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Goldstone Type (Non-Poincaré) Supergravity

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The gauge description of gravity as Higgs-Goldstone type fields is extended on supergravity.

Following the Poincaré gauge version of gravity, contemporary supergravity models are connected with the super (graded) Poincaré group. Most of these models are based on gauge coordinate supertranslations in the spirit of Kibble's gravity approach,[1] although graded affine bundles come into play, too.[2] However, Poincaré gauge versions of gravity (especially Kibble's one criticized by F. Hehl and others) fail to be quite satisfactory.[3,4] The Poincaré supergravity faces its own additional difficulties. For instance, many authors ignore the fact that by the supertranslation law coordinates X^μ are not real, but represent even elements of a Grassmann algebra.

The Poincaré gauge versions of gravity lost sight of the fact that not only gauge potentials, but Higgs-Goldstone fields appear in a gauge theory when symmetries are spontaneously broken. Einstein's gravitational field and supergravity turn out to be fields just of this type.

§ 1. Gauge gravitation theory

We consider the gauge gravitation theory as being based on Einstein's relativity and equivalence principles reformulated in the fibre bundle terms.[3)~5)]

In these terms a metric gravitational field on an orientable space-time manifold X^4 is defined as a global section g of the fibre bundle B of pseudo-Euclidean bilinear forms in tangent spaces over X^4. The bundle B is associated with the tangent bundle $T(X)$ possessing the structure group $GL^+(4, \boldsymbol{R})$, and this is isomorphic with the fibre bundle in quotient spaces $GL^+(4, \boldsymbol{R})/SO(3, 1)$. The global section h of the quotient bundle, which is isomorphic with g, describes a gravitational field in the tetrad form.

By the well-known theorems[6] for a global section h of the quotient bundle to exist, the possibility of contraction of the structure group $GL^+(4, \boldsymbol{R})$ of the tangent bundle to the Lorentz group is necessary and sufficient. It means the existence of an atlas $\Psi^g=\{u_L, \phi_L{}^g\}$ of $T(X)$ such that all transition functions of Ψ^g are elements of the Lorentz gauge group, and all metric functions $g_L=\phi_L{}^g g$ coincide with the Minkowski metric η relative to Ψ^g. Tetrad (vierbein) functions $h_L=\phi_L{}^g h$ take on values in the center of the quotient space $GL^+(4, \boldsymbol{R})/SO(3, 1)$ relative to Ψ^g.

Usually a tetrad field h is written as a section of the principal $GL(4, \boldsymbol{R})$-bundle up to multiplication of h on the right by elements of the gauge Lorentz group. This freedom reflects the nonuniqueness of the atlas Ψ^g. Let us fix the atlas Ψ^g and tetrad functions h_L, taking values in the unit element of the group $GL^+(4, \boldsymbol{R})$. Then with respect to any atlas Ψ of $T(X)$ the tetrad functions h_L read $h_L=\phi_L(\phi_L{}^g)^{-1}$. These represent matrix functions acting in the typical fibre of $T(X)$ and describing the gauge transformations from the atlas Ψ^g to the atlas $\Psi=\{u_L, \phi_L=h_L\phi_L{}^g\}$. Changes of Ψ to $\Psi'=\{u_L, \phi_L'=g_L\phi_L\}$ lead to gauge transformations $h_L\to g_L h_L$ of the tetrad field h_L.

In the fibre bundle terms the relativity principle proves to be identical with the gauge principle of covariance under the gauge group $GL(4, \boldsymbol{R})(X)$ of all transformations of atlasses of the tangent bundle $T(X)$. Thereby the gravitation theory can be built directly as the gauge theory of the space-time group $GL^+(4, \boldsymbol{R})$. But the relativity principle fails to fix the Minkowski signature $(-, -, -, +)$ of metric fields, and therefore the equivalence principle must be called into play in the gauge gravitation theory.

In the fibre bundle terms the equivalence principle can be formulated as the postulate of the existence of a reference frame where Lorentz invariants can be defined everywhere on a manifold X^4, and these would be conserved under any parallel transport. This postulate holds if the connection on the tangent bundle can be reduced to the Lorentz connection, what entails in turn the contraction of the structure group $GL^+(4, \boldsymbol{R})$ of this bundle to the Lorentz group, and consequently the existence of a gravitational field on X^4. In this fashion the equivalence principle establishes the situation of spontaneous breaking of gauge space-time symmetries down to the exact Lorentz gauge and a gravitational field can be treated as the sui generis Goldstone field corresponding to this breakdown.

