Доклады Болгарской академии наук
Comptes rendus de l'Académie bulgare des Sciences
Tome 34, № 9, 1981

PHYSIQUE
Physique théorique

RELATIVITY PRINCIPLE AND EQUIVALENCE PRINCIPLE IN THE GAUGE GRAVITATION THEORY

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(Submitted by Academician H. Hristov on March 24, 1981)

The multitude of proposed gauge gravitation theories shows that the gauge principle alone is too broad to describe the gravity in an unambiguous way.

At the same time general relativity (GR) still remains the most satisfactory gravitation theory, although its not yet definitely solved problems of energy, reference systems, singularities and some others should not be forgotten [1].

We therefore consider it reasonable to assume that the gauge gravitation theory, like GR should also be based on the relativity and equivalence principles formulated in gauge theory terms by using the fiber bundle formalism.

In this formalism a gravitation field on an orientable spacetime manifold X^4 is gefined as a global section $g(x)$ of the fiber bundle $\wedge$ of pseudo-Euclidean bilinear forms in tangent spaces over X^4. $\wedge$ is associated with the tangent bundle $T(X^4)$ possessing the structure group $GL^+(4, R)$. $\wedge$ is isomorphic to the fiber bundle in quotient spaces $GL^+(4, R)/L$, where L is the Lorentz group. We mark it Σ. Its global section $h(x)$ describes a gravitation field in the tetrad form ($g_{\mu\nu}(x) = h^a_\mu(x)h^b_\nu(x)\,\eta_{ab}$, η_{ab} is the constant Minkowski metric field), where tetrad fields $h(x)$ are determined up to local Lorentz transformations $h(x)=h(x)L(x)$, taken on the right hand side.

I. Relativity Principle. In GR the relativity principle (RP) is usually formulated as the requirement for matter field (or test particle) equations to conserve their form whatever the changes of reference frames.

In fiber bundle terms a reference frame in the gravitation theory may be defined as the choice of a certain atlas $\Psi = \{U_i, \psi_i\}$ of the tangent bundle $T(X^4)$ ($\{U_i\}$ and $\{\psi_i\}$ are consequently ranges and homeomorphisms of trivializations of $T(X^4)$). But the group of all reference frame changes is the gauge group $GL^+(4, R)(X^4)$ of sections of the principal bundle associated with $T(X^4)$.

This definition is close to that used in the tetrad formulation of GR. If an atlas of $T(X^4)$ is chosen, tetrads $\{t_x\}_i = \psi_i^{-1}(x)\{t\}$ ($\{t\}$ is the basic repere the typical fiber R^4) are erected in every point of the space-time manifold X^4 and their transformations accompany the changes of reference frames.

The conventional (general covariant) form of GR corresponds to the special case of holonomic transformations of reference frames, when the choice of an atlas of the bundle $T(X^4)$ correlates $\Psi = \{U_i, \psi_i = d\varphi_i\}$ with coordinate atlas $\Psi_X = \{U_i, \varphi_i\}$ of the manifold X^4 and this correlation is kept steady under frame changes.

Thus in the fiber bundle formalism RP in the gravitation theory may be formulated as the requirement of the covariance of matter field equations undee the gauge group $GL^{+}(4, R)(X)$. In this from the relativity principle proves to be identical with the gauge principle of the gauge theory of external symmetry group $GL^{+}(4, R)$, and consequently the gravitation theory can be built from RP directly as the gauge theory.

However, the $GL^{+}(4, R)$-gauge theory turns out to be broader than the general conception of the gravitation theory. For example, it does not distinguish the Minkowski metric forms from other possible types in tangent spaces. That is why the equivalence principle in the gravitation theory also has to be considered.

2 **Equivalence Principle.** In GR the equivalence principle (EP) supplements RP and expresses the transition to special relativity in a certain reference frame.

In geometric terms special relativity may be defined as the geometry of Lorentz invariants (in the spirit of Klein's Erlanger program). Then EP in the gravitation gauge theory may be formulated as the requirement to conserve Lorantz invariants under transitions from map to map in some reference frames and under parallel translations. This means that the connections on $T(X^4)$ being $GL^{+}(4, R)$-gange fields have to be reduced to the Lorentz gauge fields in some reference frames, and this leads to the contraction of the structure group $GL^{+}(4, R)$ of $T(X^4)$ to the Lorentz group.

