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Charge-Dependent *NN* Interaction in the *J*-Matrix Inverse Scattering Approach

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Abstract—A method for constructing interaction within the *J*-matrix inverse scattering approach in the case of charged particles is proposed. A charge-dependent nonlocal nucleon–nucleon interaction (CD JISP) has been constructed, which is a generalization of the JISP16 interaction. Interaction in pp, nn, and np partial waves is presented by small matrices in the oscillator basis with $\hbar\omega = 40$ MeV; therefore, it can be directly used in many-body calculations within the shell model and in the resonating group model. The CD JISP interaction reproduces the np- and pp-scattering data and deuteron observables with high accuracy.

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A nonlocal separable nucleon–nucleon interaction of a new type, the *J*-matrix inverse scattering potential (JISP) [1], turned out to be very promising from the point of view of completely microscopic investigations of properties of light *s* and *p* shells [2, 3]. The JISP interaction reproduces the experimental np-scattering data and the deuteron properties with an accuracy not lower than that of the modern realistic potentials CD-Bonn [4], Argonne [5], and Nijmegen [6], constructed within the meson-exchange theory: the mean-square deviation per datum for the JISP interaction is $\chi^2/d = 1.03$ for the np database of 1992 (the number of data is 2514) and 1.05 for the np database of 1999 (the number of data is 3058). The JISP interaction is represented by low-rank matrices in the oscillator basis in each partial *NN* wave, which makes it possible to extend investigations in modern microscopic models to heavier nuclei. The last version of the JISP interaction, JISP16 [3, 7], describes the properties of all light nuclei with mass numbers $A \le 16$ (binding energies, spectra, rms radii, quadrupole moments, electromagnetic transition probabilities, etc.) within the no-core shell model (NCSM) [8, 9] with a higher accuracy than the best modern models of nuclear interaction allowing for realistic nucleon–nucleon and three-nucleon forces.

The procedure of constructing interaction within the *J*-matrix inverse scattering approach includes several stages. At the first stage, to exclude the ambiguities (caused by the existence of phase-equivalent transformations), tridiagonal (quasi-tridiagonal in the case of coupled channels) interaction matrices are constructed in each partial wave using experimental scattering phase shifts as an input. The obtained tridiagonal potentials correctly describe the nucleon–nucleon scattering in all partial waves: they reproduce scattering phase shifts with high accuracy. Moreover, the scattering-state wave functions calculated with these potentials are very close to the respective functions obtained with the realistic Nijmegen II interaction [1]. However, in the case of the ${}^3s_1$–${}^3d_1$ coupled channels, the quasi-tridiagonal potential generates the *d* component of the deuteron wave function strongly suppressed at short distances. As a result, the rms radius and quadrupole moment of a deuteron do not correspond to the experimental data. The next step in the construction of the interaction is the choice of the phase-equivalent transformation parameters in the ${}^3s_1$–${}^3d_1$ coupled waves that reproduce correctly all deuteron observables.

The next stage in the construction of the interaction is fine fitting of the JISP interaction parameters by means of phase-equivalent transformations in individual partial waves on the basis of the NCSM calculations of nuclear properties. The phase-equivalent transformations of interaction do not affect the scattering phase shifts and binding energies of the two-body system; i.e., do not affect the quality of description of the nucleon–nucleon scattering. However, phase-equivalent transformations change the properties of interaction allowing to improve the description of the light nuclei observables. Following this route, we constructed the JISP6 [2] and JISP16 [3] interactions (JISP interactions with parameters chosen to describe light nuclei with $A \le 6$ and $A \le 16$, respectively).

