Quantum entanglement in optomechanical systems with dissipation

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Abstract:

Modern optomechanical devices, such as gravitational-wave detectors, in near-term period should reach such sensitivity, that it would be possible to observe quantum phenomena with macroscopic quantum objects. One of the most interesting experimental projects in such area of fundamental physics is the proof of the Einstein–Podolsky–Rosen paradox in its initial interpretation, that is for coordinate and momentum of mechanical objects. [1]. It is necessary to note that achievements of last years in the field of ultraprecise mechanical measurements stimulated in particular by the development of gravitational-wave detectors [2], allow us assert that it is possible to attain the corresponding optimal parameters for the experimental preparation of quantum oscillators in entangled state [3].

In particular, the scheme [3] for experimental proof of EPR-paradox for two mechanical degrees of freedom of Michelson interferometer has been offered. Within the bounds of this experiment the particular interest would be the estimation of conditions and requirements for mechanical and optical noise in the scheme at which it is possible to observe quantum entanglement and also estimation of optimal parameters of the initial quantum state of optomechanical system that maximize the entanglement. As model of aforementioned system we consider mechanical and optical degree of freedom of a laser interferometer and formulate the conditions for realization of experiment in which we would be able to observe quantum entanglement arising as a result of dynamic interaction between them.

Generally the analysis of such experiments is complicated due to necessity to consider influence of numerous channels of dissipation and as a consequence the decoherence of quantum state of considered system. Similar scheme was considered in [4]. Our model differs in two aspects. First, we consider time dependence of the entanglement. Second, we assume the initial squeezed states of the mechanical and optical degrees of freedom. For the considered model we calculate logarithmic negativity [5] – one of the most popular measures of a quantum entanglement as function of time. This measure allows us to determine the characteristic "lifetime" of entangled state in system with dissipation. We analyze the phenomenon of the entanglement sudden death [6] and the entanglement sudden revival [7] considering optomechanical system.

Quantum entanglement:

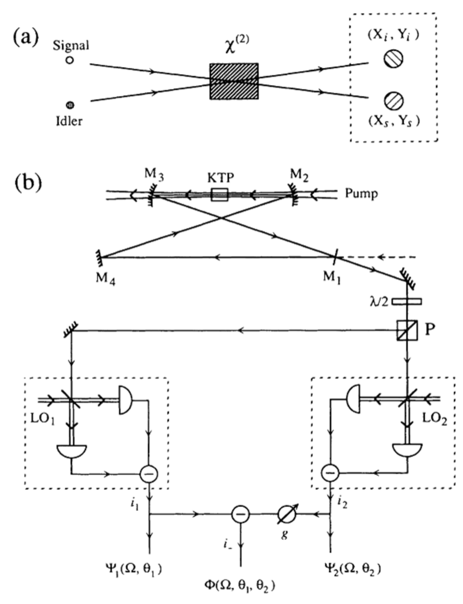
Quantum entanglement — phenomenon which does not have analogues in the classical physics, at which the quantum state of two or more subsystems cannot be described separately from each other and wave function (density matrix) of the system cannot be factorized. As a result at ensemble measurements conducted over separated space-like subsystems correlation of results is observed which does not have explanation within the limits of local classical formalism. That is entangled quantum state appears to be *quantum nonlocal.*

$$\psi(A,B) \neq \psi(A) \otimes \psi(B)$$

For mixed entangled states the situation is much more difficult: $\hat\rho \neq \sum_{k=1,2} p_k \hat\rho_k^{(1)} \otimes \hat\rho_k^{(2)},\ p_k \geq 0$