This standpoint is contrary to ideas on the Poincaré gauge description of gravity, which aimed to represent tetrad gravitational fields as gauge fields of translations due to seeming coincidence of tensor ranks of tetrad fields and translation gauge fields. Moreover, a gauge model of the Poincaré group as the fundamental dynamic group of Special Relativity was believed to supplement gauging of internal and intrinsic spin symmetries of particles. However, one faced here the specification of gauging of the Poincaré group as the dynamic group realized by differential operators. All such attempts resulted in models which were rather far from the conventional formulation of a gauge theory.[3),4)]

The conventional gauge technique can be applied for gauging of the Poincaré group, if one ignores its physical role and regards it as an abstract group.[7),8)] In this case the Poincaré gauge potentials are represented by coefficients of a general affine connection A on the principlal affine frame bundle $A(X)$. Under fixing a certain translation gauge the correspondence between A and the pair $(A_L, \hat{\theta})$ of a linear connection A_L on the linear frame bundle $L(X)$ and $\boldsymbol{R}^4$-valued 1-form $\hat{\theta}$ on the total space of $L(X)$ (or the tensor field θ of the type (1, 1) on X, or fibre mapping of $T(X)$) is established.[6)] Coefficients of tensor field θ represent homogeneous components of translation gauge potentials.

One sees at once that θ has nothing to do with h. Among other things they differ mainly in their gauge transformation laws. Tetrad functions h describing the gauge transition between the fixed atlas Ψ^g and a given atlas Ψ are transformed as $h_L \to g_L h_L$ under atlas changes, whereas gauge translation potentials θ describing the fibre mapping of $T(X)$ are transformed as $\theta_L \to g_L \theta_L g_L^{-1}$.[4),5)]

Thus the Poincaré gauge model fails to provide a gravitational field with the status of translation gauge potentials. Then one must pose the question on the physical treatment of translation gauge fields. For instance, such fields are applied to describe dislocations in the gauge theory of dislocations and disclinations,[9)] and this description can be used in the gauge field theory.[10)]

Since supergravity $(N=1)$ must include gauge gravity, one ought to construct a supergravity theory as sui generis graded generalization of a gauge gravitation theory. As a matter of experience of a gauge gravitation theory the Poincaré gauge approach fails to be quite relevant for this aim. Therefore, one is obliged to formulate supergravity in terms of super fibre bundles after the fashion of the gauge gravitation theory in our version.

§ 2. Super fibre bundles

Let us recall some notions. One calls Grassmann algebra an associative algebra Λ (with the unit element) over the field of real numbers $\boldsymbol{R}$, which possesses a canonical (finite) set of anticommutative generators $\xi_1, \cdots, \xi_L$, $[\xi_A, \xi_B]_+=0$. This is a Z_2-graded commutative algebra $\Lambda=\Lambda_0\oplus\Lambda_1$, whose even part Λ_0 (odd part Λ_1) represents a 2^{L-1}-dimensional real vector space of even (odd) products of generators ξ. An even part Λ_0 splits into $\Lambda_0=\boldsymbol{R}\oplus\Lambda_0'$, and the projection $\sigma:\Lambda\rightarrow\boldsymbol{R}$ is called the body map.

A Grassman algebra can be provided with the structures of Banach algebra and Euclidean topological space by the norm[11]

$$\|\xi\|=\sum|a^{A_1\cdots A_k}|\,,\quad \xi=\sum_{k=0}^{L}\sum_{A_1\cdots A_k}a^{A_1\cdots A_k}\xi_{A_1\cdots A_k}\,. \tag{1}$$

In a super field theory the Grassmann algebra replaces the numerical field, and superspaces over Λ replace familiar vector spaces over $\boldsymbol{R}$. We call a superspace the product $B^{n,m}=\Lambda_0{}^n\times\Lambda_1{}^m$. On the one hand, this represents the Λ-envelope of a Z_2-graded vector space $L^{n,m}=L_0\oplus L_1=\boldsymbol{R}^n\oplus\boldsymbol{R}^m$ which is obtained by multiplication of even (odd) vectors of L by even (odd) elements of Λ. A superspace $B^{n,m}$ considered as the Λ-envelope possesses $(n+m)$ basis vectors $\{\mathcal{B}_a, a=1, \cdots, n; \mathcal{B}_i, i=1, \cdots, m\}$, and coordinates of its even (odd) elements are even (odd) elements of Λ. On the other hand, a superspace $B^{n,m}$ forms a 2^{L-1} $(n+m)$-dimensional real vector space with the basis $\{\mathcal{B}_{aA}, \mathcal{B}_{iA}\}$.

