

# Width of the Giant Dipole Resonance in Medium and Heavy Nuclei

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**Abstract**—Based on an analysis of all available experimental data on photonuclear cross sections and properties of the ground and low-lying states of nuclei, the main factors that determine the width of the giant dipole resonance (GDR) are established for nuclei with the number of nucleons  $A \geq 40$ . It is shown that for nuclei with  $A > 120$ , the main reason for the increase in the GDR's width in comparison to the magic value (4–5 MeV) is the deviation of the nuclear shape from spherical (the Danos–Okamoto effect). The main factor behind the increase in the GDR's width for nuclei with  $A$  from 40 to 120 relative to the magic value is dipole-quadrupole friction, i.e., the decay of the doorway dipole states into states of more complex nature, due to the coupling of the doorway states to quadrupole vibrations of the nuclear surface.

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## INTRODUCTION

The giant dipole resonance (GDR), caused by the interaction of electric dipole ( $E1$ ) photons with atomic nuclei, dominates in cross sections of the absorption of photons by nuclei in the energy range of up to 40–50 MeV. Almost 70 years of studying this phenomenon have shown its fundamental role in understanding the dynamics of high-energy nuclear excitations [1–3]. One of the most important characteristics of the GDR is its shape (the energy dependence of the photoabsorption cross section). Owing to multi-year experimental investigations performed at different laboratories using beams of bremsstrahlung and quasi-monochromatic radiation, an extensive set of data on photonuclear cross sections has been accumulated for hundreds of isotopes over the periodic table [4]. Access to this data is provided by the Centre for Photonuclear Experiments Data of the Skobel'tsyn Institute of Nuclear Physics, Moscow State University [5]. The accuracy of the investigations allows us to trace how the GDR evolves as a whole upon a change in the numbers of protons ( $Z$ ), neutrons ( $N$ ), and nucleons ( $A = Z + N$ ) in the nucleus, to observe patterns of variation in its cross section, position in energy, width, and gross- and intermediate structure upon moving from one nucleus to another.

Figure 1 illustrates a situation with the cross section, position, shape, and width of the GDR for nuclei with different mass numbers  $A$ . It can be seen that the nuclear photoabsorption cross section grows as mass number  $A$  increases. Integrated with respect to energy, the nuclear photoabsorption cross section  $\sigma_{\text{int}}$  in the

GDR region is described with good accuracy by the expression

$$\sigma_{\text{int}} = \int_0^{30 \text{ MeV}} \sigma_{\gamma} dE \approx 60 \frac{NZ}{A} \text{ MeV} \cdot \text{mb}. \quad (1)$$

From Fig. 1, the characteristic shift of the GDR maximum  $E_{\text{max}}$  (center of gravity) toward lower energies with the growth of  $A$  can be seen. It corresponds to the relation

$$E_{\text{max}} \approx 75A^{-1/3} \text{ MeV} \approx 75 \frac{1.2}{R} \text{ fm} \cdot \text{MeV}, \quad (2)$$

where nucleus radius  $R = 1.2A^{1/3}$  fm.

From Fig. 1, it is also evident that the GDR width varies within wide limits: from 4–5 MeV to 2–3 tens of MeV. The energy interval in which a cross section exceeds half its maximum value is considered to be the GDR width, denoted below as  $\Gamma$ . The GDR width is one of its most important characteristics. Experimental data on this quantity provide valuable information on the physics of the GDR and features of its excitation and decay. Without knowing how the GDR width is formed for nuclei belonging to different areas of the periodic table, we cannot clearly understand how the GDR is created and relaxes.

The factors that determine the GDR widths of light (up to  $A = 40$ ) nuclei are fairly well known. At the same time, these factors for medium and heavy nuclei require further research. This work is devoted to analyzing experimental widths of the GDR in medium and heavy nuclei and revealing the effects that determine this width.

