Application of Viscous Vortex Domains Method for Solving Flow-Structure Problems

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Abstract

Non-iterative method has been developed for solving the coupled problem of flow-structure interaction. For simulation of an incompressible flow the two-dimensional numerical method of viscous vortex domains (VVD) based on Navier-Stokes equations has been used [1], [2]. The body motion equations are incorporated into the system of fluid dynamics equations, that allows to solve the coupled problem without iterations. In present study this method is applied for simulating oscillations of one and two pendulums in a fluid under gravity.

Keywords: *flow-structure interaction, Navier-Stokes equations, physical pendulums, damped oscillations*

1 Problem statement

Problem of physical pendulum in viscous medium is being solved for the demonstration of the VVD method abilities. Physical pendulum, represented by the 10% thick plate, is "pivoted" at one end (point O) and free to rotate around it without friction (see fig. 1). The problem being solved is two-dimensional, so the span of the plate is infinite and all dimensional values are implied for one unit of span. The pendulum is affected by gravity and hydrodynamic forces. Since the rotation axis is fixed, the pendulum motion is governed by the second Newton's law:

$$J\ddot{\alpha} = M + M_g,$$

where M is the moment of hydrodynamic forces about the point O, and J is the moment of inertia of plate about O.

The torque of gravity force M_g equals to

$$M_g = mgh\sin\alpha,$$

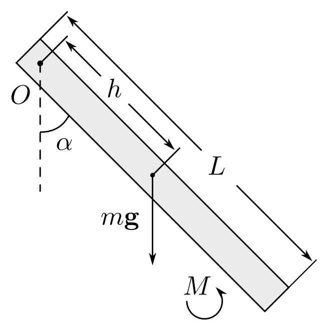

Figure 1. Schematic of the pendulum.

where h is the distance between center of mass and suspension point O, $m = (\rho - \rho_0)S$, where ρ and ρ_0 are the densities of the plate and the fluid respectively, S is the cross-sectional area of the pendulum.

The fluid resistance force is being calculated directly without using any phenomenological model and for this purpose the Navier-Stokes equation is being solved.

Union yields

$$\begin{cases} J\ddot{\alpha} = -mgh\sin(\alpha) + M \\ \dfrac{d\mathbf{V}}{dt} = -\dfrac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{V} \\ \nabla\mathbf{V} = 0 \\ M = -\mathbf{e}_z \cdot \oint_C [\mathbf{r} \times (-P\mathbf{n})]dl \\ \mathbf{P} = 2\rho_0\nu\mathbf{W} - pE \end{cases} \tag{1}$$

where $\mathbf{V}$ is the fluid velocity, ν — kinematic viscosity. Last two equations in system 1 represent the moment of the hydrodynamic forces M, which is integral of pressure p and tangent forces $\mathbf{W}$ over the plate surface.

On the plate surface the no-slip condition is set, and at infinity — the no-perturbations condition $\mathbf{V} = 0$. Initially the pendulum and the medium stated at rest. The plate initially is deviated 90°.

2 Method of solving

For solving this problem the method of viscous vortex domains (VVD) is used [1], [2]. The VVD method is a vortical (mesh-free) method for the viscous incompressible flow simulation in Lagrange coordinates. It is based on the theorem following from the Navier-Stokes equations that circulation of the velocity over some contour in a viscous fluid is conserved if the points of the contour move with velocity $\mathbf{u}$ [3]:

$$\mathbf{u} = \mathbf{V} + \mathbf{V}_d; \quad \mathbf{V}_d = -\nu \nabla \mathbf{\Omega}/\Omega; \quad \mathbf{\Omega} = [\nabla \times \mathbf{V}], \tag{2}$$

where $\mathbf{V}$ is fluid velocity, $\mathbf{V}_d$ — diffusive velocity, ν — kinematic viscosity of fluid.

The method VVD is similar to the Diffusion Velocity method [3]. The two methods differ in discrete formulas that are used for calculating the diffusion velocity. Those of the VVD method are well-founded and include no arbitrary parameters, they allow to simulate more accurately the vorticity evolution than one does in the manner of [3], especially near surfaces. The VVD method describes properly the boundary layer, and allows to calculate the friction force at the body surfaces.