Experimental np-scattering phase shifts were used as an input in construction of the JISP16 interaction (and earlier versions); as a result, the NCSM calcula-

Binding energy E_d, d-state probability, rms radius $\langle r^2\rangle^{-1/2}$, quadrupole moment$Q$, and asymptotic normalization constants $\mathcal{A}_s$ and $\eta = \mathcal{A}_d/\mathcal{A}_s$ generated by the CD JISP, JISP16, Nijmegen II, and CD Bonn interactions in comparison with the experimental data [14, 15]

Potential	E_d, MeV	d-state probability, %	$\langle r^2\rangle^{-1/2}$, fm	Q, e fm^2	$\mathcal{A}_s$, fm$^{-1/2}$	$\eta = \frac{\mathcal{A}_d}{\mathcal{A}_s}$
CD JISP	–2.224575	4.086	1.9708	0.2761	0.8846	0.0256
JISP16	–2.224575	3.978	1.9643	0.2886	0.8629	0.0252
CD-BONN	–2.224575	4.85	1.966	0.270	0.8846	0.0256
Nijmegen-II	–2.224575	5.635	1.968	0.2707	0.8845	0.0252
Experiment [14]	–2.224575(9)	5.67(11)	1.9676(10)	0.2859(3)	0.8845(8)	0.0253(2)
Experiment [15]	–2.224589	–	1.9635 1.9560 1.950		0.8781	0.0272

tions of nuclear observables in [2, 3, 7, 10, 11] were performed within the charge-independent nuclear force approximation; i.e., in each partial wave, the nuclear JISP interaction in nn, np, and pp pairs was assumed to be identical (in the case of the pp interaction, the Coulomb potential was added). However, it is known that nuclear forces depend on the charge of interacting particles. Our investigations [12] showed that the JISP interaction is not an exception: the pp-scattering phase shifts calculated within this interaction differ from the experimental data, the violation of charge independence is most pronounced in the $^1s_0$ and $^3p_0$ partial waves. The purpose of this study is to construct a charge-dependent JISP (CD JISP) interaction which reproduces not only the deuteron observables and np-scattering phase shifts but also the experimental data on pp scattering in all partial waves. The formalism of construction of the JISP interaction in the case of neutral particles was described in detail in [1]; hence, we will focus on the pp interaction within the CD JISP approach. Below, we use the notations of [1].

In construction of the CD JISP interaction, we use the same main parameters of the inverse scattering approach in the J-matrix formalism as in the case of JISP interaction [1]: the oscillator parameter $\hbar\omega$ = 40 MeV and the sizes of the matrices of nonlocal potentials are determined by the total oscillator quanta number $2N + l$, which is equal, respectively, to 8 and 9 in even and odd partial waves with a total angular momentum up to $j = 4$. In construction of the CD JISP, the values of the np- and pp-scattering phase shifts [13] are used as an input.

The construction of the nonlocal CD JISP interaction describing np scattering is exactly the same as the constructing of the JISP. The only difference from the formalism reported in [1] is the allowance for the effects of relativistic kinematics: instead of the nonrelativistic relation between the kinetic energies of interacting nucleons in the center-of-mass system, $E_{\rm cm}$, and the laboratory system, $E_{\rm lab}$,

$$E_{\rm cm} = \frac{m_a}{m_a + m_b} E_{\rm lab}, \tag{1}$$

the following relativistic formula is used:

$$E_{\rm cm} = (m_a c^2 + m_b c^2)\left[\sqrt{\frac{2E_{\rm lab} m_b c^2}{(m_a c^2 + m_b c^2)^2} + 1} - 1\right]. \tag{2}$$

Since nucleon–nucleon phase shifts in the range of relatively low laboratory energies (0–350 MeV) are only used to construct the CD JISP interaction, the effects of relativistic kinematics are insignificant. They lead only to a small variation in the matrix elements of the CD JISP interaction. The scattering phase shifts in both versions completely coincide in all partial waves, and the scattering state wave functions slightly differ only at high energies. It should be noted that the difference from the functions calculated with the realistic Nijmegen II potential is seen only at short distances and is more pronounced in the case of s-wave scattering.

The CD JISP interaction somewhat differs from the JISP one only in the $^3s_1$–$^3d_1$ coupled wave. Due to the presence of the bound state, the deuteron binding energy E_d and the asymptotic normalization constants $\mathcal{A}_s$ and $\eta = \mathcal{A}_d/\mathcal{A}_s$ [1] are used as additional input data. The values of the normalization asymptotic constants used to construct the CD JISP np interaction are close to the modern experimental data and differ from the values used in construction of the JISP16 potential (see table). However, this circumstance almost does not affect the scattering characteristics: the δ_s and δ_d phase shifts calculated with CD JISP and JISP16 coincide, small variations are observed only in the behavior of the mixing parameter revealing a small difference in the tensor parts of the JISP16 and CD JISP interactions responsible for s- and d-channel mixing.