The GDR width reaches its maximum value in the lightest nuclei ( $A \leq 14$ ):  $\approx 30$  MeV. As  $A$  grows, there is a trend toward compression of the area of the concen-

tration of main  $E1$  transitions. In  $1d-2s$  shell nuclei ( $A = 16-40$ ), the GDR width varies within the interval 5–20 MeV. In nuclei with  $A = 50-140$ , the GDR width is 4–12 MeV. In nuclei with  $A \geq 140$ , the GDR width lies within the interval 4–8 MeV. The GDR width reaches its minimum value in spherical nuclei with a magic number of protons and/or neutrons. For these, the GDR can be represented by a single resonance with a width at half height (the full width at half maximum) of 4–5 MeV. Below, this width is denoted  $\Gamma_0$  and referred to as the “magic” width. Examples of the GDR with magic width can be seen in Fig. 1 (nuclei of  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ , and  $^{208}\text{Pb}$ ). Factors determining the GDR magic width are discussed below.

The GDR width is determined by (1) the width of direct decay ( $\Gamma^\uparrow$ ) of doorway particle-hole ( $1p1h$ ) states with the escape of a nucleon into the continuum, (2) the width of the spread in energy of doorway states, and (3) the width of decay ( $\Gamma^\downarrow$ ) of doorway states into the states of more complex nature ( $2p2h$ ,  $3p3h$ , ...).

The width of the spread in energy of doorway states (the second factor of the previous list) is in turn due (1) the spread in energy of the  $E1$  transitions from one shell, (2) the spread in energy of  $E1$  transitions from different shells (GDR configurational splitting), (3) the splitting of  $E1$  transitions in isospin (GDR isospin splitting), and (4) the splitting of  $E1$  transitions due to the nucleus being nonspherical (GDR deformation splitting).

Before analyzing the GDR width of medium and heavy nuclei, let us recall the situation with the GDR width of light nuclei. A main factor of GDR broadening in light nuclei ( $A < 50$ ) as compared to their magic width (4–5 MeV) is configurational splitting, i.e., splitting in the energy of electric dipole transitions of nucleons from different shells [6]. Another important factor of GDR broadening for light nuclei is the splitting of the  $E1$  states in isospin [7, 8]. The GDR isospin and configurational branches are largely formed by the same  $E1$  transitions. The GDR configurational splitting of light nuclei is thus sustained by its isospin splitting. The roles of configurational and isospin splitting in the formation of the GDR width in stable massive nuclei become insignificant.

To reveal the physics behind the GDR width of medium and heavy nuclei ( $A \geq 40$ ), reliable experimental data were needed on the cross sections of nuclear photoabsorption in the region of excitation energies up to  $\sim 40$  MeV. The author has analyzed all of the experimental material concerning cross sections of the photodisintegration of atomic nuclei. Data from experiments with both bremsstrahlung and quasi-monochromatic photons were used, along with either the most accurate directly measured photoabsorption cross sections or total photonucleon cross sections constructed from the results of photoproton and photonucleon experiments that the author considered