Basing on the above-mentioned theorem, flow region with non-zero circulation is presented with number of domains (small regions with finite volumes), which move with velocity $\mathbf{u}$ and so their circulation remains constant. The actual boundaries of every domain are not tracked, but coordinates of the only tracking point in every domain are saved. The domain's circulation is known either from boundary conditions or from initial conditions. At every time step new domains are generated on the bodies surfaces and the system of linear algebraic equations is being solved to find circulations of that domains and to satisfy no-slip condition.

Each body is represented with contour with N_k segments, k - the number of the body, $k = 1, 2, ..., N_{\text{body}}$. New domains are generated at the nodes of the contours at each time step. Their circulations must be calculated to satisfy the boundary conditions. There are N unknown circulations g_i^{new}, $N = \sum_k N_k$, and N_{body} unknown angular velocities ω_k at each time step. For all segments of all the bodies we write down the equations of impenetrability conditions

$$\sum_{i=1}^{N} \mathbf{n} \cdot \mathbf{v}_{n_k}(\mathbf{r}_i, g_i^{\text{new}}) + \sum_{k'} \beta_{n_k,k'} \omega_{k'} = -\sum_{j=1}^{N_{\text{old}}} \mathbf{n} \cdot \mathbf{v}_{n_k}(\mathbf{r}_j, g_j), \quad n_k = 1, ..., N_k, \quad k' = 1, ..., N_{\text{body}} \tag{3}$$

where

$$\beta_{n_k,k'} = \mathbf{n} \cdot \left(\mathbf{w}_{n_k,k'} - \delta_{k,k'} [\mathbf{e}_z \times (\mathbf{r}^*_{n_k} - \mathbf{r}_O)] \right),$$

$\mathbf{v}_{n_k}(\mathbf{r}, g)$ is the velocity, induced on n_k-th segment of the k-th body by the domain located at the point $\mathbf{r}$ with circulation g. To calculate the induced velocity the Bio-Savart law is used. In our scheme $\mathbf{v}_{n_k}(\mathbf{r}, g)$ is calculated as an average velocity induced over n_k-th segment, $\mathbf{r}^*_{n_k}$ is the center of the n_k-th segment, $\mathbf{r}_O$ is a suspension point.

In dependence of the numerical scheme these equations for closed contour can be approximately linear. In our numerical scheme they are strongly linearly dependent, therefore one of the equations for each body is excluded, and additional condition is imposed on the amount of circulations $\sum\limits_{i=i_{k_1}}^{i_{k_2}} g_i^{\text{new}} + 2S_k(\omega - \omega^{\text{old}}) = 0$ for the contour of every body, where i_{k_1} and i_{k_2} are the first and last segment numbers of k-th body contour, S_k is its cross section area.

The no-slip conditions are satisfied automatically as there are no attached vortices at the surface due to the diffusion velocity.

Since the coupled problem is being solved and bodies angular velocities are unknown we include bodies dynamics equations for each body into the system. For k-th body it is as follows

$$J_k \dot{\omega}_k = M_k + m_k [(\mathbf{r}_{\text{com},k} - \mathbf{r}_{O,k}) \times \mathbf{g}] \tag{4}$$

where $\dot{\omega}_k = (\omega_k - \omega_k^{\text{old}})/\Delta t$ is the angular acceleration of a body and ω_k^{old} is angular speed at the previous time step, $r_{\text{com},k}$ is the center of mass, and M_k is the hydrodynamic force moment.

In [1] the following expression of a moment of hydrodynamical forces has been derived.

$$\frac{M_k}{\rho_0} = \frac{1}{\rho_0} J_k \dot{\omega}_k + \frac{1}{2\Delta t} \sum_{i=i_{k_1}}^{i_{k_2}} g_i^{\text{new}} \cdot (\mathbf{r}_i - \mathbf{r}_O)^2 - \frac{1}{2} \sum_{i=i_{k_1}}^{i_{k_2}} \left((\mathbf{r}_i - \mathbf{r}_O) \cdot \Delta \mathbf{l}_i \right) \cdot \left(\omega^{\text{old}} [\mathbf{e}_z \times (\mathbf{r}_i - \mathbf{r}_O)] \right)^2$$

It is remarkable that the fluid torque depends on unknown circulations linearly. Substituting these expressions into (4) and adding to the equations (3) we obtain the closed system of linear equations. Solving this linear system at every time step we obtain all unknown angular velocities and circulations of the new domains without iterations. Moving the bodies at the calculated velocities, and all the vortices at the velocity $\mathbf{u}$ according to (2) we obtain the solution of the coupled equations of bodies dynamics and the Navier-Stokes equations.

