Международная конференция по прикладной математике и информатике, посвященная 100-летию со дня рождения академика А.А. Дородницына

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Abstracts

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Международная конференция по прикладной математике и информатике, посвященная 100-летию со дня рождения академика А.А. Дородницына (ВЦ РАН, Москва, 7–11 декабря 2010 г.): Тезисы докладов.

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larity of Legendre transformation is avoided and so is the necessity of using generalized Hamiltonian formalism in Dirac sense. There are some, both formal and physical arguments for admitting such terms. There is no continuous limit transition between both models.

Usually, problems of infinite dimension obscure some simple structural ideas. To avoid this, we work in finite-dimensional manifolds and finite-dimensional spaces of “wave functions”. In the physicists language, one deals with finite-level systems. The very idea of passing over to infinite dimension, including the “usual” nonlinear Schroedinger equation with dynamical scalar product is briefly outlined, but rather heuristically, without a sufficient mathematical correctness.

Our ideas are different from what is usually meant by physicists as an analysis of classical-quantum relationships. They have not to do with quantization, limit transitions, quasi-classical WKB-limit in traditional sense, method of stationary phase and so on. Mathematically, the point is that equations we postulate for describing some quantum phenomena are, at least from the formal point of view, considered in terms of Hamiltonian systems theory. The dichotomy “quantum-classical” disappears in a sense, becomes diffused. One can suppose that physically this might be perhaps a proper procedure when describing nano-physical problems, where one deals with a convolution, overlapping of what is traditionally meant as quantum or classical.

Independently on possible quantum applications, our models may be useful for describing collective and internal degrees of freedom of classical systems, including applications in condensed matter theory.

Numerical investigation of limit cycles with dry friction

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There are many papers and books devoted to the frictional self-excited oscillations, or stick-slip phenomenon [1]. The first explanation of this
Phenomenon was given by Lord Rayleigh during the years 1877–1878 [2]. This phenomenon was completely investigated for mechanical systems with one degree of freedom. For two (and more) degrees of freedom, many questions remain unanswered till now. We mention only recent papers [3–5], which relate to this paper and deal with limit cycles of predefined structure. In paper [6] an investigation is based on the one-dimensional map with successive points of transfer from stick to slip mode of friction.

We suggest a numerical algorithm, based on a two-dimensional map, for searching all possible limit cycles and their parametric investigation. We consider a double oscillator, one mass of which is in contact, with a moving with a constant velocity \( V \), platform [4]. The equations of motion are

\[ m_1 \ddot{x}_1 = -(k_1 + k_2) x_1 + k_2 x_2 - F_i \]
\[ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) - F_i \]

with the absolute coordinates \( x_1, x_2 \) referenced from equilibrium positions of masses \( m_1, m_2 \) in the absence of the friction force \( F_i \). The Coulomb dry friction law is accepted with the presumption that the limit of the stick friction force \( R \) is greater than the slip friction force \( S \) (\( R > S \)).

The motion of the mass on the platform acquires one of the three distinct modes: (0) sticking \( \dot{x}_2 = V \), \( F_0 = k_2 (x_2 - x_1) \), (1) slipping with forestalling \( \dot{x}_2 > V \), \( F_1 = -S \), (2) slipping with a lag \( \dot{x}_2 < V \), \( F_2 = S \).

The switching conditions between any two of these modes are consistent and defined by the equalities \( \dot{x}_2 = V \) and \( k_2 |x_2 - x_1| = R \).

It is shown, that the system permits limit cycles necessarily including the modes (0) and (2) and perhaps (1). So any limit cycle has to include one or more switching (0)–(2) from mode (0) to mode (2). This switching is defined by the equalities \( \dot{x}_2 = V \), \( k_2 (x_2 - x_1) = R \). For short we shall use the word “turn” to indicate any motion between two successive switchings (0)–(2).

For finding the limit cycles, we choose the initial conditions at the moment of switching (0)–(2), for which the equalities \( \dot{x}_2 = V \), \( k_2 (x_2 - x_1) = R \) are fulfilled. Then only two independent initial conditions remain at this instant of time. We can write this conditions in the vectorial form \( (x_1(0), \dot{x}_1(0))^T = (x_0^0, \dot{x}_0^0)^T = z^0 \). If the value of vector \( z^k = f^k (z_0^0) \) after \( k \) turns coincides with its initial value \( z^0 \), then this initial value defines the limit cycle with \( k \) turns, and \( z^0 \) is a fixed point of the map.
$f^k$. For limit cycles with one turn and more, some symmetry properties of the phase trajectories are demonstrated analytically.

A numerical algorithm for global searching for all limit cycles (fixed points of the map $f^k(z)$), and an algorithm for constructing domains of existence and stability (stability of a fixed point of the map $f^k(z)$) of particular types of limit cycles have been developed. A classification of one turn and two turns limit cycles types has been carried out.

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References