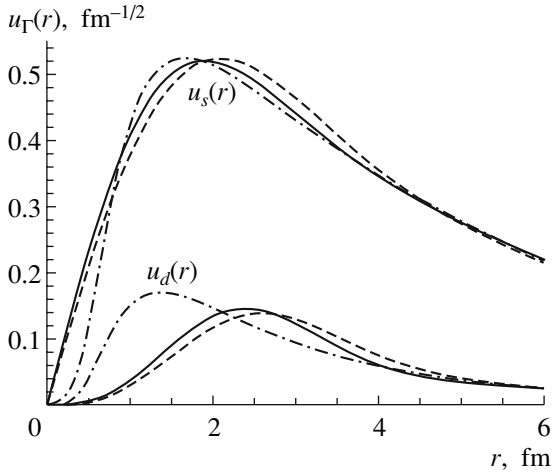

Fig. 1. Radial deuteron wave functions calculated with the CD JISP potential (solid lines), JISP16 potential (dashed lines), and Nijmegen II potential [6] (dot-dashed lines).

The choice of new values of the asymptotic constants affects the parameters of phase-equivalent transformations, which are necessary to get description of the deuteron observables. In contrast to the JISP16 interaction, where it was sufficient to use only one phase-equivalent transformation (mixing the basis oscillator functions 0s and 0d with the parameter $\beta = -11°$), in the case of CD JISP, it was necessary to use three phase-equivalent transformations mixing the basis functions (0s, 0d), (0s, 1s), and (0d, 1d) with the parameters $\beta = -11°$, 3.1°, and 8.3°, respectively. As a result, the calculated values of the rms radius and quadrupole moment of a deuteron are in good agreement with the experimental data (see table, where the results of the calculations with the JISP16 and realistic Nijmegen II and CD-Bonn interactions are also given for comparison). The deuteron wave functions calculated with CD JISP and JISP16 are close (Fig. 1); however, in contrast to the scattering state wave function, they significantly differ from the deuteron functions obtained with the Nijmegen II potential.

The CD JISP interaction for nn scattering can be constructed in a similar way. However, since the experimental values of nn-scattering phase shifts are unknown, the nn-scattering phase shifts calculated with the realistic potential Nijmegen II [6] are used as an input for the construction of neutron–neutron potentials. The thus constructed nn potentials reproduce with high accuracy the Nijmegen scattering phase shifts in the energy range up to $E_{\text{lab}} = 350$ MeV. The nn-scattering wave functions for the obtained potentials are very close to the Nijmegen functions, as in the case of np scattering.

Let us now consider in more detail the method of constructing the CD JISP interaction, describing pp scattering. In this case, it is necessary to take into account the long-range Coulomb interaction, and the formalism proposed in [1] cannot be used directly.

In the J-matrix method with oscillator basis functions, the phase shift is given by the relation [16]

$$\tan\delta_l = \frac{W_b(j_l, f_l) - W_b(n_l, f_l)\tan\delta_l^{\text{Sh}}}{W_b(j_l, g_l) - W_b(n_l, g_l)\tan\delta_l^{\text{Sh}}}, \tag{3}$$

where the quasi-Wronskian

$$W_b(j_l, f_l) \equiv \left\{ \frac{d}{dr}[j_l(kr)] f_l(\zeta, kr) - j_l(kr)\frac{d}{dr} f_l(\zeta, kr) \right\}\Bigg|_{r=b} \tag{4}$$

($W_b(n_l, f_l)$, $W_b(j_l, g_l)$, and $W_b(n_l, g_l)$ are defined similarly);

$$f_l(\zeta, kr) = \frac{1}{r}\sqrt{\frac{\pi}{2}} F_l(\zeta, kr),$$
$$g_l(\zeta, kr) = -\frac{1}{r}\sqrt{\frac{\pi}{2}} G_l(\zeta, kr); \tag{5}$$