3 Results

The results of computational experiments are shown on fig 2, 3. Fig 2 represents the angle of pendulum deviation on time. One can see the well-known damped oscillations dependency. But with the closer look one can see several differences: fig. 3 shows period of each oscillation depending on its amplitude. Experimental data is plotted with points. One can see the period to diminish with amplitude decrease. This fact is explained by analysis of the following equation for high-amplitude oscillations:

$$J\ddot{\alpha} = -mgh\sin\alpha \quad (5)$$

The equation (5) is not linear, but it still can be solved analytically. One can obtain, that the period of oscillations at large amplitude equals to

$$T(A) = 4\sqrt{\frac{J}{mgh}} \cdot K\left(\sin\frac{A}{2}\right), \quad K(k) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}. \quad (6)$$

$T_0 = \lim_{A\to 0} T(A) = 2\pi\sqrt{J/mgh}$ is the harmonic oscillations period. The dependency (6) has been confirmed experimentally in [4]. The comparison of analytical solution (6) with results of VVD experiment shows good agreement for the first 4 points (the beginning of the experiment). The later oscillations period differ from the theoretical line up to 3%. The difference is caused by the fact, that VVD method solves Navier-Stokes equations directly without using empirical models and parameters, and fluid flow, which is developed during simulation, affects pendulum.

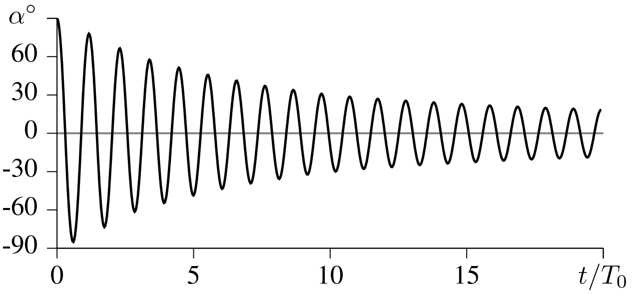

Figure 2. Deviation angle of plate looks similar to the harmonic oscillations.

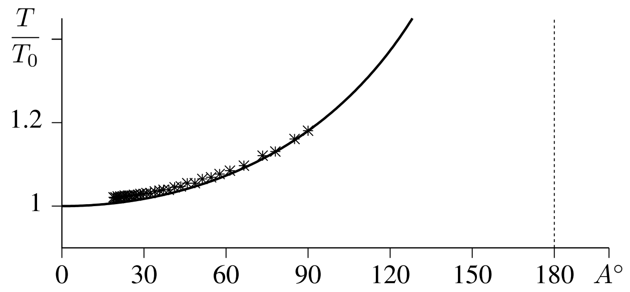

Figure 3. Oscillations period. Analytical solution (6) (solid lime) is compared to computational experiment by VVD method (points).

Using information about deviation and angular speed the energy of pendulum has been calculated at every time moment

$$E(t) = mgh\sin\alpha(t) + \frac{J\dot{\alpha}^2(t)}{2}. \quad (7)$$

The dependency of energy on time is shown on fig. 4. Due to medium influence the pendulum energy does not remain constant. It is transferred to fluid, where fraction of it remains in a form of vortices, and other fraction dissipates into heat. The damping of a physical pendulum has been investigated in [5] on a metre stick. That study has shown, that real-world damping depends not only on pendulum velocity, but on the square of velocity too.

$$J\ddot{\alpha} + mgh\sin\alpha + c_1\dot{\alpha} + c_2\dot{\alpha}|\dot{\alpha}| = 0 \quad (8)$$

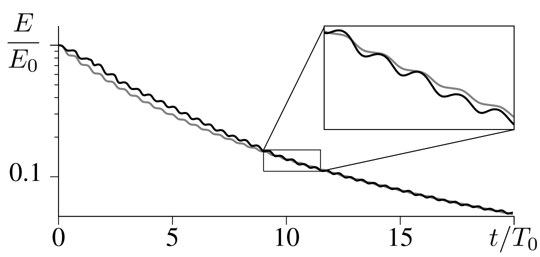

Figure 4. Total energy of plate. While average decrease is rather monotonous, closer look shows the presence of recuperation.

The article [5] and present study has got significant distinction in subjected body: a metre stick has been considered in that paper, while in the present study we consider two-dimensional plate. According to damping model, proposed in [5], resistance coefficients $c_1 = 0.006$ and $c_2 = 0.038$ were chosen basing on the VVD experimental data. The obtained model curve is shown on fig. 4 in gray. Despite similarity of computational and phenomenological curves in general, zoom region indicates an important distinction: energy in computational experiment is not monotonous. In other words the phenomenon of recuperation takes place. In the beginning of motion the coefficient of recuperation is about 5% and it grows with time, reaching value $\sim 25\%$ in the end of experiment.

