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Numerical modeling of heat and moisture transfer processes in a system lake – soil

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Abstract

A one-dimensional model of the shallow reservoir thermodynamics either describing physical processes of the heat and moisture transport in the underlying soil layer is constructed. The model simulates seasonal variations and year-to-year variability of thermal and hydrological regime of the "lake-soil" system, including the ice and snow layer formation. Using as input data long-term meteorological observations, a number of numerical experiments is performed. The comparison of numerical results with available observational data has showed that the model reproduces thermal and hydrological regimes of shallow lakes in Western Siberia and Yakutat satisfactory.

1. Introduction

Due to increasing power of modern computers and development of technologies of parallel programming, mesoscale and global atmospheric models are characterized by increasing spatial resolution and using of full vertical momentum equation instead of hydrostatic relation. These tendencies lead to new problems in parameterization of subgrid processes, among which an important role belongs to interaction of atmosphere with various types of land surface. A crucial point here is interaction between atmosphere and large number of small hydrological objects, e.g. lakes and wetlands. This interaction is especially important for Northern regions of Eurasia (Western Siberia, Karalee, Finland) and North America (Canada) with high density of small lakes and wetlands. Moreover, recent experiments with climate models have indicated climate in these regions among most sensitive to anthropogenic emissions of greenhouse gases. To take into account response of land hydrology on climate change and its feedback loop on climate correctly, surface layer block in climate model must quantify effect of “hydrological heterogeneity”. Therefore intercomparison studies of different approaches to this problem using standard set of meteorological data from surface stations gain a particular importance. As an example of such studies one could mention a project, aimed to compare various parameterization schemes of surface processes, PILPS-2(e) with emphasis on hydrological processes at high latitudes.

There is a particular interest to simulate heat and moisture transfer processes in a system water reservoir – soil, where water reservoir could be either a small lake or a wetland. Until recently, wetlands were parameterized in climate models by means of specification of land surface features without any description of water body dynamics and thermodynamics, the role of which in mass and heat exchange with atmosphere is not clear enough. However, some aspects of wetland – atmosphere interaction have been studied intensively (e.g., see studies of water balance of wetlands in case of irrigation [1], wetland hydraulics [4], methane generation and transport [5]).

Lakes modify structure of surface atmospheric layer, since that changing heat, moisture and momentum fluxes. In most climate models and NWP systems effect of small and shallow lakes is parameterized in oversimplified way, e.g. under assumption, that lakes are well mixed. But in nature lakes are stratified with depth most part of a year. To take into account density stratification effect on mixing in most realistic way, one should utilize modern theories of turbulent transport, that are, however, rather expensive computationally. The last argument becomes even more valuable, if we consider lake water and heat regime at large time scales, including underlying soil thermodynamics.

In this issue we present a compromise approach to parameterization of lake and wetlands effect, that captures either much of essential physics of heat and moisture transfer, or effective realization algorithms. The issue consists of two parts. First one describes physical problem and model description, and second one demonstrates some results of numerical experiments using long-range meteorological data sets from synoptic stations. In conclusion main results of the investigation are summarized.

2. Model description

An essential feature (see experimental studies [8], [12]) of shallow lakes is small horizontal heterogeneity of thermodynamic parameters. This makes possible to take into account only transfer processes in vertical direction. Obviously, at considered time scales one-dimensional approach could be applied either to soil beneath a lake or to ice and snow covers, forming in cold season.

a. Heat and moisture transfer in water body

Lake temperature is calculated according to one-dimensional heat diffusion equation. Vertical coordinate, z, is directed downward, and level $z = 0$ is located at free surface of a lake, so that solution domain is $[0, h]$, where $h = h(t)$ is lake depth, and t is time. It is convenient to use a new spatial coordinate $\xi = z / h$ instead of z. In (ξ, t) coordinates heat diffusion equation has the following form

$$c\rho\frac{\partial T}{\partial t} = \frac{1}{h^2}\frac{\partial}{\partial \xi}\left(\lambda\frac{\partial T}{\partial \xi}\right) + c\rho\frac{dh}{dt}\frac{\xi}{h}\frac{\partial T}{\partial \xi} - c\rho\frac{1}{h}\frac{dh_0}{dt}\frac{\partial T}{\partial \xi} - \frac{1}{h}\frac{\partial S}{\partial \xi} + M \qquad (1)$$