A superspace is provided with the norm and the Euclidean topology induced by the norm (1). Functions on superspaces, differentiation with respect to Grassmann coordinates, supersmooth (superanalitic) functions and mappings are defined on the analogy of the ordinary case, but with a glance to a certain specification.[11)~13)] We only remark that functions on a superspace $B^{n,m}$, which take on values in the Grassmann algebra, can be considered as mapping of the vector space $\boldsymbol{R}^{2^{L-1}}(n+m)$ into the vector space $\boldsymbol{R}^{2^L}$. Accordingly, the differentiation of such functions with regard to Grassmann coordinates can be rewritten via their derivatives with regard to real coordinates, when these derivatives obey the generalized version of the Cauchy-Riemann condition.

An (n, m)-dimensional supermanifold $M^{n,m}$ is defined as a Banach manifold endowed with an atlas $\Psi_M=\{u_L, \varphi_L: u_L\rightarrow B^{n,m}\}$ whose transition functions are supersmooth[11),12)] One constructs the super tangent bundle $T(M^{n,m})$ over the supermanifold $M^{n,m}$. The typical fibre of this bundle is the superspace $B^{n,m}$, and the structure group of $T(M^{n,m})$ is the group of automorphisms of $B^{n,m}$.

This is the super Lie group $L(n, m)$ of invertible matrices

$$M=\begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

where A and D are square $(n\times n)$ and $(m\times m)$ matrices consisting of even Grassmann elements, and B, C are rectangular matrices consisting of odd Grassmann elements. A matrix M is invertible as soon as matrices σA and σD (where σ is the body mapping) are invertible. The super Lie group $L(n, m)$ represents an ordinary Lie group included in the group $GL(2^{L-1}(n+m), \boldsymbol{R})$.

Thereby a supermanifold $M^{n,m}$ and a super tangent bundle $T(M^{n,m})$ may be represented as a certain 2^{L-1} $(m+n)$-dimensional real manifold and the tangent bundle over it, whose transition functions obey the special condition of the Cauchy-Riemann type.

Let us describe the "super" analogue of a space-time manifold.

§ 3. Super space-time symmetry

A super Lie group SG is an abstract group admitting the parametrization by Grassmann elements, which endows this group with superanalytic manifold. The analogy of a Lie algebra for a super Lie group is graded Lie module of left-invariant derivations at the unit element of SG.[11] The even part of this module (which consists of the tangent space at the unit element of SG) represents an ordinary Lie algebra, which equals the Lie algebra of the super Lie group SG as a Lie group.

Hereinafter we restrict ourselves to the case of super Lie groups whose Lie albebras can be obtained as Λ-envelopes of Lie superalgebras (graded Lie algebras). This case exhausts symmetries of a super field theory.

One calls Lie superalgebra any Z_2-graded algebra $A=A_0\oplus A_1$ endowed with products [,] satisfying the following axioms:

$$[I, I'] = -(-1)^{\alpha(I)\alpha(I')}[I', I],$$

$$[I, [I', I'']] = [[I, I'], I''] + (-1)^{\alpha(I)\alpha(I')}[I', [I, I'']],$$

$$I \in A_{\alpha(I)}, \quad I' \in A_{\alpha(I')}.$$

The even part of a Lie superalgebra forms a Lie algebra, whereas the odd part A_1 forms a representation of this Lie algebra, what enables one to classify simple Lie superalgebras after the fashion of the Lie algebra classification.[14]

Irredusible linear representations of Lie superalgebras A are realized in Z_2-graded vector spaces L by matrices

$$\begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix}$$

for even elements and

$$\begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix}$$

for odd elements. This representation space L is a sum of spaces of irreducible representations of the Lie albebra A_0, and odd elements of A_1 perform transitions between these representations.

Since, roughly speaking, A is a superalgebra of generators of a super Lie group, we see that the study of A exhausts the description of symmetries of a super field theory.

Point out that a supergravity theory admits only real superalgebras and their real representation at least in an even sector.