4 Interference of two pendulums

The interference of two pendulums has been modeled. In viscous fluid two plates are placed one above another. Each pendulum is 10% thick and can swing without friction around suspension point. The pendulum above is 500 times denser than fluid, density of the bottom plate is being varied so that ratio of masses $\bar{m} = m_b/m_t$ changes from 0.02 to 1, where index b indicates bottom pendulum and t — top. The ratio of lengths of two pendulums $\bar{l} = l_b/l_t$ is varied from 0.25 to 2. The gap between two pendulums is fixed and equals to the top plate thickness $0.1l_t$. The dimensionless parameter of the problem is $\mathrm{Re} = \sqrt{gl_t^3}/\nu = 1000$. Initially the pendulum above is deviated $90°$ and the plate below is in equilibrium state. Both two pendulums and fluid at the beginning of the experiment are at rest. There is no coupling between plates. The motion of bottom pendulum is the result of fluid-structure interaction, which in its turn is caused by top pendulum motion.

The result of numerical simulation is depicted of figures 5, 6, 7. The fig. 5 shows the interference of pendulums of the same length. Since the oscillations period is proportional to the square root of pendulum length and hence coincide for both plates there is only one frequency visible in the angles plot. The top pendulum occurred to be low-sensitive to the variance of bottom pendulum properties. Amplitude of the upper plate at the end of experiment altered $\pm 5\%$ only. The bottom pendulum behavior turned out to be much more complicated. Since it is initially at rest and not connected to the top pendulum, deviations of bottom plate indicate pure fluid-structure interaction. One can distinguish the difference between "light" and "heavy" plates behavior. For mass ratio 0.02 and 0.1 amplitude of the bottom pendulum is not monotonous (fig. 5a, b), while cases of the heavy pendulum with ratio 0.4 and 1 indicate steady increase in lower plate amplitude. This increase is not infinite, but the moment it starts to diminish is not fit in fig. 5c, d. The oscillations of top and bottom pendulums are not synchronous, there is a quarter-period phase shift, which is caused by specific displacement of vortices: the vortex, generated by the upper pendulum motion (black region on the right side of bottom plate on fig. 5c), contains the low pressure region and acts on a bottom pendulum only during the top plate is on the right (or left). In other words the fluid adds a delay in pendulums interference.