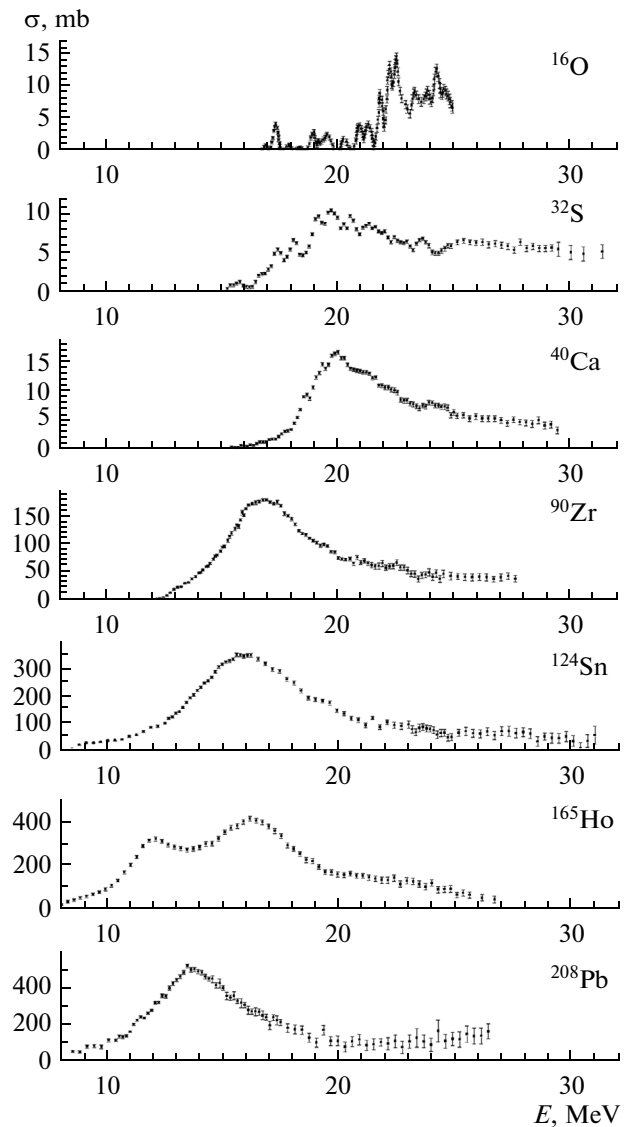
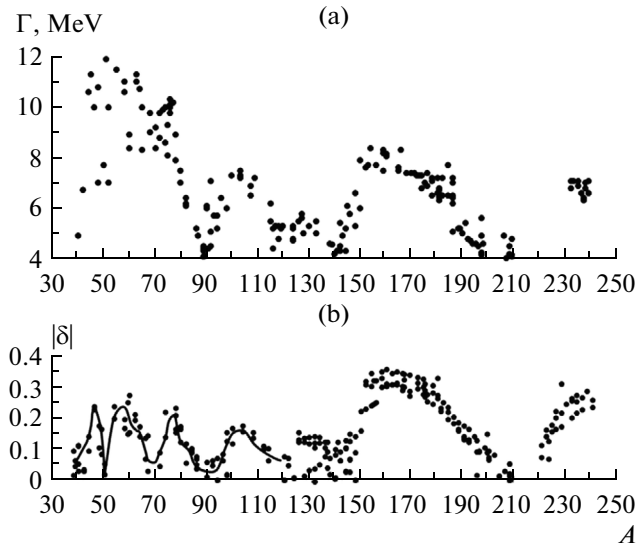


Fig. 1. Dependences of photoneutron cross sections on photon energy for nuclei with different masses.

trustworthy [9]. For nuclei with  $A > 65$ , the total photoneutron cross sections  $\sigma(\gamma, n) + \sigma(\gamma, 2n) + \sigma(\gamma, np) + \sigma(\gamma, 3n)$  giving the best approximation to the photoabsorption cross sections for massive nuclei in the GDR were used. In total, around 200 photonuclear cross sections for more than 120 nuclides were subjected to analysis. The main data source was the Center for Photonuclear Experiment Data of the Skobel'syn Institute of Nuclear Physics, Moscow State University [5]. The obtained systematics of GDR widths for nuclei with  $A \geq 40$  can be seen in the upper part of Fig. 2. The two left-hand points are for  $^{40}\text{Ca}$  and  $^{42}\text{Ca}$ .

From the data presented in Fig. 2, it follows that the GDR widths for nuclei with  $A > 40$  are on average notably less than those for light nuclei. Immediately



**Fig. 2.** Systematics of (a) experimental widths  $\Gamma$  of giant resonance and (b) experimental magnitudes of deformation parameters  $|\delta|$  for nuclei with  $A \geq 40$ .

after the calcium isotopes, they reach their highest (for the mass-number range under study) values of  $\approx 12$  MeV, while for  $A > 80$ , they nowhere exceed 8.5 MeV. For nuclei with a magic number of nucleons and those close to them,  $\Gamma \approx 4$  MeV. Since configurational splitting and isospin splitting have no appreciable effect on GDR width in medium and heavy nuclei and  $\Gamma^\uparrow$  and spread in energy of doorway  $1p1h$  transitions from one shell not large, only width  $\Gamma^\downarrow$  of doorway states decay into states of more complex nature ( $2p2h$ ,  $3p3h$ , ...) and the splitting of  $E1$  transitions due to the nonsphericity of a nucleus in the ground state (the Danos–Okamoto effect) are left for further analysis of factors determining the supermagic width of the GDR [10, 11].

To analyze the effect of this last factor on the GDR width, magnitudes  $|\delta|$  of the quadrupole deformation parameter obtained from the experimental data are given in the bottom part of Fig. 2 (the value of GDR deformation splitting depends on  $|\delta|$ ). Deformation parameters  $\delta$  were derived from the electric quadrupole moments [12] and systematics [13].

