1/6.5)\, T_D \tag{20}$$

(T_D Debye temperature).

For ultrarelativistic gas one has

$$T_D = T_F (2^{\frac{2}{3}} p_F / 3xc)^{\frac{1}{2}} , \qquad T_c / T_F \gg 1 , \tag{21}$$

if $p_F/xc \gg 1.6 \cdot 10^2$, $p_F = (3\pi^2 h^3 n)^{\frac{1}{3}}$, $n \gg (2\alpha)^3 \cdot 10^{48}\ \text{cm}^{-3}$.

The critical magnetic field is given by

$$H_c = \frac{2.6}{\sqrt{x}}\, h n^{\frac{2}{3}} \qquad (x = \alpha M_B) ,$$

at n given by (21)

$$H_c \sim 1.5\, \alpha^2 \cdot 10^{24}\ \text{gauss} . \tag{22}$$

The ratio of gas pressure to magnetic field pressure is $\beta = p_g/(H^2/8\pi) \sim 8.6\, xc/p_F$, so that one has $\beta < 1/18$.

One sees that for ultrarelativistic quarks gas pressure will not be of great importance, so that at the great density $n > \alpha^3 \cdot 10^{49}\ \text{cm}^{-3}$, the transition to the superconducting state is not opposed by the magnetic field. Due to the sudden expulsion of the magnetic field, one can expect some kind of explosion which can play a role in the explosions of galactic nuclei or play some part in the evolution of quark-stars. One can guess that in stellar objects, where both baryons and quarks are present, the transition to the superconducting state can take place at a density of baryons

$$n_B > 3\alpha^2 \cdot 10^{50}\ \text{cm}^{-3} . \tag{23}$$

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It was a pleasure to have the possibility to discuss some of the results considered with Profs. D. S. Kothari and F. C. Auluck and we may use this occasion to express our thanks for the hospitality extended to one of us in India.