At first we discuss the super analogies of the spatial rotation $SO(3)$ and the covariant spatial transformation algebra $GL(3, \boldsymbol{R})$. One ought to discern two kinds of $SO(3)$:

$$\{M_1, M_2, M_3: [M_i, M_j]=\varepsilon_{ijk}M_k\},$$

$$\{S_+=iM_1-M_2,\ S_-=iM_1+M_2,\ S_3=iM_3\}.$$

These are isomorphic real subalgebras of the complex algebra $SL(3, \boldsymbol{C})$. The first describes space rotations, and the second is the so-called quantum mechanic albebra of spin operators.

The minimal superalgebra including $SO(3)$ is OSP (1,1). Its even part consists of the Lie albebra $SP(2)$, which equals $SO(3)$ in the quantum mechanic form $\{S_+, S_-, S_3\}$, and its odd part contains two generators Q^+, Q . The commutation relations of OSP (1, 1) read:

$$[S_3, Q^+]=\frac{1}{2}Q^+, \qquad [S_3, Q^-]=-\frac{1}{2}Q^-,$$

$$[S_+, Q^+]=0, \qquad [S_+, Q^-]=Q^+,$$

$$[S_-, Q^+]=Q^-, \qquad [S_-, Q^-]=0,$$

$$[Q^+, Q^+]=-\frac{1}{2}S_+, \qquad [Q^-, Q^-]=\frac{1}{2}S_-, \qquad [Q^+, Q^-]=\frac{1}{2}S_3 \tag{2}$$

plus commutation relations for S_+, S_-, S_3. The odd elements Q realize the fundamental spin 1/2 representation of $SO(3)$.

The superalgebra OSP (3, 1) possesses representations by real matrices in the scalar-spinor graded space $L^{1,2}$, and in the vector-spinor graded space $L^{3,2}$ describing the spatial part of a super space-time. Unfortunately the quantum mechanic algebra $SO(3)$ cannot be included in the real algebra $GL(3)$, and the superalgebra $OSP(1, 1)$ fails to be relevant for a supergravity theory.

The superalgebra $OSP(3, 1)$ turns out to be quite satisfactory. Its even part is the direct sum of the Lie algebra $SO(3)$ with generators $\{M_1, M_2, M_3\}$ and the Lie algebra $SP(2)=SO(3)$ with generators $\{S_+, S_-, S_3\}$. Its odd part contains three pairs of generators $\{Q_i^+, Q_i^-, i=1, 2, 3\}$. The commutation relations of $OSP(3, 2)$ read:

$$[M, S]=0, \qquad [M_i, Q_j^{\pm}]=\pm\varepsilon_{ijk}Q_k^{\pm},$$

$$[Q_i^+, Q_{j+i}^+]=0, \qquad [Q_i^-, Q_{j+i}^-]=0,$$

$$[Q_i^-, Q_{i+i}^+]=\frac{1}{4}\varepsilon_{ijk}M_k$$

plus commutation relations for $\{M_1, M_2, M_3\}$, $\{S_+, S_-, S_3\}$, and commutation relations (2) for each index i of Q_i^+, Q_i^-. The pair of odd elements $\{Q_i^+, Q_i^-\}$ for each index i realizes the fundamental spin 1/2 representation of the algebra $\{S_+, S_-, S_3\}$, and the triplet $\{Q_1^+, Q_2^+, Q_3^+\}(\{Q_1^-, Q_2^-, Q_3^-\})$ realizes the vector (covector) representation of the algebra $\{M_1, M_2, M_3\}$.

The superalgebra OSP (3, 1) possesses the minimal-dimensional representation in the graded space $L^{3,2}$ by (5×5) matrices where non-zero elements a_{ij} read:

$$M_1=\{a_{32}=-a_{23}=1\}, \qquad M_2=\{a_{13}=-a_{31}=1\}, \qquad M_3=\{a_{21}=-a_{12}=1\},$$

$$S_+=\{a_{45}=1\}, \qquad S_-=\{a_{54}=1\}, \qquad S_3=\left\{a_{44}=-a_{55}=\frac{1}{2}\right\},$$

$$Q_i^+=\left\{a_{i5}=a_{4i}=\frac{1}{2}\right\}, \qquad Q_i^-=\left\{a_{5i}=-a_{i4}=\frac{1}{2}\right\}.$$

The even subspace of $L^{3,2}$ is the space of the vector representation of $SO(3)=\{M_1, M_2, M_3\}$, and the odd subspace of $L^{3,2}$ is the space of the spin $1/2$ representation of $SO(3)=\{S_+, S_-, S_3\}$.