The symbols are: c is heat capacity of water, ρ - its density, λ - eddy diffusivity, T - temperature, $\frac{dh_0}{dt}$ - water balance at free surface of lake, S - solar radiation flux, penetrated to depth z, M is temperature change rate due to buoyancy mixing (convection). Eddy diffusivity is parameterized by the following empirical algorithm: if lake is covered by ice, then $\lambda = 1.5$ W/(m*K) and during warm season λ is assumed to be a linear function of wind speed in surface layer

$$\lambda = 10\lambda_0 + \frac{V}{V_0}(\lambda_{\max} - \lambda_0),$$

where λ_0 is molecular diffusivity, V – wind speed, V_0 – wind speed at which eddy diffusivity reaches its maximum $\lambda_{max} = 150$ W/(m*K).

To calculate solar radiation, penetrated under upper water surface, a generally accepted exponential relation is used:

$$S(\xi) = S(0)\exp(-\alpha_e h\xi),$$

where α_e is extinction coefficient. For obtaining M (in conditions of unstable density stratification) a following procedure of vertical mixing is used: with fixed time interval the total immediate redistribution of heat content of model layers is performed, so that water density grows with depth. In this procedure density is assumed to be dependent only on temperature according to simplified version of well-known UNESCO formula:

$$\rho = \rho_0 (1 + 8.0 * 10^{-5} + 5.88 * 10^{-5} T - 8.11 * 10^{-6} T^2 + 4.77 * 10^{-8} T^3),$$

$$\rho_0 = 1000 kg / m^3.$$

and T is expressed is Celsius degrees.

At the top boundary of lake temperature is found from heat balance equation, at the bottom (lake – soil boundary) continuity of temperature and heat flux are used. Lake depth is defined from water balance equation:

$$\frac{dh}{dt} = r - E - R_s - R_b$$

where r is precipitation rate, E - evaporation rate, R_s - surface runoff, R_b - water exchange with underlying soil. Soil runoff is not considered.

If during integration surface temperature of water becomes less than 0 ˚C, initial ice cover with thickness of 1 cm is formed. To obtain temperature of ice layer heat diffusion equation (1) is solved, excluding radiative and convective terms. Heat exchange coefficient in ice λ_s is assumed to be constant, and it equals 2.2 W/(m*K). At the water – ice boundary temperature equals to melting point temperature 0 ˚C. Ice melting rate at top boundary of ice layer is defined by heat balance at this surface, and melted water is added to water layer.

b. Heat and moisture transfer in snow cover

During cold season precipitation forms snow cover. In the model it is characterized by two variables: temperature and liquid water content. Their evolution is considered in *(z, t)* coordinates according to equations [3]:

$$\begin{aligned} c_{sn}\rho_{sn}\frac{\partial T}{\partial t} &= \frac{\partial}{\partial z}\lambda_{sn}\frac{\partial T}{\partial z} + \rho_{sn} L F_{fr}, \\ \frac{\partial W}{\partial t} &= -\frac{\partial \gamma}{\partial z} - F_{fr}. \end{aligned} \tag{2}$$

The symbols means the following: L – latent heat of melting, F_{fr} - rate of freezing, W - liquid water content, γ - filtration flux of liquid water. The model also takes into account densification of snow due to gravitation. Thermodynamic coefficients in system (2) are calculated using empirical formulas, that are described in [3]. At the top boundary of snow heat balance equation is used, at the bottom (snow – ice boundary) continuity of temperature and heat flux are applied.