Figure 2 shows the correlation between  $\Gamma$  and  $|\delta|$  for nuclei with  $A > 90$ . It is especially obvious for  $A > 120$ –130, where nuclei with high static deformation are grouped. For heavy nuclei with high quadrupole deformation (with the deformation parameter  $|\delta| > 0.20$ ), the gross structure of GDR appears in the form of two maxima, as in the case of  $^{165}\text{Ho}$  nucleus (see Fig. 1). This nucleus is an axially symmetric prolate ellipsoid with  $|\delta| \approx 0.26$ . Since the nucleus has two characteristic dimensions (the minor ( $a$ ) and major ( $b$ ) semi-axes expressed in fm), it also has to two resonance energies

(frequencies) of electric dipole oscillations, according to Eq. (2):  $E_a \approx 75 \frac{1.2}{a}$  MeV and  $E_b \approx 75 \frac{1.2}{b}$  MeV, resulting in the distinct double-humped shape of the photonuclear absorption cross section. This is the so-called Danos–Okamoto effect. It is evident that it would influence the GDR’s general shape and width. In the simplest case, when there are no other factors that have an appreciable impact on the gross structure of the GDR, its general shape for a nonspherical axial nuclear ellipsoid with semi-axes  $a$  and  $b$  directed perpendicular to the axis of nucleus symmetry and along it, respectively, can be presented as a superposition of two spherical resonances with  $\Gamma_0 = 4$ –5 MeV separated in energy by the quantity

$$\begin{aligned} \Delta E = E_a - E_b &\approx 75 \cdot 1.2 \left( \frac{1}{a} - \frac{1}{b} \right) \text{MeV} \\ &\approx 75 A^{-1/3} |\delta| \text{MeV}, \end{aligned} \quad (3)$$

where deformation parameter  $\delta = \frac{b-a}{R}$  (which is associated with the frequently used parameter  $\beta_2$  by relation  $\delta = 0.945\beta_2$ ), while  $\bar{R} = \sqrt{ab}$  is the geometric mean of the semi-axes of a nonspherical nucleus for which the estimate  $\bar{R} = 1.2A^{1/3}$  fm is valid.

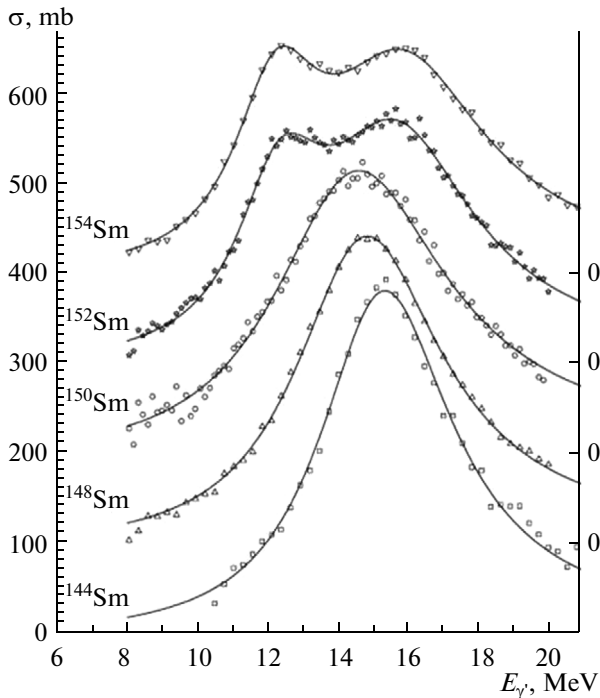
The GDR width of a nonspherical nucleus must therefore grow up to

$$\Gamma = \Gamma_0 + \Delta\Gamma, \quad (4)$$

where  $\Delta\Gamma$  grows as  $\Delta E$  increases. Here if  $\Delta\Gamma$  is proportional to  $\Delta E$ , then it must also be proportional to  $|\delta|$ . The correlation between the value (magnitude) of quadrupole deformation parameter  $\delta$  and giant resonance width  $\Gamma$ , and probably even the proportionality between them, must therefore be a consequence of the Danos–Okamoto effect.