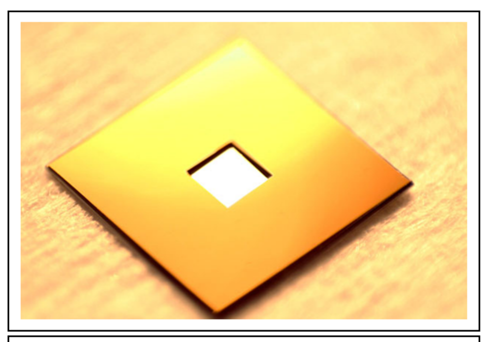
Einstein – Podolsky – Rosen paradox

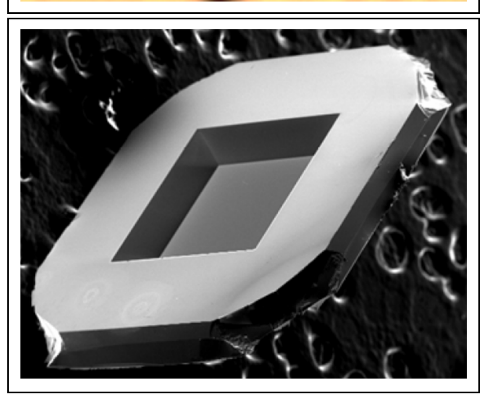
In 1935 in their most famous and most cited article "Can quantum-mechanical description of physical reality be Considered Complete?"Albert Einstein, Boris Podolsky and Nathan Rosen have formulated their well-known paradox, which subsequently has caused set of questions relating to the interpretation of quantum mechanics. For this long time this paradox was basically in the interest of those who were engaged in interpretation of quantum mechanics. But recently the experimental proof became possible. For optical continuous variable (CV) systems the experimental proof was given only in 1992 by professor Kimble and others [8]. We are interested in verification of EPR-paradox for mechanical CV systems as the proof of macroscopic quantum mechanics which appears to be the key problem of the modern physycs.



(a) Scheme for realization of EPR-paradox by nondegenerate parametric amplification, with the optical aplitudes (X_s, Y_s).
(b) Principal components of experiment.

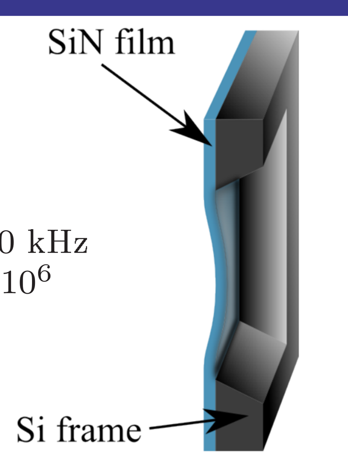
SiN membrane — almost ideal object for the macroscopic quantum mechanics:

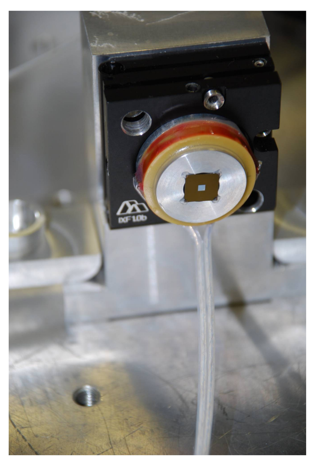




Mechanical properties:
Mass of the oscillator: $m = 50$ ng
Size: $1,5mm \times 1,5mm$
Thickness: 75nm
Resonant frequency: $\omega_m/2\pi = 500$ kHz
Mechanical quality factor: $Q_m = 10^6$

Optical properties:
Power reflectance: $\sim 33\%$





The mass of membrane is extremely small but it can be considered as a macroscopic object. Due to interaction of light with membrane by radiation pressure membrane and light, which is quantum object, appear to be in the entangled state. One of the disadvantages of membrane is not enough high reflectivity since it reduces the optomechanical coupling.

Entanglement measures:

In the quantum information field entanglement is precious physical resource which we need to quantify, like energy or entropy. So the question arises: "How much is this state entangled?". The answer for this question would be the *entanglement measure*, which is monotone by the entanglement functional of quantum state and associates to each density operator a real positive number $\hat\rho$: $\hat\rho \longrightarrow E(\hat\rho) \in \mathbb{R}^+$.

Logarithmic negativity — monotone, additive, but not convex functional of quantum entanglement defined as [5]:

$$E_N(\hat\rho) = \ln||\hat\rho^{T_B}||_{tr}, \quad ||\hat A||_{tr} = \mathrm{Tr}\,(\sqrt{\hat A^\dagger \hat A}),$$

where $\hat\rho^{T_B}$ — partially transposed (with respect to the system B) density operator of bipartite system A and B.