This contraction is necessary and sufficient for a global section of the quotient bundle Σ, and hence of the metric bundle $\wedge$, to exist. The existence of a tetrad or metric gravitation field anywhere on X^4 results from EP.

In turn the presence of a gravitation field entails the usual postulates of the equivalence principle in GR. Thus there is a holonomic reference frame, where the gravitation metric field becomes of the Minkowski type and its Christoffel symbols vanish in a space-time point, although, since the Lorentz gauge fields contain also torsion components, the whole connection in general case does not go to zero in this frame. But let us recall that there also exists a reference frame, where the whole connection vanishes in a point, yet the gravitation tetrad field may be present.

EP in the gauge gravitation theory defines a certain Klein-Chern geometry of invariants on the total spaces and on the sheave of sections of associated with $T(X^4)$ bundles. The contraction of the structure group $GL^{+}(4, R)$ of $T(X^4)$ to the Lorentz group and consequently to its maximal compact subgroup $SO(3)$ means the existence of atlases of $T(X^4)$ such that every Lorentz or $SO(3)$ invariant does not change under transition from map to map of these atlases. For example, the $(3+1)$-decomposition proves to be possible in all points of X^4.

This enables us to interpret the geometrical aspects of the gravity in the spirit of Klein's Erlanger programme, although Fock, Bondi, Havas and some other authors deny the presence of any symmetries in the gravitation theory.

3. **Gravity as a Field of the Goldstone Type.** It should be emphasized that the equivalence principle formulated in the gravitation gauge theory permits to view the gravitation field as a field of the Goldstone type in the gauge theory of external symmetries [2,3].

In a gauge theory Goldstone and Higgs fields are known to appear when ever the symmetry is spontaneously broken. In that case the structure group G of a certain vector fiber bundle λ, whose sections $\{\varphi\}$ describe some multiplet of matter fields, is contracted to its subgroup H. A global constant section $\varphi_0(X) = V_0$ of λ then exists, where v_0 is a non-zero H-fixed point of the typical fiber V of λ. This field is usually interpreted as the vacuum or ground state and then minor fluctuations $\{q\}$ near it are considered.

These fluctuations take the known form $q=q_0+q_H+\sigma$, where values of q_H lie in the H-invariant subspace of V and $(q_0+\sigma)$ are sections of the bundle in quotient spaces G/H associated with λ. Then σ prove to be Goldstone fields and (q_0+q_H) are called conventionally Higgs fields in the gauge theory with spontaneous symmetry breaking. However, in gauge theories of internal symmetries Goldstone fields σ can be removed by a certain gauge transformation.

In the gauge gravitation theory the contraction of the structure group $GL^+(4, R)$ of $T(X^4)$ to L resulting from EP describes a situation, which is analogous to spontaneous symmetry breaking. It entails the existence of a global section of the quotient bundle Σ with the typical fiber $GL^+(4, R)/L$, and the single L-fixed point of this fiber space is the Minkowski metric. Then, by analogy with the case of the spontaneous breaking of internal symmetries, one may imagine the Minkowski metric field as possessing the sense of the Higgs field q_0, but the small deformations from it will play the role of Goldstone fields σ.

These deformations are identified with the presence of the gravitation field, which therefore may be considered as a field of the Goldstone type. But here, in contrast to Goldstone fields of internal symmetries, the gravitation field cannot be removed by any gauge, because the gauge transformations of external ymmetries also act on operators of partial derivatives, which are vectors, $\left\{\partial_\mu=\frac{\partial}{\partial x^\mu}\right\}$ of tangent spaces. But these vectors retain the sense of derivatives only in holonomic frames. In nonholonomic frames $\left\{\partial_a=h_a^\mu\frac{\partial}{\partial x^\mu}\right\}$, and consequenty under arbitrary gauge transformations, not all tetrad coefficients are concealed in the regauge connection, as in the internal symmetry case, i. e. the gravitation field remains.

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