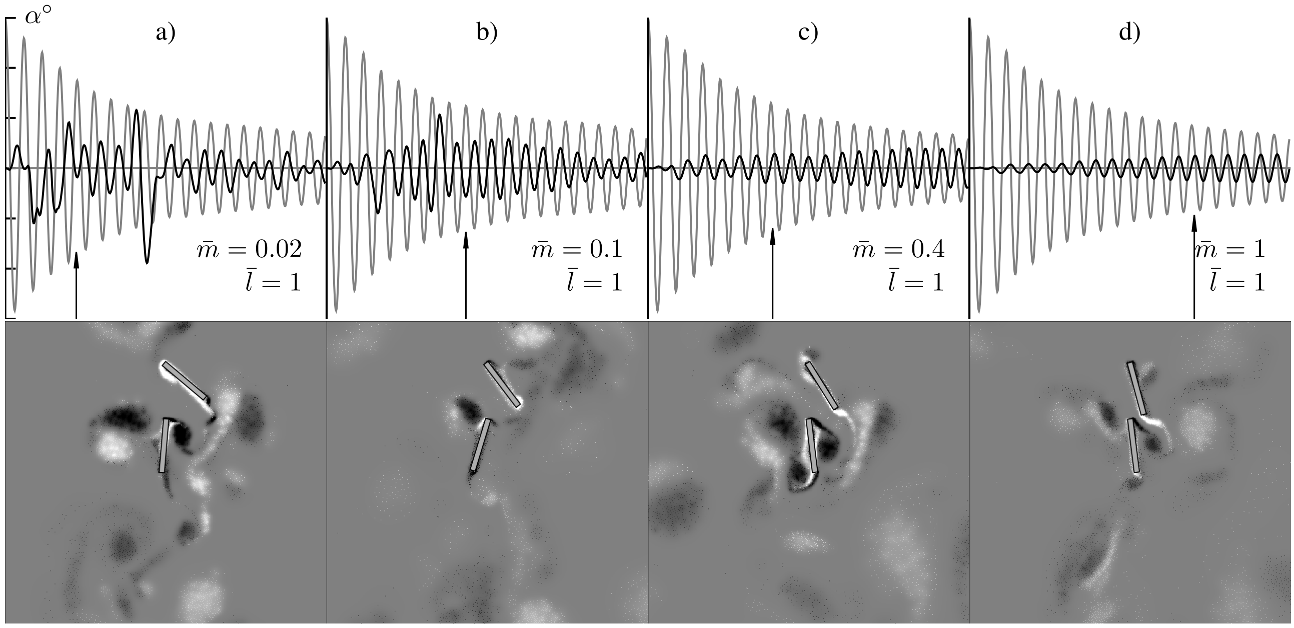

Figure 5. Oscillations of two pendulums of the same length and various mass ratios. Top pendulum, initially deviated $90°$ induces oscillations of bottom pendulum by fluid interaction. Upper row — deviation angle of top (gray) and bottom (black) pendulums. Lower row — corresponding vorticity field. Light regions are for positive values (counter-clockwise rotation) and dark is for negative. Arrows indicate moment they were taken at.

"Light" pendulums (mass ratio 0.02 and 0.1) of various lengths (fig. 6) indicate stochastic behavior. Due to small mass they are strongly affected by vortices generated by the top plate. Especially it is observable for pendulum of length $\bar{l} = 0.25$ (fig. 6a, b): small length also increases its sensitivity to the fluid flow because vortices size in general exceeds the pendulum size. The fundamental frequency of bottom pendulum now differs from the top one. Neglecting the amplitude influence, one can write down frequencies ratio as $\bar{f} = f_b/f_t = \sqrt{l_t/l_b} = \sqrt{1/\bar{l}}$. Both frequencies are observable on the angles plot. Despite two pendulums are not coupled directly, the frequency of vortices generation equals to the top plate

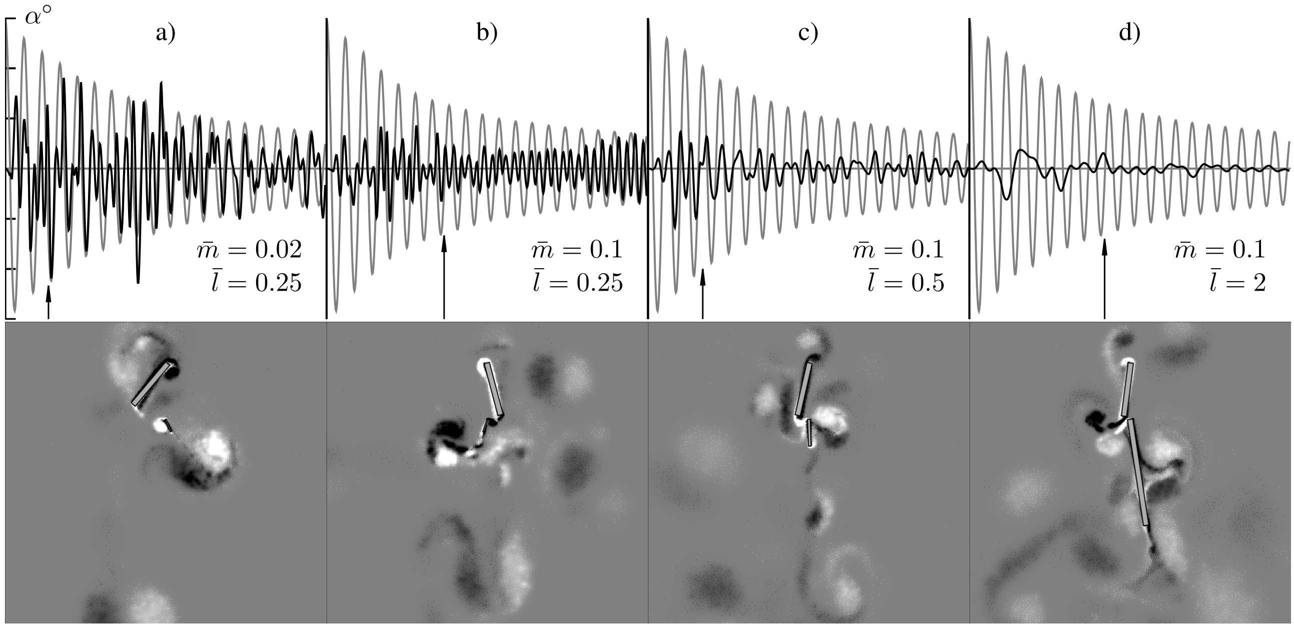

Figure 6. Oscillations of two pendulums. The case of "light" pendulum below.