Theorem. *If $\hat\rho$ — is a density operator of a bipartite Gaussian state of 1×1 modes, characterized by its correlation matrix $\mathbb{V}$, then*

$$E_N(\hat\rho) = \max\{-\ln(\nu), 0\},$$

where ν is the minimum symplectic eigenvalues of the partially transposed matrix $\Lambda\mathbb{V}\Lambda$, $\Lambda = \mathrm{diag}[1,1,1,-1]$.

For Gaussian states nonzero Logarithmic negativity is both necessary and sufficient condition for entanglement, as it was shown by Simon *et. al.* [9]. Particularly for bipartite Gaussian state characterized by covariation matrix which can be written in a block form like:

$$\mathbb{V} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \text{ where } \quad A = \begin{pmatrix} \langle\Delta\hat x_A^2\rangle & \langle\Delta\hat x_A \circ \Delta\hat p_A\rangle \\ \langle\Delta\hat x_A \circ \Delta\hat p_A\rangle & \langle\Delta\hat x_A^2\rangle \end{pmatrix},$$

$$B = \begin{pmatrix} \langle\Delta\hat x_B^2\rangle & \langle\Delta\hat x_B \circ \Delta\hat p_B\rangle \\ \langle\Delta\hat x_B \circ \Delta\hat p_B\rangle & \langle\Delta\hat x_B^2\rangle \end{pmatrix}, \quad C = \begin{pmatrix} \langle\Delta\hat x_A \circ \Delta\hat x_B\rangle & \langle\Delta\hat x_A \circ \Delta\hat p_B\rangle \\ \langle\Delta\hat p_A \circ \Delta\hat x_B\rangle & \langle\Delta\hat p_A \circ \Delta\hat p_B\rangle \end{pmatrix},$$

where $\hat x_{A,B}$ and $\hat p_{A,B}$ — coordinate and momentum of a particles A and B correspondingly, and $\langle\hat\alpha\rangle \equiv \mathrm{Tr}[\hat\rho\hat\alpha]$, $\Delta\hat\alpha \equiv \hat\alpha - \langle\hat\alpha\rangle$, and $\hat\alpha \circ \hat\beta \equiv \frac{1}{2}[\hat\alpha\hat\beta + \hat\beta\hat\alpha]$. Then the logarithmic negativity is determined by a simple formula:

$$E_N = \max\left[-\ln\sqrt{\frac{\Sigma - \sqrt{\Sigma^2 - 4\,\mathrm{Det}\,\mathbb{V}}}{2}}, 0\right], \text{ where } \Sigma = \mathrm{Det}\,A + \mathrm{Det}\,B - 2\,\mathrm{Det}\,C.$$

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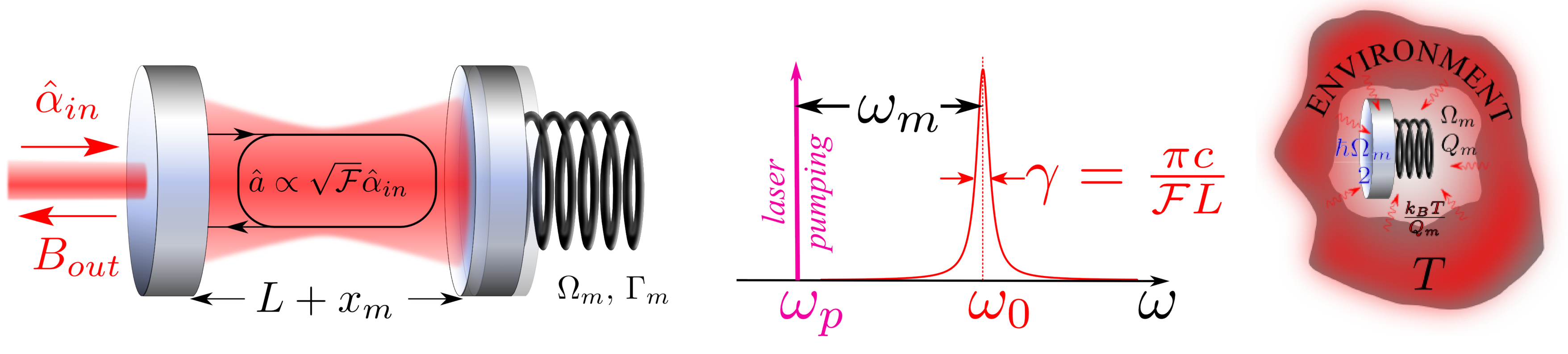
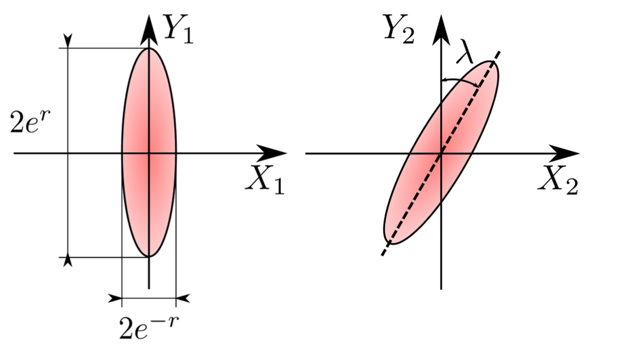
Quantum entanglement in optomechanical system:



We consider ordinary optomechanical system consisting from optical mode of Fabry-Perot resonator with eigenfrequency ω_0 and bandwidth γ and a mirror which is considered to be a mechanical oscillator with eigenfrequency ω_m and damping $\Gamma_m = \omega_m/Q_m$ (shown on picture). The resonator is pumped by laser with pumping frequency detuned from resonance $\omega_p = \omega_0 + \Omega$. The Hamiltonian of optomechanical system is:

$$\frac{\hat{\mathcal{H}}}{\hbar} = \underbrace{\omega_0\hat a^\dagger\hat a}_{\hat{\mathcal{H}}_{\rm opt}/\hbar} + \underbrace{\omega_m\hat b^\dagger\hat b}_{\hat{\mathcal{H}}_{\rm mech}/\hbar} + \underbrace{G_0(\hat b + \hat b^\dagger)\hat a^\dagger\hat a}_{\text{OM interaction}} + \underbrace{i\sqrt{\gamma}(\hat\alpha_{in}^\dagger\hat a e^{i\omega_p t} - h.c.)}_{\text{optical pumping}} + \underbrace{i\sqrt{\Gamma_m}\int\frac{d\omega}{2\pi}(\hat b\hat\beta_\omega^\dagger - h.c.)}_{\text{mech. dissipation}} + \frac{\hat{\mathcal{H}}_{\rm bath}}{\hbar},$$

where $G_0 = \omega_0 x_{\rm ZPF}/L$ — coupling considered of optomechanical interaction, $x_{\rm ZPF} = \sqrt{\hbar/(2m\omega_m)}$ — amplitude of zero point fluctuations of oscillator, $\hat a$, $\hat b$ — quantum annihilation operators of optical and mechanical modes correspondingly, $\hat\alpha_{in}$ — annihilation operator of pumping photons, $\hat\beta_\omega$ — annihilation operator of thermostat phonons, corresponding to mechanical dissipation, a $\hat{\mathcal{H}}_{\rm bath} = \int\frac{d\omega}{2\pi}\hbar\omega\hat\beta_\omega^\dagger\hat\beta_\omega + \int\frac{d\omega}{2\pi}\hbar\omega\hat\alpha_\omega^\dagger\hat\alpha_\omega$ — Hamiltonian of evolution of thermostats (mechanical and optical).



Initial state of optical and mechanical subsystems

This system is nonlinear, but with powerful enough optical pumping it could be linearized and the Hamiltonian in the interaction picture with $\hat{\mathcal{H}}_0 = \hbar\omega_p\hat a^\dagger\hat a$ is equivalent to the Hamiltonian of coupled harmonic oscillators with dissipation:

$$\frac{\hat{\mathcal{H}}_{\rm eff}}{\hbar} = \Omega\hat a^\dagger\hat a + \omega_m\hat b^\dagger\hat b + G_{\rm eff}(\hat b + \hat b^\dagger)(\hat a + \hat a^\dagger) + \frac{\hat{\mathcal{H}}_{\rm diss}}{\hbar} + \frac{\hat{\mathcal{H}}_{\rm bath}}{\hbar}$$

where $G_{\rm eff} = G_0 A$ — linear constant of optomechanical interaction, $A = \sqrt{P_c L/(\hbar\omega_p c)}$ — classical amplitude of the optical mode of the resonator, associated with c circulating power P_c.