c. Heat and moisture transfer in soil

To describe heat and water transport in a soil under a lake the model from [2] is used. This model accounts for temperature, water, ice and vapor content in soil. Since a soil under a lake should be saturated by liquid water or by frozen – in case of seasonal freezing or permafrost – vapor content in soil is neglected. If one also neglects water flux due to temperature gradient, which is very small, a system from [2] takes the form:

$$\rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_s \frac{\partial T}{\partial z}\right) + \rho_s L F_{fr},$$
$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial z}\lambda_W \frac{\partial W}{\partial z} - \frac{\partial \gamma}{\partial z} - F_{fr}, \qquad (3)$$
$$\frac{\partial I}{\partial t} = F_{fr}.$$

where λ_W is water diffusivity, and I is ice content. The system describes processes of heat and moisture diffusion, filtration of liquid water, as well as freezing/melting of water. For quantification of filtration and diffusivity coefficients, an approach from [10, 11] is used. At the top boundary of soil (water – soil interface) continuity of temperature and heat flux are applied, and water flux as calculated as a function of saturation of top soil layers. At the lower boundary (in current version it is 100 m depth) heat and moisture fluxes are set to zero.

d. Heat balance at the surface of lake

Heat balance equation is used in the model to calculate surface temperature (temperature of water, ice or snow) and it is as follows

$$S(1-\alpha) + E_a - E_s - H - LE = -\frac{\lambda}{h}\frac{\partial T}{\partial \xi}. \qquad (4)$$

where S – is net solar radiation, E_a – atmospheric downward radiation, E_s – radiation of surface, H and LE are heat fluxes and latent heat flux, respectively, a – surface albedo. In spring and autumn, when snow at unfrozen lake or rain at ice cover may occur, energetic effect of these processes also taken into account in equation (4). All fluxes, taking part in (4) could be calculated using standard set of measurements of synoptic stations.

Net solar radiation is calculated according to Kondratiev formula [7]:

$$S^* = \frac{S}{1 + \varepsilon\tau / \sin h_0}\left(1 - c_{sh} n\right)$$

where ε is empirical function of sun height, τ is optical thickness for integral flux, that is assumed to be 0.105, n – number of clouds (maximum 1), c_{sh} = 0.5607 is empirical coefficient, h_0 – sun height. Solar radiation intensity on the horizontal surface at the top boundary of atmosphere is calculated according to well-known formula:

$$S^* = S_0^*\left(\sin\varphi \sin\delta + \cos\varphi\cos\varphi\cos\theta\right)$$

where S_0^* - constant solar flux intensity, φ - latitude, δ - longitude, θ - sun angle.

Atmospheric downward radiation is parameterized as a function of air temperature and humidity at 2 m height and number of clouds[7]:

$$D = \varepsilon_a \sigma T_2^4 \left(1 + c_{\lg} n^2\right),$$
$$\varepsilon_a = c_e e_2^{1/7} \exp\left(\frac{350}{T_2}\right),$$
$$c_{\lg} = 0.22,$$
$$c_e = \begin{cases} 0.15, T_2 < 273.15 \\ 0.14, T_2 \geq 273.15. \end{cases}$$

In this equation T_2 is temperature at 2 m, e_2 is water vapor pressure at 2 m, σ is Stefan – Boltsman constant.

To yield heat flux and latent heat flux aerodynamic method with coefficients according to Monin – Oboukhov [6] similarity theory is employed. The corresponding equations have the following form:

$$H = -c_p \rho_a C_H |V_2| (\Theta_2 - \Theta_s),$$
$$LE = -\rho_a L C_E |V_2| (q_2 - q_s),$$

where c_p is specific heat at constant pressure, ρ_a – air density, C_H and C_E are dimensionless exchange coefficients for temperature and vapor respectively, Θ_2 and q_2 are potential temperature and specific humidity at 2 m, respectively, Θ_s and q_s their values at the surface, V_2 is wind speed at 2 m.