The influence of the Danos–Okamoto effect on the shape of a massive nucleus GDR is illustrated in Fig. 3, which shows the photoneutron cross sections of samarium isotopes [14]. We can see that with an increase in the number of neutrons and in deformation parameter  $\delta$ , the GDR width grows from 4.3 MeV for almost spherical  $^{148}\text{Sm}$  up to 8.4 MeV for strongly prolate  $^{154}\text{Sm}$ , developing into GDR splitting for two heaviest isotopes. The influence of the Danos–Okamoto effect in heavy nuclei is well known [14–16]. The correlation between quantities  $\delta$  and  $\Gamma$  was demonstrated in [17, 18].

The use of all available information on GDR shape and the quadrupole deformation parameters of nuclei with  $Z \geq 50$  allows us to investigate with the greatest possible accuracy the influence of quadrupole deformation on GDR characteristics, and to answer in particular the question of whether nonsphericity is indeed the sole reason for the GDR broadening of heavy nuclei and how far the correlation between

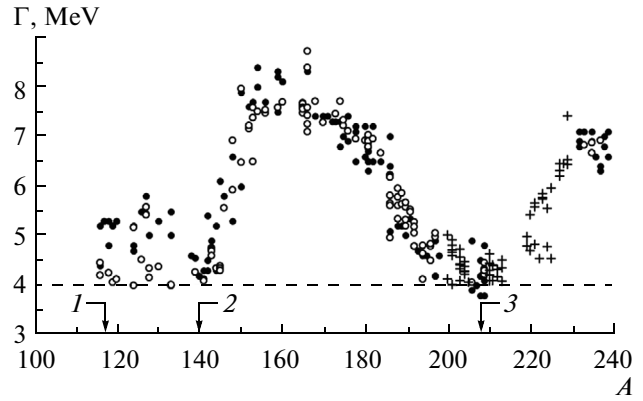


**Fig. 3.** Dependences of photoneutron cross sections of samarium isotopes on photon energy [14]. For ease of comparison, the cross sections are shifted upward by 100 mb (the right-hand ordinate scale) upon an increase in an isotope's mass number. Quadrupole deformation parameters  $\delta$  of the isotopes are in parentheses:  $^{144}\text{Sm}$  (0.08);  $^{148}\text{Sm}$  (0.13);  $^{150}\text{Sm}$  (0.18);  $^{152}\text{Sm}$  (0.29);  $^{154}\text{Sm}$  (0.32).

GDR width and the magnitude of deformation parameter extends.

Figure 2 shows the perfect correlation between GDR widths and the magnitudes of deformation parameters for  $140 < A < 240$ . This correlation is shown even more persuasively by Fig. 4, in which for the same isotopes the widths of experimental photoneuclear cross sections (black dots) are given, along with the GDR widths calculated from the magnitudes of quadrupole deformation parameters  $|\delta|$  using Eq. (4), where  $\Gamma_0 = 4$  MeV, while  $\Delta\Gamma = 11|\delta|$  MeV (white dots). Within the scatter of dots of both types, the data coincide over the entire  $140 < A < 240$  range of mass numbers.

The degree of correlation between these quantities is so high that there is no doubt the Danos–Okamoto effect is responsible for the broadening of the GDR of nuclei with  $A > 120$ –130. Note that the use of the well-known dependence between nucleus radius  $R$  and giant resonance energy  $E$  ( $E \approx 75A^{-1/3}$  MeV  $\approx 75 \frac{1.2}{R}$  MeV) yields GDR broadening approximation  $\Delta\Gamma \approx 12.5|\delta|$  MeV, which is close to the one from fitting for the entire set of experimental data ( $\Delta\Gamma = 11|\delta|$  MeV).