Solving the standard Heisenberg-Langevin equations we calculate covariation matrix $\mathbb{V}(t)$ of second moments of coordinates and momentums of oscillators for every moment of time t taking noises into account, assuming that mechanical thermostat is at temperature $T \neq 0$, and optical thermostat relating to pumping field is at zero temperature (laser noises are assumed to be quantum). As the initial state $\mathbb{V}(0)$ we consider covariation matrix of dual-mode squeezed state:

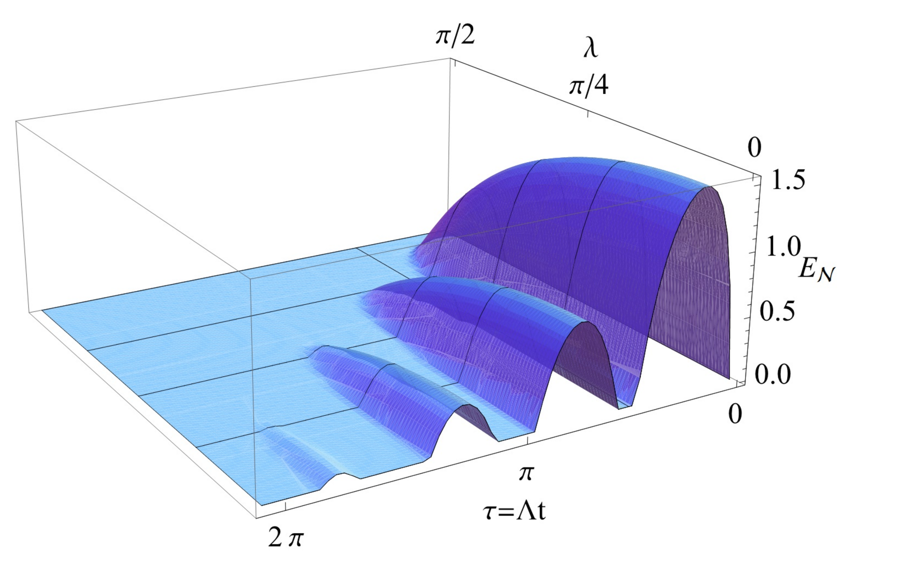
$$\mathbb{V}(0) = \begin{pmatrix} S(r_1, 0) & 0 \\ 0 & S(r_2,\lambda) \end{pmatrix}, \quad \text{where} \quad S(r,\lambda) \equiv \begin{pmatrix} \cosh 2r - \sinh 2r\cos 2\lambda & \sinh 2r \sin 2\lambda \\ \sinh 2r\sin 2\lambda & \cosh 2r + \sinh 2r\cos 2\lambda \end{pmatrix}$$

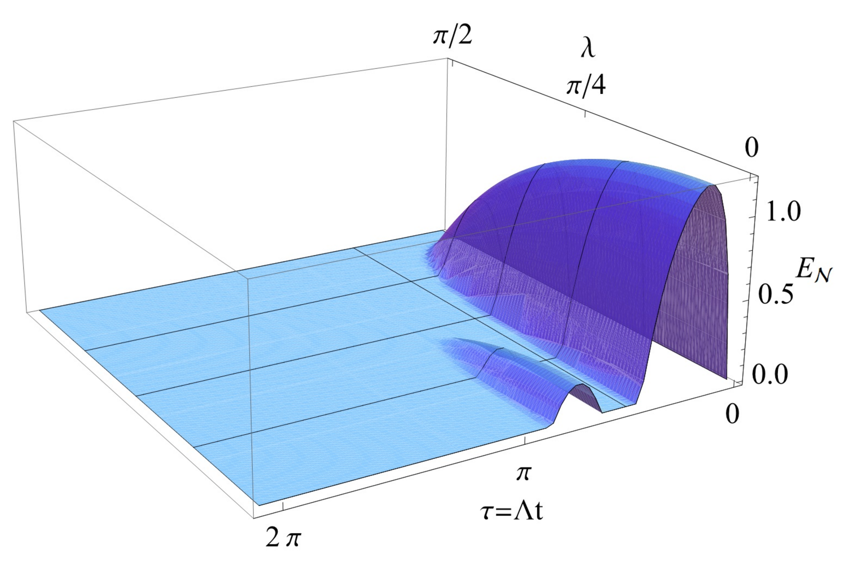
Owing to that the initial state obviously is Gaussian and dynamics of system is linearized the knowledge of a matrix $\mathbb{V}(t)$ completely defines the condition of the state at any moment of time.