2. Model verification

To verify the model numerical experiments using standard set of meteorological measurements were conducted. Experimental data was extracted from NDP048 archive, that is accessible in Internet (http://cdiac.esd.ornl.gov/ftp/ndp048). It contains long-term measurement data for 225 synoptic stations of former USSR. In this issue numerical experimentation results for Kolpashevo (Tomsk region) and Yakutsk are presented. In last case modeled values were compared with natural observations on lake Syrdakh (20 km North from Yakutsk).

In experiments with data from Kolpashevo, a thermodynamic regime of 2 m depth lake, that is typical for Tomsk region, was studied during a period 1936 – 1984. We havn't got any data of measurements at lakes, since that model skill to simulate snow surface temperature has been studied. Comparison with observations is made under an assumption, that snow surface temperature above lake (calculated by the model) is close to those measured on the nearby land station. Comparison results prove this assumption, as it seen on the Fig. 1: modeled and experimental temperature curves are in good correspondence. Accuracy of snow measurements at Kolpashevo station is 0.5 deg C. However, in monthly average, the model provides surface temperature 2 C less, than observed. This is caused, to our opinion, by inadequate surface fluxes provided by surface layer scheme in stable stratification, that is especially important during cold season in continental climate.

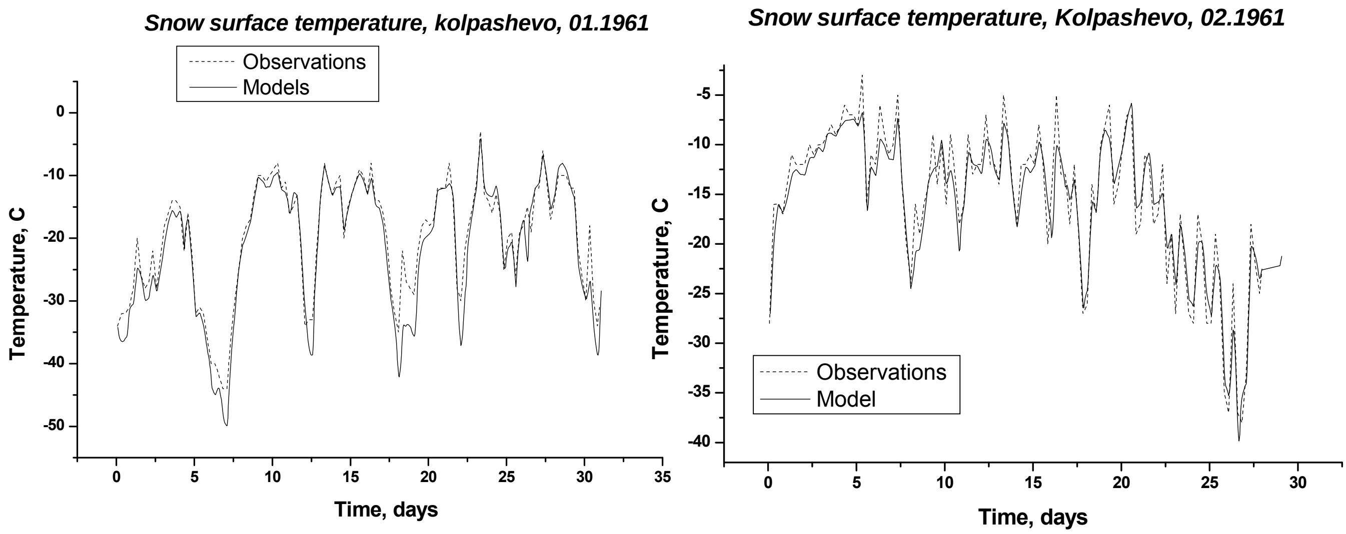

Fig. 1. Snow surface temperature modeled and measured at synoptic station (Kolpashevo, Western Siberia).