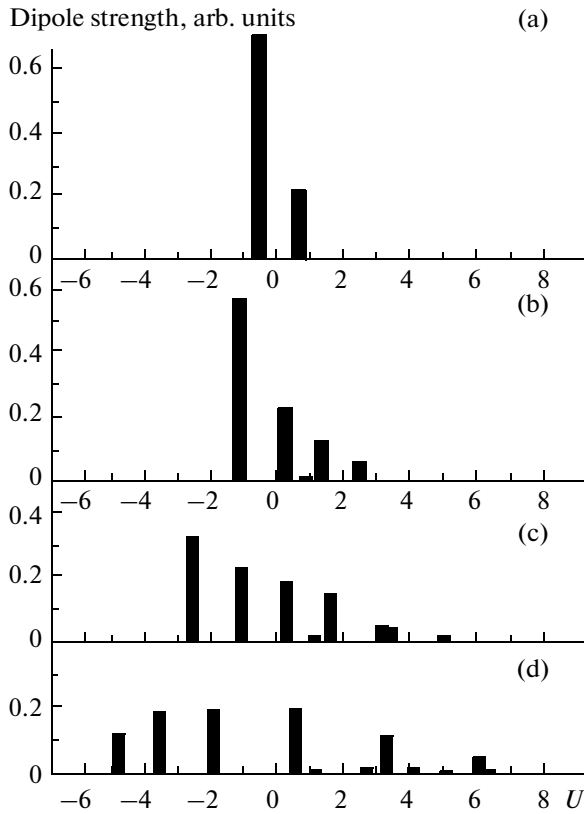


**Fig. 4.** Widths  $\Gamma$  of the giant resonance in heavy nuclei ( $A > 117$ ). Black dots correspond to the widths of experimental photoneuclear cross sections. White dots and crosses mark the widths derived from the magnitudes of the quadrupole deformation parameters using the formula  $\Gamma = (4 + 11|\delta|)$  MeV. The white dots denote nuclei with the measured photoneuclear cross sections. The crosses denote nuclei of the mass-number ranges ( $A = 200$ – $205$  and  $A = 210$ – $230$ ), for which photoneuclear cross sections are unavailable. The dashed line corresponds to the width of giant resonance in magic (spherical) nuclei. The domains of nuclear magicity are (1)  $Z = 50$ ; (2)  $N = 82$ ; (3)  $Z = 82$ ,  $N = 126$ .

On the one hand, data on the values of quadrupole deformation and GDR widths complement each other; on the other hand, they clearly demonstrate the link between giant resonance broadening  $\Delta\Gamma$  and deformation parameter magnitudes  $|\delta|$ . The compared quantities are minimal in the domains of nuclear magicity ( $Z = 50$ ,  $N = 82$  and  $Z = 82$ ,  $N = 126$ ) and reach their highest values at the midpoint between these domains.

The degree of correlation between the compared data allow us to predict the GDR width for the mass-number ranges ( $A = 200$ – $205$  and  $A = 210$ – $230$ ) where photoneuclear cross sections are unavailable due to the absence of stable isotopes but there are data on electric quadrupole moments (and hence on deformation parameters too). For these predictions, we use the formula  $\Gamma = (4 + 11|\delta|)$  MeV. The corresponding values are marked with crosses in Fig. 4. Using these additional values, Fig. 4 yields the pattern of GDR width behavior upon variation in the mass number for the entire set of investigated heavy nuclei ( $120 < A < 240$ ).

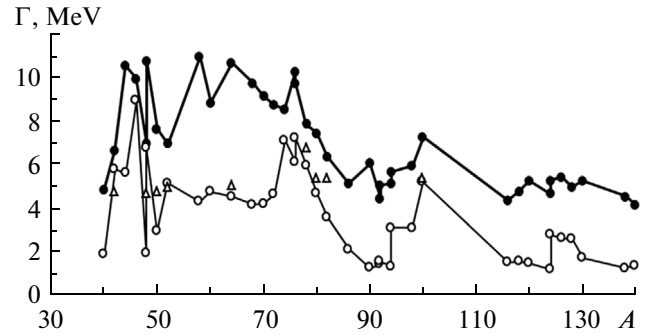
Let us now turn to nuclei with mass numbers from 40 to 90. As follows from Fig. 2, there is no correlation between  $\Gamma$  and  $|\delta|$  for these nuclei, which excludes the Danos–Okamoto effect as the main factor of GDR broadening for nuclei of this mass range. Nuclei in the indicated range are relatively soft vibrational nuclei; many have shapes close to spherical. In these nuclei, the GDR can broaden due to coupling between the  $E1$  oscillations and nuclear surface vibrations (especially quadrupole vibrations; or, in other words,