The Lie algebra $SO(3)=\{M_1, M_2, M_3\}$ can be included into the Lie algebra $GL(3, \boldsymbol{R})$, and the superalgebra $OSP(3, 1)$ can be included into the Lie superalgebra $L(3, 2)$. The even part of $L(3, 2)$ is the direct sum of Lie algebras $GL(3, \boldsymbol{R})$ and $GL(2, \boldsymbol{R})$. Odd generators of $L(3, 2)$ are $\{Q_i^+, Q_i^-, \tilde{Q}_i^+, \tilde{Q}_i^-, i=1, 2, 3\}$. Generators $\{M, S, Q\}$ and $\{M, S, \tilde{Q}\}$ compose two graded subalgebras $OSP(3, 1)$ of $L(3, 2)$. The even generators completing $OSP(3, 1)$ to $L(3, 2)$ are yielded by products $[Q, \tilde{Q}]$.

The superalgebra $L(3, 2)$ possesses the natural representation in the graded space $L^{3,2}$. The even part of $L^{3,2}$ is the spatial vector space of the natural representation of the Lie algebra $GL(3, \boldsymbol{R})$, and the odd part of $L^{3,2}$ is the spinor space realizing the spin $1/2$ representation of the Lie algebra $SO(3)=SP(2)$ and the dilatation operator $D=\{a_{44}=a_{55}=1/2\}$. In comparison with Q, matrices $\tilde{Q}$ on $L^{3,2}$ read: $\tilde{Q}_i^+=\{\tilde{a}_{4i}=-\tilde{a}_{i5}=1/2\}$, $\tilde{Q}_i^-=\{\tilde{a}_{i4}=\tilde{a}_{5i}=1/2\}$.

In the fashion of the spatial supersymmetries the super space-time symmetries are examined. The subalgebra $OSP(4, 2{:}\ 1)$ seems to be the most convenient for describing super space-time symmetries. Its even part is the direct product of the Lorentz algebra $SO(3, 1)$ and the symplectic algebra $SP(2, 2)$ which is isomorphic to the algebra $SO(4, 1)$ and includes the Lorentz algebra $SO(3, 1)$. The odd part of $OSP(4, 2; 1)$ is composed of generators Q_a^i, $a=1, \cdots, 4$; $i=1, \cdots, 4$, which form the 4-vector representation of $SO(3, 1)$ along the indices a, and the 4-spinor representation of $SO(3, 1)\subset SP(2, 2)$ along the indices i.

The superalgebra $OSP(4, 2; 1)$ possesses the minimal-dimensional representation by real matrices in the graded space $L^{4,4}$, which is the direct sum of the 4-vector space X^4 and the 4-spinor space Θ^4. The odd generators Q_a^i are realized in $L^{4,4}$ by matrices

$$Q_a^i=\left\{a_{a,4+i}=\frac{1}{2}(\eta_{aa})^i,\ a_{4+k(i),a}=\frac{1}{2}(-\eta_{aa})^{k(i)},\quad k(i)=i-(-1)^i\right\}.$$

Imbedding of the Lie algebra $SO(3, 1)$ into the Lie algebra $GL(4, \boldsymbol{R})$ induces imbedding of the Lie superalgebra $OSP(4, 2; 1)$ into the Lie superalgebra $L(4, 4)$ possessing the natural representation in the graded space $L^{4,4}$. The even part of $L(4, 4)$ is the direct sum of two algebras $GL(4, \boldsymbol{R})$, and the odd part contains generators Q_a^i, $\tilde{Q}_a^i$, where, in comparison with Q, matrices $\tilde{Q}$ on $L^{4,4}$ read:

$$\tilde{Q}_a^i=\{\tilde{a}_{a,4+i}=\eta_{aa}a_{a,4+i},\ \tilde{a}_{4+k(i),a}=a_{4+k(i),a}\}.$$

The even generators completing $OSP(4, 2; 1)$ to $L(4, 4)$ are produced by products $[Q, \tilde{Q}]$.