Syrdakh lake is relatively small, located near to similar in depth and horizontal size lakes, that temporarily connected by small rivers during high water periods. It is stretched from NW to SE; its length is 2 km, and width is 1 km. Average depth of lake is 4.5 m, and maximum is 12 m.

Ice cover forms in the first part of October, and disappears in the end of May. The lake is located above permafrost of 280-320 m thickness, with melted area beneath lake bottom – so called talik.

Water and heat regimes of Syrdakh were modeled for period from 1970 to 1980, for which meteorological data set was available as well as results of experimental studies, conducted in 1977-1978 [8]. Evolution of water, ice and snow layer thickness, calculated by the model is presented on Fig. 2. Comparison of modeled and observational data yields the following results [8, 12]:

- maximal modeled freezing of lake occurs during winters with minimal snow cover, and, conversely, minimal freezing – during winters with maximal snow cover; this matches observationally proven fact;
- freezing depths, provided by the model, lie in interval 0.7 – 1.5 m, that is common for lakes of Central Yakutat;
- ice cover forms in the beginning of October, and melts in the end of May, that also corresponds to observations;
- evaporation from lake during warm season, calculated by the model, is ~400 mm, that is close to measured value of 450 mm;

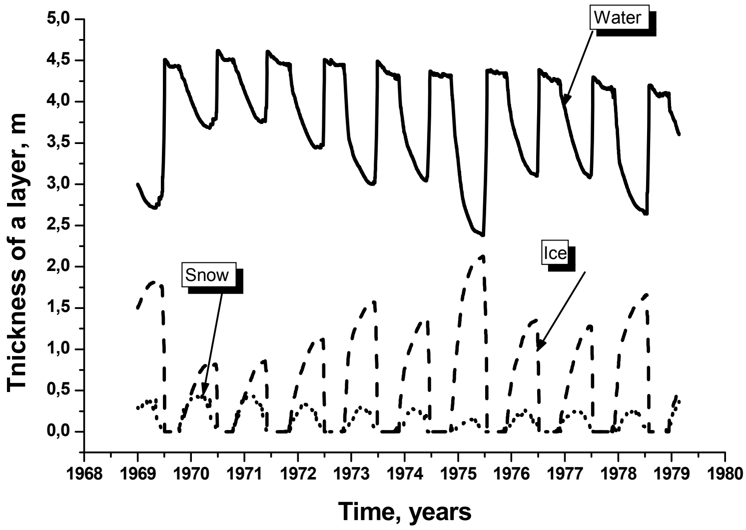

Fig. 2. Long-term temporal variability of snow, ice and liquid water layers of lake Syrdakh, modeled.

Figure 3 provides distribution of thermoisoplets in soil layer under Syrdakh, demonstrating modeled temporal evolution of talik and permafrost. It is seen, that talik exists for ~20 years (1964 - 1985), and its lower boundary is oscillating in annual cycle from 1.2 to 2 m.