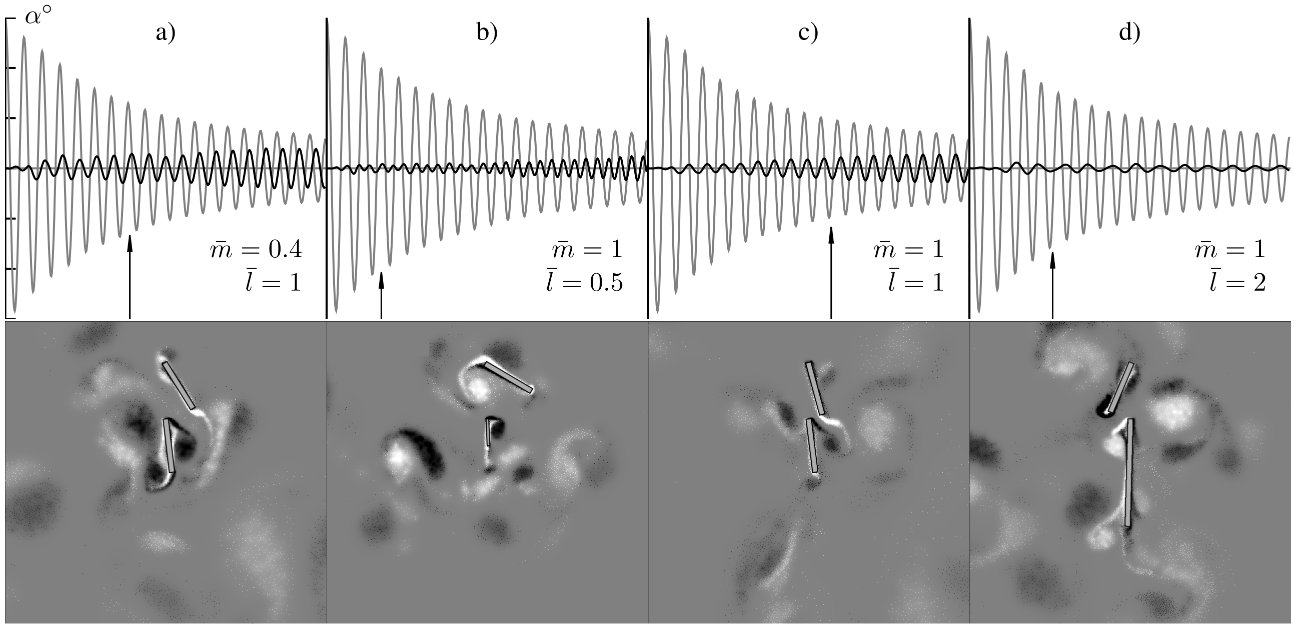

Figure 7. Oscillations of two pendulums. The case of "heavy" pendulum below.

oscillations frequency and therefore it appears in bottom plate motion.

Interference of pendulums in case, when the lower one is "heavy" enough ($\bar{m} = 0.4$, 1) looks more regular. In all these cases bottom pendulum oscillates at his own frequency even when it does not coincide with the frequency of top one.

5 Conclusions

It is shown that the presented method is effective for solving the coupled problems of flow-structure interaction of several bodies and fluid. Application of the method is demonstrated on oscillations of one and two pendulums in viscous fluid under the action of gravity. Studying of the damping oscillations of one pendulum indicated a good agreement between the period of high-amplitude oscillations with the precise analytical solution. It has been shown that the time dependence of the deviation angle can be approximately described by a model in which the resistance of the speed is the sum of linear and quadratic functions, however, a full coincidence can not be achieved, since the phenomenon of energy recuperation takes place and the fluid returns the pendulum a part of energy and resistance coefficient becomes negative.

Studying the oscillations of two pendulums interacting through a fluid motion showed that this interaction is complex nonlinear and can not generally be simulated with simplified models but only as solving of flow-structure problem.

This work is carried out with financial support from the Russian Foundation for Basic Research (project No 12-01-00985a).

All computations are performed on Lomonosov supercomputer `http://parallel.ru/`.

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