§ 4. Super Minkowski space

The Grassman envelope $B^{4,4}$ of the graded space $L^{4,4}$ seems to be the best model for the super expansion of the standard Minkowski space. The super *Lie* group OSP (4, 2; 1) whose Lie algebra is the Grassman envelope $OSP_\Lambda(4, 2; 1)$ of the Lie superalgebra OSP (4, 2; 1) replaces the Lorentz group. The supergroup $OSP(4, 2; 1)$ keeps the invariance of the bilinear form

$$H(\mathscr{B}, \mathscr{B}') = \eta_{\alpha\alpha'} X^\alpha X^{\alpha'} + a_{ij}\theta^i\theta^{j'}$$
$$= -X^i X^{1'} - X^2 X^{2'} - X^3 X^{3'} + X^4 X^{4'} + \theta^1\theta^{2'} - \theta^2\theta^{1'} + \theta^3\theta^{4'} - \theta^4\theta^{3'} \qquad (3)$$

on the elements $\mathscr{B} = X^\alpha \mathscr{B}_\alpha + \theta^i \mathscr{B}_i$ of the superspace $B^{4,4}$. Coordinates X^α and θ^i of elements $\mathscr{B}$ are even and odd Grassmann elements respectively, and the form (3) takes on values in the even part of the Grassmann algebra Λ.

The $SO(3, 1)$ and $SP(2, 2)$ generators multiplied in even parameters turn the form H into zero by definition, and one can verify the same section of generators Q multiplied in odd parameters.

§ 5. Supergravity

Let us examine a supermanifold $M^{4,4}$ and a super tangent bundle $T(M^{4,4})$ over $M^{4,4}$. This bundle possesses the typical fibre $B^{4,4}$. The structure group of $T(M^{4,4})$ is the supergroup $L(4, 4)$ whose Lie algebra represents the Grassmann envelope of the Lie superalgebra $L(4, 4)$.

Following the treatment of the equivalence principle in the gauge gravitation theory considered by us, let us require the contraction of the structure group $L(4, 4)$ of the bundle $T(M^{4,4})$ to the super Lie group $OSP(4, 2{:}\ 1)$. Since a supermanifold $M^{4,4}$, a super tangent bundle $T(M^{4,4})$ and structure groups $L(4, 4)$, $OSP(4, 2; 1)$ are an ordinary manifold, an ordinary fibre bundle and Lie groups respectively for this structure group contraction to take place a global section G of the associated fibre bundle in quotient spaces $L(4, 4)/OSP(4, 2; 1)$ must exist (where $L(4, 4)$ and $OSP(4, 2; 1)$ are considered as Lie groups).

This quotient space is isomorphic to the space of all bilinear forms on a superspace $B^{4,4}$, which can acquire the canonical form (3) via $L(4, 4)$ transformations. Then on the fashion of our treatment of the gauge gravitation theory a global section G may be treated as a supergravity field.

The definition of a supergravity field in the fibre bundle terms in the same way as a gravity field opens the door to build a supergravity theory as the gauge theory of the super-group $L(4, 4)$ which is spontaneously broken down to the super Lorentz group OSP (4, 2; 1). For instance, one may build supergeometry (super Christoffel symbols, supertorsion, supercurvature, etc.) directly by analogy with the conventional gravitation theory, but with a glance to the certain specification connected with the odd Grassmann algebra generators.

Supergravity must include gravity. This inclusion is induced by the immersion of the real field R into a Grassmann algebra and by projection of a Grassmann algebra onto R by the body map. The body map of a superspace $B^{n,m}$ onto R^n is defined by the equivalence relation $\sigma\mathscr{B} = \sigma\mathscr{B}'$, $\mathscr{B}, \mathscr{B}' \in B^{n,m}$ on a superspace $B^{n,m}$. But the definition of this relation on a supermanifold faces difficulties. Really, let (u, ψ) and (u', ψ') be two charts on a supermanifold, whose intersection is not connected, i.e.,

$$u \cap u' = V_1 \cup V_2, \ V_1 \cap V_2 = \phi .$$

Denote

$$\psi_{1,2} = \psi \downarrow V_{1,2} , \qquad \psi'_{1,2} = \psi' \downarrow V_{1,2} .$$

Let $\mathcal{B}_1 \in V_1$, $\mathcal{B}_2 \in V_2$ be two σ-equivalent elements relative to (u, ψ), i.e., $\sigma\psi_1\mathcal{B}_1 = \sigma\psi_2\mathcal{B}_2$. But these may be nonequivalent relative to (u', ψ') because $\sigma\psi_1'\psi_1^{-1}\sigma \neq \sigma\psi_2'\psi_2^{-1}\sigma$ in general. A supermanifold possesses such an atlas that σ-equivalence is defined everywhere on a supermanifold,[15] but the quotient of a supermanifold by this equivalence relation fails to be even a topological manifold in general. Moreover this equivalence relation is destroyed by transition to other atlases of a supermanifold.

The body map problem is common to all models utilizing supermanifolds, and this is the serious difficulty for physical outcomes of supersymmetry formalism to consider it as quite satisfactory.

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