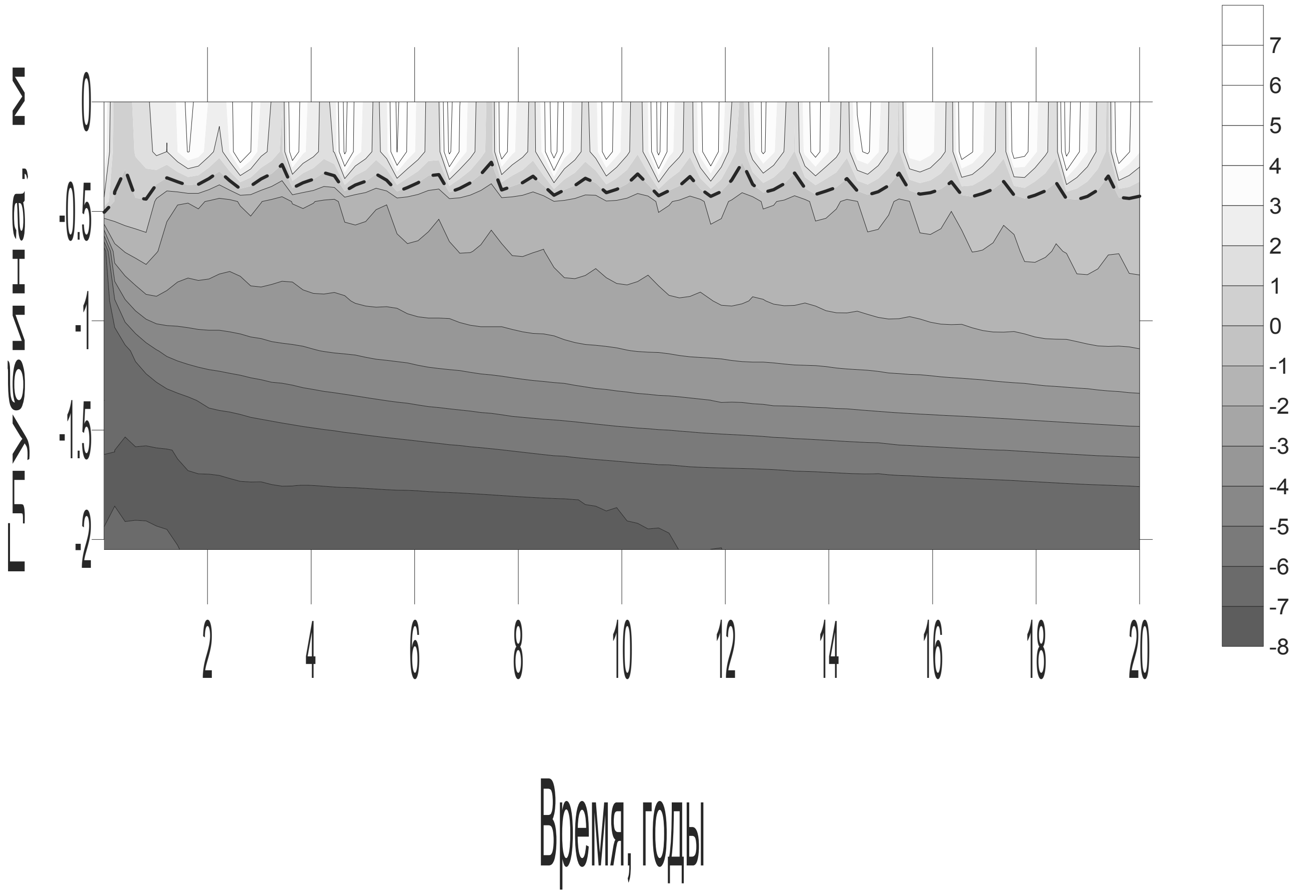

Fig. 3. Long-term dynamics of temperature under lake Syrdakh, modeled. Vertical axis is $-\log_{10}z$, where z, m; horizontal axis is time, years. Dashed line indicates 0 °C isoterm.

3. Conclusion

The major result of the work presented is creation of thermodynamic model of lake, interacting with atmosphere and underlying soil. Heat and water diffusion, filtration of liquid water due to gravitation, freezing/melting, evolution of ice and snow covers, heat and moisture exchange with atmosphere are considered. Therefore, the main processes, responsible for daily and annual variability of state of lake – soil system, are taken into account. Comparison of numerical experiments output with observational data for Syrdakh lake in Central Yakutat proved, that the model has a good skill in simulating of the following parameters: average lake freezing depth, dates of forming and destruction of ice cover, average evaporation in warm season. Moreover, the model simulates talik beneath a lake, which existence is confirmed by observations. Comparison of modeled time series of snow surface temperature with observed data shows a good correspondence.

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