Now knowing covariation matrix of system and having set the initial state we can define logarithmic negativity $E_{\mathcal{N}}(\hat\rho(t))$ as monotone and additive entanglement measure for Gaussian state.

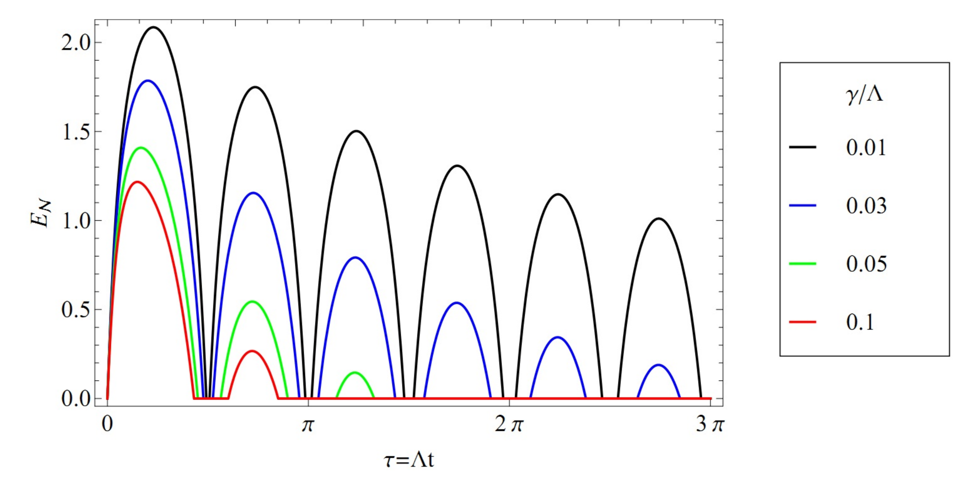
Aim of our work: **Formulate the requirements for system of two coupled harmonic oscillators with dissipation at which it would be possible to set up an experiment of the entanglement observation.**

Results:





Logarithmic negativity E_N as a function of time $\tau = \Lambda t$ ($\Lambda = 20\gamma$ — sloshing frequency) and initial squeeze angle of oscillators λ (squeezing — 10 dB) at various values of temperature of the mechanical heat bath T: *left* — $T = 1K$, *right* — $T = 2K$, optical frequency $\omega_o = 1.77\times10^{15}s^{-1}$, finesse: $F = 10^5$, length of Fabry-Perot resonator $L = 10$ cm, circulating power $P_c = 1$ W



Logarithmic negativity E_N as a function of time at various values of γ/Λ, squeezing — 10 dB, $T = 1K$, $\lambda = 0$.

Conclusion:

We can see that entanglement in system of coupled oscillators: (E_N):

1. oscillates with oscillates with the quadruple sloshing frequency(4Λ);
2. maximal at parallel squeezing of the initial states: $\lambda = 0$;
3. entanglement monotonously increases with increase of squeezing parameters of the initial states r_1 and r_2 for both oscillators;
4. entanglement decreases that faster, than it is more parameter $\Theta = 2k_BT/(Q_m\hbar\omega_m)$ [10];
5. entanglement decreases that faster, than it is more ratio of sloshing frequency to damping rate: Λ/γ.

The phenomenon of entanglement sudden death (ESD) and sudden revival (ESR) was discovered for entangled qubits. We investigate it for a continuous variable system. Two initially entangled and afterward interacting subsystems(mechanical and optical) can become completely disentangled in a finite time and than the initially unentangled subsystems can be entangled after a finite time despite the fact that the coherence between them exists for all times. We have analyzed and shown that sudden death and sudden revival of the entanglement is the result of dynamics of system. This ESD and ESR constitutes yet another distinct and counter-intuitive trait of entanglement. For parameters mentioned above we calculated characteristic "lifetime" of entangled state ($\tau \sim 1\mu s$) which is sufficient for experimental observation of entangled state.

The made estimations allow us to assert about possibility of attainment of the corresponding optimal parameters for the experimental preparation of quantum oscillators in entangled state.