$F_l(\zeta, kr)$ and $G_l(\zeta, kr)$ are, respectively, regular and irregular Coulomb functions; and $j_l(kr)$ and $n_l(kr)$ are, respectively, the spherical Bessel and Neumann functions. In Eq. (3), δ_l^{Sh} is the phase shift by an auxiliary short-range potential V^{Sh} obtained from the initial one by truncating the Coulomb potential V^{EM} at $r = b$; i.e.,

$$V^{\text{Sh}} = \begin{cases} V = V^{\text{Nucl}} + V^{\text{EM}}, & r \le b \\ 0, & r > b; \end{cases} \qquad b \ge R_{\text{Nucl}}. \tag{6}$$

The method of constructing the CD JISP interaction in the case of scattering of charged particles is as follows. First, using the relation

$$\tan\delta_l^{\text{Sh}} = \frac{W_b(j_l, f_l) - W_b(j_l, g_l)\tan\delta_l}{W_b(n_l, f_l) - W_b(n_l, g_l)\tan\delta_l}, \tag{7}$$

which is inverse to Eq. (3), the auxiliary phases δ_l^{Sh} are calculated from the experimental pp-scattering phase shifts in each partial wave using some fixed value of the parameter b. Since these phases correspond to scattering by the short-range potential (6), we can apply the formalism [1] and construct a tridiagonal matrix representing the nonlocal interaction V_{Nl}^{Sh}. Obviously, this nonlocal interaction includes partially the long-range Coulomb interaction and, therefore, cannot be considered as the nuclear CD JISP pp interaction.

In a similar way, with the same value of the parameter b, one can construct the tridiagonal matrix V_{Nl}^{0}, which reproduces the phase shifts of charged particle

scattering by the truncated Coulomb potential. To construct V^0_{Nl}, one should use the phase shifts $\delta^{\rm Sh}_l$ obtained by the formula (7) with $\delta_l = 0$ as an input. One might expect that the nonlocal potential V^0_{Nl} contains the same long-range Coulomb interaction as $V^{\rm Sh}_{Nl}$. As a result, the difference of these two nonlocal potentials,

$$V^{pp}_{Nl} = V^{\rm Sh}_{Nl} - V^0_{Nl}, \qquad (8)$$

can be used as the CD JISP pp interaction.

Note that the V^{pp}_{Nl} interaction dependence on the parameter b is much weaker than the dependence of the auxiliary potentials $V^{\rm Sh}_{Nl}$ and V^0_{Nl}.

Using the proposed procedure, we constructed the CD JISP pp potentials in the partial waves $^1s_0$, $^3p_0$, $^3p_1$, $^1d_2$, $^3f_3$, and $^1g_4$ and in the coupled channels $^3p_2$–$^3f_2$ and $^3f_4$–$^3h_4$. The sizes of matrices of the obtained nonlocal potentials in various partial waves are the same as the sizes of matrices of the CD JISP nn and np potentials. The optimal value of b, the truncation parameter of the long-range Coulomb interaction in the coordinate space, was found to be equal to the classical turning point of the oscillator basis function with $n = N : b = 2r_0\sqrt{N + l/2 + 3/4}$ (r_0 is the oscillator radius).

The pp-scattering phase shifts calculated with the obtained CD JISP interaction, are shown in Figs. 2 and 3. It is seen that the *NN* interaction charge symmetry violation is most pronounced in the $^1s_0$ and $^3p_0$ partial waves. In other partial waves, including coupled *pf* and *fh* waves, the results of the calculations with the CD JISP pp and np potentials are fairly close.