**Fig. 5.** Distribution of the strengths of  $E1$  transitions, as predicted by the dynamic collective model [19–22] for nuclei with different softness  $S$ . The horizontal scale is expressed in nondimensional units  $U = \frac{E - E(1^-)}{E_1(2^+)}$ , where

$E$  is the energy of nucleus excitation,  $E(1^-)$  is the energy of GDR maximum, and  $E_1(2^+)$  is the energy of the first  $2^+$  level. The data corresponds to the following values of softness  $S$ : (a) 1.69; (b) 2.81; (c) 4.50; (d) 7.50.

through dipole–quadrupole friction). Considering the coupling between  $E1$  oscillations and quadrupole vibrations means allowing for the decay of doorway dipole  $1p1h$  excitations into excitations of more complex nature ( $2p2h$ ,  $3p3h$ , ...). In the shape of the GDR, this is realized through an intermediate structure; in its width, through component  $\Gamma^\downarrow$ . The  $\Gamma^\downarrow$  fraction in the full GDR width for nuclei heavier than calcium is  $>50\%$  and rapidly grows upon an increase in  $A$ , reaching  $\approx 90\%$  for nuclei with  $A \approx 200$ . The dynamic collective model (DCM) [19–22] is used to consider the dipole–quadrupole friction during GDR formation in vibrational nuclei. This yields the splitting of collective  $E1$  excitation into transitions whose number and energy spread grow along with the dipole–quadrupole friction. The spread of  $E1$  transitions that occurs in this case determines the GDR broadening in non-magic nuclei.



**Fig. 6.** Comparison of experimental widths  $\Gamma$  of the giant resonance (black dots) and widths predicted by the dynamic collective model (white dots and triangles) for nuclei with  $A = 40$ – $140$ .

It is convenient to characterize the degree of coupling between the dipole and surface quadrupole

vibrations using softness parameter  $S = \langle \beta \rangle \frac{E(1^-)}{E_1(2^+)}$ ,

where  $\langle \beta \rangle$  is the rms amplitude of surface vibrations,  $E(1^-)$  is the energy of the GDR maximum, and  $E_1(2^+) = \hbar(C/B)^{1/2}$  is the energy of a surface phonon (where  $C$  is the stiffness coefficient,  $B$  is the mass coef-

ficient). Parameters  $E_1(2^+)$  and  $\langle \beta \rangle$  can be found using the data on low-energy levels of even–even nuclei: the energy of exciting first level  $2^+$  and the reduced probability of  $E2$  transition from this level to the ground state. The effect softness has on the GDR shape and width as predicted by the dynamic collective model is illustrated by Fig. 5, where the dipole-strength distributions are shown for different  $S$ . The horizontal scale of energies is expressed in nondimensional units

$U = \frac{E - E(1^-)}{E_1(2^+)}$ , where  $E$  is the energy of nucleus excitation.

Using softness parameters  $S$  found from the experimental data, we estimated GDR widths using Fig. 5 for nuclei with  $A = 40$ – $140$ . The widths were found from the expression  $\Gamma_{\text{DCM}} = E_1(2^+) \cdot \Delta(U)$ , where  $\Delta(U)$  is the interval in which the  $E1$  transitions lie on the horizontal scale in Fig. 5.

The estimated widths (white dots) and their experimental values (black dots) are given in Fig. 6. The GDR widths obtained by the authors of the dynamic collective model for a number of nuclei are indicated with white triangles. From a comparison of the experimental and theoretical data, it follows that in the interval of mass numbers  $A = 40$ – $120$ , dipole–quadrupole friction is the main factor behind the increase in GDR width, relative to magic value  $\Gamma_0 = 4$  MeV. In the indicated mass-number interval, the GDR width grows on average 3–5 MeV due to this friction. The

residual part of the experimental width can be attributed to magic width  $\Gamma_0$ .

Finally, let us consider the GDR magic width (4–5 MeV). Analysis shows that in light nuclei, it is shaped in comparable fractions by the spread in energy of  $1p1h$ -transitions from one shell and  $\Gamma^\uparrow$ . In heavy nuclei, this occurs predominantly via  $\Gamma^\downarrow$ .

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