It should be noted that the CD JISP pp potentials are less accurate in description of the experimental pp-scattering phase shifts than the np potentials in description of np-scattering phase shifts. The accuracy of description of the experimental partial phase shifts $\delta^{\rm exp}_l(E_i)$ can be estimated as a mean-square deviation per datum of the theoretical phase shifts from the experimental ones (in construction of CD JISP, we used $N = 53$ phase shifts in each partial wave as an input):

$$\chi^2/N = \frac{1}{N}\sum_{i=1}^{N}\left(\delta_l(E_i) - \delta^{\rm exp}_l(E_i)\right)^2. \qquad (9)$$

In the $^1s_0$partial wave, the value of $\sqrt{\chi^2({\rm np})/N}$ in the case of np scattering is 0.143°, while for the pp scattering $\sqrt{\chi^2({\rm pp})/N} = 0.241°$. In the $^3p_0$ partial wave, these values are $\sqrt{\chi^2({\rm np})/N} = 0.050°$ and $\sqrt{\chi^2({\rm pp})/N} = 0.098°$. In other partial waves, the situation is similar: the accuracy of reproduction of the pp-scattering phase shifts is worse than that of np-scattering phase shifts by a factor of 1.5–2.

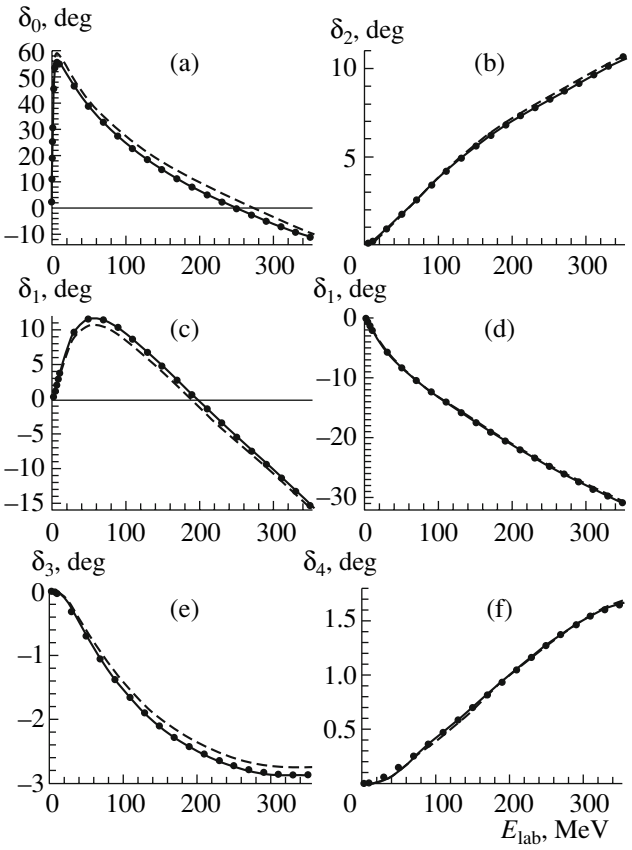

Fig. 2. pp-scattering phase shifts in the (a) $^1s_0$, (b) $^1d_2$, (c) $^3p_0$, (d) $^3p_1$, (e) $^3f_3$, and (f) $^1g_4$ partial waves. The experimental phases used as an input in construction of the potentials, are shown by circles; the solid and dashed lines depict the calculations with the CD JISP pp and np interactions, respectively.

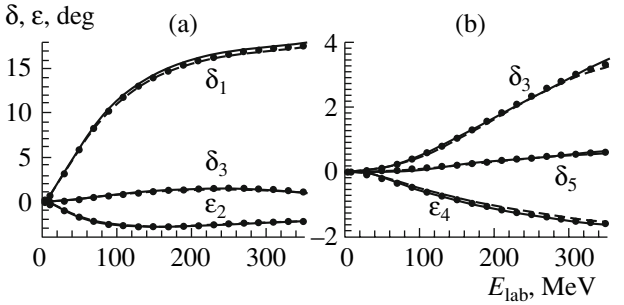

Fig. 3. Partial phase shifts and mixing parameters of pp scattering in the (a) $^3p_2$–$^3f_2$ and (b) $^3f_4$–$^3h_4$ coupled waves. See Fig. 2 for details.

Concluding, we note that the results reported here are only the first stage of construction of the charge-dependent nucleon–nucleon interaction. It is necessary

to obtain more accurate description of proton–proton scattering phase shifts. Furthermore, we plan to perform systematic NCSM calculations of the observables in light *s*- and *p*-shell nuclei with the CD JISP interaction. Such investigations are needed for fine tuning of the parameters of the constructed interaction using the phase-equivalent transformations following the ab exitu approach formulated in [3].

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